



Can Correspondence Analysis Serve as a Ranking Method? A Comparison of the CA-WLD Approach With Multi-Criteria Decision-Making Techniques

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Abstract

This study evaluates correspondence analysis (CA) as a ranking tool for multi-criteria decision-making (MCDM). The proposed Correspondence Analysis–Weighted Load-Distance (CA-WLD) method converts alternative–criterion distances in the CA space into a weighted decision score. In a Monte Carlo simulation of 20,242 contingency tables, nine established MCDM techniques agree closely, whereas CA-WLD yields distinct information that depends on the interaction of weighting with data structure; a max-min baseline shows that this is not merely non-compensation. A controlled perturbation analysis shows that CA-WLD withholds first place from a classical leader whose lead rests on volume or on above-competitor concentration, and endorses it once that concentration falls or its peak criterion is weighted sufficiently. Applied to a real site-selection problem (241 respondents), CA-WLD diagnoses the popularity bias of count-based methods. It is a complementary diagnostic tool, not a rival.

Keywords: *correspondence analysis, multi-criteria decision making, ranking, non-compensatory evaluation, Monte Carlo simulation*

1. Introduction

Multi-criteria decision making (MCDM) encompasses a set of methods aimed at determining the optimal alternative under multiple, often conflicting, criteria [20]. MCDM is conventionally divided into two branches: multi-attribute decision making (MADM), which evaluates a finite set of discrete alternatives, and multi-objective decision making (MODM), which generates solutions over a continuous feasible region [30]. The present study belongs to the MADM branch, as it addresses the ranking of a finite set of alternatives. Numerous techniques such as TOPSIS, VIKOR, PROMETHEE, AHP, MOORA, ARAS, and EDAS have been developed and applied across a broad range of fields, from facility location selection to supplier evaluation [5]. Each method is based on a different decision logic. For example,

TOPSIS is based on the distances to the positive-ideal and negative-ideal solutions, PROMETHEE on pairwise superiority relations, and AHP on weights derived from pairwise comparisons.

This richness in the number of methods also brings with it an important debate. To what extent do different MCDM techniques produce similar rankings when applied to the same decision problem? There is no complete consensus on this issue in the literature [6]. However, large-scale comparison studies reveal that the methods yield closely aligned rankings under most conditions. Zanakis et al. [33] compared numerous methods via simulation and found that AHP derivatives as well as TOPSIS yielded results similar to the weighted sum (SAW) method, while differences between methods became more pronounced as the number of alternatives increased. Similarly, it has been reported that when the distribution of criterion values is narrow, the vast majority of methods produce highly overlapping rankings [15]. This situation highlights two points. First, since established methods yield similar results across a wide set of conditions, the contribution of a new method cannot be sought merely in producing a different ranking. Second, the differentiation between methods is not random but depends on the structure of the problem. This study directly addresses both points.

Correspondence analysis (CA) is a multivariate technique that maps the structure of relationships among categorical variables in a low-dimensional space [14]. The method positions row and column categories using coordinates based on chi-square distance. In this way, closely situated points represent strong associations, while distant points indicate weak associations [1]. CA has been widely used in perceptual mapping and categorical data analysis, particularly in marketing and the social sciences [3, 16]. However, CA is primarily a descriptive and exploratory technique. It explains and visualizes the relationship among variables but does not rank options for decision-making purposes.

The use of correspondence analysis in the context of decision-making is not entirely new. Stewart [29] proposed correspondence analysis as a descriptive approach to multi-criteria decision making and evaluated this technique as an alternative to factor analysis in small decision matrices. Correspondence analysis has also been used for seriation, that is, arranging objects according to frequency patterns. However, these uses have essentially remained at a descriptive and exploratory level. Correspondence analysis has (i) not been transformed into a weighted formulation that produces explicit decision scores comparable to established MCDM techniques for the alternatives, (ii) not been systematically compared with these techniques under different data structures, weighting methods, and problem sizes, and (iii) not been validated by comprehensive simulation evidence. The gap this study aims to fill is precisely this.

The main purpose of this study is to demonstrate that correspondence analysis is not merely a method that explains relationships among variables, but can also be used for ranking and decision-making, similarly to MCDM techniques, through proximity and distance-based scoring formulations derived from the geometry of correspondence analysis. In the proposed approach, the distances from each alternative to each criterion in the CA space are transformed into a proximity (benefit) value, and a decision score is obtained by combining these with the criterion weights. The second and definitive aim of the study is to reveal the validity and behaviour of this approach not through a single numerical example, but with a detailed simulation experiment.

The study positions the proposed method not as a competitor to established MCDM techniques that will replace them, but as an approach that complements them. Accordingly, the method is systematically compared with nine established MCDM techniques within a broad simulation design. The comparison

is conducted along three axes: (i) data structure (the intensity of edge/specialized profiles and popularity asymmetry), (ii) weighting method (objective methods such as entropy and CRITIC, as well as external weights), and (iii) problem size (number of alternatives and criteria). In this way, it is determined when the method overlaps with established techniques, when it diverges, and what additional information it provides when it diverges. The main source of the additional information produced by the proposed method is a non-compensatory evaluation logic. The majority of multi-criteria methods are compensatory: a weakness of an alternative on one criterion can be offset by its strength on another criterion, and the alternative can still achieve a high composite score [26]. For example, an unbalanced option with a very high value on a single criterion and low values on the others may rank high with methods such as the weighted sum. In non-compensatory approaches, however, this compensation is not possible and imbalance between criteria is penalized [19]. In the simulation results, the proposed method exhibits this second behaviour by ranking unbalanced alternatives lower compared to classical techniques. This distinction between compensatory and non-compensatory evaluation is an established topic in the multi-criteria evaluation literature [21].

In this context, the study seeks to answer the following research questions (RQ):

- RQ1. Does the correspondence-analysis-based ranking method produce ranking information that differs from that of established MCDM techniques, and if so, what is the magnitude of this difference?
- RQ2. How does this divergence change according to the structure of the data (the density of edge or specialized profiles)?
- RQ3. What is the effect of the weighting method (objective and external) on divergence?
- RQ4. What is the effect of problem size (number of alternatives and criteria) on divergence?
- RQ5. Considering all these findings, under what conditions should the proposed method be used?

The remainder of the study is organized as follows. Section 2 presents the theoretical framework based on correspondence analysis and the multi-criteria decision-making literature. Section 3 explains the mathematical formulation of the proposed CA-WLD method, the techniques compared, and the simulation design. Section 4 presents the simulation findings within the framework of the research questions. Section 5 applies the method to a real decision problem, the selection of a shopping mall site in Çorum. Section 6 discusses the interpretation of the findings and the conditions for the use of the method, and Section 7 summarizes the results.

2. Theoretical framework

2.1. Correspondence analysis

Correspondence analysis (CA) is a descriptive and exploratory multivariate technique that visualizes the structure of relationships between categorical variables in a low-dimensional space based on a contingency table [1, 13]. The method transforms the rows and columns of the table into relative frequency distributions called profiles and positions these profiles on a common map [9]. In this respect, correspondence analysis has been widely used for perceptual mapping and categorical data analysis in marketing and the social sciences [3, 16].

The geometry of correspondence analysis is based on the chi-square distance. Two points that are close to each other on the map have similar profiles, while the distance of a point from the centroid indicates how much that profile diverges from the average profile [14]. The total of the weighted squared distances of the profiles to the centroid is referred to as inertia, which represents the total variability in the table and is numerically equal to the ratio of the chi-square statistic to the sample size [16]. The proportion of inertia explained by each principal axis indicates the extent to which the low-dimensional map represents the data. As this proportion increases, the map reflects the actual data structure more reliably [13]. Additionally, the chi-square metric balances the contribution of low-frequency categories, thereby ensuring that infrequently observed features are represented meaningfully on the map [9].

Depending on how the coordinates of row and column points are scaled, two types of maps are distinguished. In a symmetric map, both rows and columns are displayed in principal coordinates. In this case, row-to-row and column-to-column distances approximately correspond to chi-square distances. However, the distance between row and column points cannot be interpreted directly [13]. In contrast, in an asymmetric (row-principal) map, one set of points is positioned in principal coordinates, while the other is placed in standard coordinates. Thus, the proximity of a row point to a column point can be interpreted as the strength of the association between the two [3]. This distinction is critical to the method proposed in the study, as meaningfully measuring the proximity between the alternative and the criterion requires the use of the asymmetric map.

2.2. Multi-criteria decision making

Multi-criteria decision making (MCDM) is a field that examines decision problems in which a set of alternatives is evaluated according to multiple, and often conflicting, criteria [20]. The objective in these problems is to rank, classify, or select the most suitable alternative. Numerous MCDM methods have been developed in the literature, and these methods have been evaluated together in comprehensive comparative studies [31].

Methods differ according to how they integrate performance on the criteria [30]. One group of methods aggregates all criterion values into a single composite score. Approaches based on weighted summation and TOPSIS, which relies on distances from the positive-ideal and negative-ideal solutions, are examples of this logic [30]. Another group consists of outranking methods that evaluate alternatives through pairwise comparisons, with PROMETHEE and ELECTRE representing this approach [4]. Distance- or reference-point-based methods (e.g., TOPSIS and VIKOR), on the other hand, rank alternatives according to their geometric distances to the ideal and anti-ideal points [22]. This latter group is conceptually closely related to the distance-based logic of the method proposed in the study.

In this study, the proposed method is compared with nine established MCDM techniques (TOPSIS, VIKOR, PROMETHEE II, WASPAS, ARAS, EDAS, MARCOS, COCOSO, and MABAC), which represent different aggregation logics. The mathematical details and sources of these methods are presented in Section 3.

2.3. Distance-based assessment and the load-distance approach

A common approach in location and allocation problems is to evaluate alternatives based on distance. The load-distance logic assesses a location according to the sum of the distances to service points, each

weighted by the relevant load or demand, and prefers the location with the lowest weighted distance [7]. This principle forms the basis of classical location methods.

The same distance-based logic is also observed in multi-criteria decision making. Methods such as TOPSIS and VIKOR rank the alternatives according to their distances from the positive-ideal and negative-ideal reference points, and identify the best alternative as the one nearest to the desired point [22]. Thus, distance serves as a common evaluation criterion both in location selection and in multi-criteria ranking.

In this study, the proposed method also adopts a distance-based logic. However, it measures distance not in a physical space, but in the relational space generated by correspondence analysis. The distances from the alternatives (row points) to the criteria (column points) are combined with the criterion weights to yield a decision score. The “load-distance” phrase in the method’s name refers to this weighted distance basis. In this way, the classical load-distance logic is transferred to a relational space derived from categorical preference data.

2.4. Criteria weighting

In multi-criteria decision making, the weights assigned to criteria directly affect the outcome and are divided into two groups according to the source of weight determination. External methods determine the weights based on the judgments of decision makers or experts. The Analytic Hierarchy Process (AHP) is the most common example of this approach and evaluates the criteria through pairwise comparisons [25, 30]. Objective methods, on the other hand, derive the weights directly from the structure of the data in the decision matrix without intervention from the decision maker [10].

The two most common objective methods are entropy and CRITIC. The entropy method is based on the variability of the values within a criterion. The more the scores of the alternatives differ in a given criterion, the more information that criterion conveys and the higher weight it receives [10, 27]. In addition to variability, the CRITIC method also takes into account the conflict (correlation) between criteria. Thus, it reflects both the intensity of contrast and the relationships among criteria in the weights [8].

The choice of weighting method is not merely a technical detail. Different weighting schemes can produce different rankings for the same problem, and results are sensitive to the weights [23]. Therefore, the behaviour of a ranking method cannot be evaluated independently of the weighting approach used. In this study, the proposed method is tested under both objective (entropy and CRITIC) and external weighting schemes, and the impact of weighting selection on the results is systematically examined.

2.5. Compensatory and non-compensatory assessment

In compensatory methods, weak performance of an alternative on one criterion can be offset by superior performance on another criterion. This may be desirable in certain contexts; however, it means that the method itself, rather than the decision maker, is making a trade-off decision unconsciously [26]. In contrast, non-compensatory approaches do not allow such substitution between criteria and directly penalize imbalance across criteria [19]. In the non-compensatory approach they developed for composite index construction, Mazziotta and Pareto [19] use a variability penalty to highlight alternatives with a more balanced profile across criteria, even if their means are identical. This mechanism systematically lowers

the composite score of alternatives that exhibit an excessively concentrated (unbalanced) performance profile across criteria.

The practical consequence of non-compensatory behaviour in decision problems is as follows: an alternative that performs very well on a single criterion but falls short on others may rise to the top in compensatory methods, whereas in a non-compensatory approach, it cannot offset this advantage [21].

In this study, the simulation findings of the proposed method indicate that a similar mechanism is at work. Alternatives with a high Herfindahl concentration value on their criteria (see Section 3.6), that is, those that concentrate their performance predominantly on a single criterion, tend to receive lower rankings under the proposed method. This behaviour is a natural consequence of the method reflecting the proximity structure in the relational space rather than a compensatory aggregation operation. Schär et al. [26], emphasizing that the choice between compensatory and non-compensatory evaluation is determined by the decision context, demonstrate that the compensatory property of a method has significant practical implications for both the final ranking and its interpretation.

3. Method

3.1. Mathematical basis of correspondence analysis

Correspondence analysis (CA) is a multivariate statistical method that represents the relationship between the row and column categories in a contingency table in a low-dimensional space [13]. In this section, the CA algebra on which the proposed CA-WLD method is based is summarized.

Given a contingency table N with m rows (alternatives) and n columns (criteria), the correspondence matrix P is obtained by dividing all cell frequencies in the table by the sum of all cell frequencies n_{++} :

$$P = \frac{1}{n_{++}} N \quad (1)$$

Here, n_{++} denotes the sum of all frequencies in the table, while P represents the matrix of relative frequencies (masses).

Row and column mass vectors are defined as the row and column sums of the P matrix:

$$r = P\mathbf{1}_n, \quad c = P^\top \mathbf{1}_m \quad (2)$$

Here, r is the row masses vector (the relative weights of the alternatives), and c is the column masses vector (the relative weights of the criteria). The symbol $\mathbf{1}_n$ denotes a vector of ones of length n .

The core of correspondence analysis is formed by the matrix of standardized residuals S , which measures deviations from the independence model:

$$S = D_r^{-1/2} (P - rc^\top) D_c^{-1/2} \quad (3)$$

Here D_r and D_c are the diagonal matrices of the row and column masses, respectively, and $rc^\top$ is the matrix of expected relative frequencies under independence.

Singular value decomposition (SVD) is applied to the S matrix:

$$S = U\Sigma V^\top \quad (4)$$

Here, U and V denote orthonormal matrices containing the left and right singular vectors, respectively, and Σ denotes the diagonal matrix of singular values in descending order.

The variance explained by each dimension (principal inertia) is equal to the square of the corresponding singular value:

$$\lambda_k = \sigma_k^2 \quad (5)$$

Here, λ_k denotes the principal inertia of the k -th dimension.

Alternatives (rows) are placed on the principal coordinates and criteria (columns) on the standard coordinates to create an asymmetric (row-principal) map:

$$F = D_r^{-1/2} U \Sigma \quad (6)$$

$$G = D_c^{-1/2} V \quad (7)$$

Here, F ($m \times K$) denotes the principal coordinates of the alternatives, and G ($n \times K$) denotes the standard coordinates of the criteria. This asymmetric arrangement allows for a direct interpretation of the Euclidean distances between row and column points [14].

The contribution of a dimension to total inertia (explained inertia ratio) is used to assess the representational power of the analysis:

$$\tau_k = \frac{\lambda_k}{\sum_j \lambda_j} \quad (8)$$

Here, τ_k is the proportion of inertia explained by the k -th dimension. In CA-WLD, the number of dimensions to be included in the distance calculation is determined according to the cumulative proportion of explained inertia.

3.2. Formulation of the CA-WLD method

The proposed CA-WLD (Correspondence Analysis–Weighted Load-Distance) method comprises two complementary formulations that convert the relational space provided by correspondence analysis into a decision score: proximity-based Formulation A (CA-WLD-A) and ideal-point-based Formulation B (CA-WLD-B). The implementation of the method follows these steps.

Step 1: Preparation of the contingency table. The input of the method is an $m \times n$ contingency table showing the preference or frequency of use between m alternatives (rows) and n criteria (columns). By normalizing the cell frequencies, the row and column mass vectors and the corresponding profile matrices are obtained [13]. All criteria are treated as benefit-type throughout: a higher cell frequency indicates a stronger alternative–criterion association and is therefore more desirable. Problems containing cost-type or mixed-scale criteria require a prior transformation, as discussed in Section 6.4.

Step 2: Construction of the asymmetric CA map. By applying SVD to the contingency table, row and column coordinates are computed. Row points (alternatives) are positioned in principal coordinates, while column points (criteria) are positioned in standard coordinates. This asymmetric (row-principal)

arrangement allows for the direct interpretation of the proximity between row and column points. The closer a row point is to a column point, the stronger the association it has with that criterion [9, 16].

Step 3: Calculation of alternative–criterion distances. The Euclidean distance between each alternative i and each criterion j is calculated using the coordinates on the asymmetric CA map:

$$d_{ij} = \sqrt{\sum_{k=1}^K (F_{ik} - G_{jk})^2} \quad (9)$$

Here, F_{ik} and G_{jk} denote the coordinates of alternative i and criterion j on the k -th dimension, and K denotes the number of dimensions included in the calculation, which is determined on the basis of the cumulative explained inertia.

Geometric rationale. The use of the asymmetric (row-principal) map is necessary for the direct interpretation of row–criterion distances. In correspondence analysis, distances can only be interpreted directly when points are positioned in the same space. In symmetric maps, since rows and columns are on different scales, row–column distance may be misleading [13]. In the row-principal configuration, the criteria form the vertices of the row profile space with their standard coordinates, and each alternative is located as the weighted average (barycentre) of these vertices, weighted by its own profile values. This relationship is expressed by the transition formula: the principal coordinates of the alternatives are equal to the sum of the standard coordinates of the criteria weighted by the row profile [13, 14].

Within this framework, the Euclidean distance between an alternative and a criterion represents the relative proximity of the alternative to that criterion, that is, the salience of that criterion in its profile: the closer an alternative is to the vertex of a criterion, the larger the share of that criterion in its profile. The choice of Euclidean distance as the distance metric is due to the fact that barycentric positioning is defined by this metric. Direction-based measures such as inner product or cosine similarity disregard magnitude (mass) information, while chi-square distance is defined for row-to-row comparisons and cannot be directly applied to row–column relationships. For this reason, CA-WLD is based on the weighted Euclidean distance consistent with barycentric geometry.

Step 4a: Proximity-based score (Formulation A, CA-WLD-A). This formulation, which is based on a weighted summation structure, calculates a weighted proximity score for each alternative. First, alternative–criterion distances are normalized using the min–max method per criterion, then proximities (one minus the normalized distance) are summed with weights:

$$S_i^A = \sum_{j=1}^n w_j (1 - \tilde{d}_{ij}) \quad (10)$$

Here, S_i^A denotes the proximity-based score of alternative i , w_j denotes the weight of criterion j , n denotes the number of criteria, and $\tilde{d}_{ij}$ denotes the min–max normalized value of the distance in Equation (9) on a criterion-by-criterion basis. The score is maximized: an alternative positioned close to significant criteria receives a high score. This formulation resembles the weighted sum (SAW) structure [30]. Although the aggregation structure is similar to a weighted sum, calculating the score on the basis of distances in the correspondence analysis profile geometry imparts sensitivity to imbalance to the

method (see Section 2.5).

Step 4b: Weighted ideal-point distance (Formulation B, CA-WLD-B). Formulation B, which is based on the ideal-point logic, calculates the weighted Euclidean distance score for each alternative:

$$S_i^B = \sqrt{\sum_{j=1}^n w_j d_{ij}^2} \quad (11)$$

Here, S_i^B represents the weighted ideal-point distance score of alternative i , and w_j and d_{ij} denote the criterion weight and the alternative–criterion distance from Equation (9), respectively. The score is minimized. This formulation extends the TOPSIS ideal-point distance logic to the CA space [22] and can be positioned as a TOPSIS-like, distance-based framework based on ideal-point logic. Formulation B ranks alternatives that are strong in a single criterion but weak in others (unbalanced profile) lower than compensatory methods. This characteristic reinforces the method’s sensitivity to imbalance and its relatively non-compensatory behaviour (see Section 2.5).

Step 5: Ranking. Both formulations are applied independently. In Formulation A the alternative with the highest score is ranked as the best, while in Formulation B the alternative with the lowest score is ranked as the best. In this study, proximity-based Formulation A was used as the main CA-WLD method, while the ideal-point-based Formulation B was reported as a robustness check. The rankings produced by the two formulations are highly consistent (mean Spearman $\rho = 0.84$; see Section 4), indicating that the findings are not sensitive to the specific choice of formulation.

The steps presented in this section, from preparing the contingency table to ranking, are consolidated into a single procedure in Algorithm 1. The algorithm explicitly displays the inputs (contingency matrix, weight vector, and inertia threshold), the intermediate calculations (correspondence analysis coordinates and alternative–criterion distances), and the outputs (CA-WLD-A and CA-WLD-B scores and their rankings).

3.3. Comparison methods

To position CA-WLD against established practice, nine widely used MCDM techniques representing different aggregation logics were selected as the comparison set. The methods come from four families: value aggregation (WASPAS, ARAS, COCOSO), reference point or distance (TOPSIS, VIKOR, EDAS, MARCOS), boundary approximation (MABAC), and outranking (PROMETHEE II). Table 1 summarizes the selected methods.

TOPSIS [17] is based on the principle that the best alternative should be closest to the positive-ideal solution and farthest from the negative-ideal solution. VIKOR [22] produces a compromise solution in the presence of conflicting criteria. It maximizes group benefit while minimizing individual regret. PROMETHEE II [4] calculates a net flow score through pairwise comparisons based on preference functions. WASPAS [35] is a hybrid approach that combines the weighted sum and weighted product models. ARAS [34] evaluates alternatives as a ratio to the ideal best solution. EDAS [18] ranks alternatives based on their average deviations from the mean solution. MARCOS [28] uses distances to both ideal and anti-ideal reference points together. COCOSO [32] combines three separate aggregation strategies to produce

Input: Contingency matrix $N \in \mathbb{R}^{m \times n}$; weight vector $w \in \mathbb{R}^n$ with $\sum_j w_j = 1$; cumulative inertia threshold $\theta \in (0, 1]$

Output: Score vectors s^A, s^B ; rankings π^A, π^B

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 $P \leftarrow N / (1_m^\top N 1_n)$ ; // correspondence matrix
 $r \leftarrow P 1_n$ ;  $c \leftarrow P^\top 1_m$ ; // row and column masses
 $S \leftarrow D_r^{-1/2} (P - r c^\top) D_c^{-1/2}$ ; // standardized residuals
 $(U, \Sigma, V) \leftarrow \text{SVD}(S)$ ;
 $F \leftarrow D_r^{-1/2} U \Sigma$ ; // row (alternative) principal coordinates
 $G \leftarrow D_c^{-1/2} V$ ; // column (criterion) standard coordinates
 $K \leftarrow \min\{k : \sum_{l \leq k} \lambda_l / \sum_l \lambda_l \geq \theta\}$ ; // dimension selection
for  $i \leftarrow 1$  to  $m$ ,  $j \leftarrow 1$  to  $n$  do
     $d_{ij} \leftarrow \sqrt{\sum_{k=1}^K (F_{ik} - G_{jk})^2}$ ; // alternative-criterion distance
end
for  $j \leftarrow 1$  to  $n$  do
     $\tilde{d}_{.j} \leftarrow \text{min-max-normalize}(d_{.j})$ ; // per criterion
end
for  $i \leftarrow 1$  to  $m$  do
     $s_i^A \leftarrow \sum_{j=1}^n w_j (1 - \tilde{d}_{ij})$ ; // CA-WLD-A (proximity)
     $s_i^B \leftarrow \sqrt{\sum_{j=1}^n w_j d_{ij}^2}$ ; // CA-WLD-B (ideal point)
end
 $\pi^A \leftarrow \text{sort}(s^A, \text{descending})$ ;  $\pi^B \leftarrow \text{sort}(s^B, \text{ascending})$ ;
return  $(s^A, s^B, \pi^A, \pi^B)$ 

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Algorithm 1. The CA-WLD procedure.

an integrated compromise score. MABAC [23] ranks alternatives by positioning them according to the boundary approximation area.

Table 1. Nine established MCDM methods used in the comparison

Method	Family	Decision logic	Source
TOPSIS	Reference point / distance	Closest to positive-ideal, farthest from negative-ideal	[17]
VIKOR	Compromise / distance	Group benefit max., individual regret min.	[22]
PROMETHEE II	Outranking	Net flow with preference function	[4]
WASPAS	Value aggregation (hybrid)	Weighted sum + weighted product	[35]
ARAS	Value aggregation	Ratio to the ideal best solution	[34]
EDAS	Reference point / distance	Deviations from the average solution	[18]
MARCOS	Reference point / distance	Distance from ideal + anti-ideal references	[28]
COCOSO	Value aggregation	Combination of three aggregation strategies	[32]
MABAC	Border approximation	Distance from border approximation area	[23]

The nine techniques listed above are all compensatory: a deficiency on one criterion can be offset by strength on another. Because the behaviour of CA-WLD is itself imbalance-sensitive, restricting the comparison to compensatory methods alone would leave an obvious question unanswered, namely whether the divergence reported below is simply a restatement of non-compensation. A non-compensatory reference is therefore included. The max-min (Wald-type) rule ranks first the alternative whose *worst* criterion performance is highest, so that a single weak criterion cannot be compensated [17]. Two further variants are examined: a weighted max-min, in which the criterion weights multiply the normalized performances

before the minimum is taken, and an augmented max-min, which adds a small multiple of the mean performance to the minimum in order to reward more balanced alternatives. All three are computed on the criterion-wise maximum-normalized decision matrix. These references are reported separately in Section 4.6 and are *not* included in the MCDM consensus, which by construction represents the compensatory family.

3.4. Weighting schemes

To examine the effect of the weighting method on ranking results (RQ3), three different weighting schemes were applied in each simulation replication.

Entropy weighting is based on the Shannon [27] information entropy principle. The weight of each criterion is determined in proportion to the degree of variability among the alternatives for that criterion. High variability indicates high information content and thus implies a higher weight [8].

The CRITIC (Criteria Importance Through Intercriteria Correlation) method [8] considers both intra-criterion contrast intensity (standard deviation) and inter-criteria conflict (correlation-based). Criteria with higher variability and low correlation with other criteria receive higher weights.

External (fixed) weighting refers to weights that are not derived from data and reflect the preferences of the decision maker. Unlike objective methods (entropy, CRITIC), these weights are independent of the data structure. In the simulation, external weights were sampled independently from the Dirichlet distribution ($\alpha = 3$) in each replication. In this way, weight vectors that fluctuate realistically but remain independent of the data were obtained. In the real-data application, external weights were taken as the decision-maker weights determined by AHP [25, 30].

3.5. Simulation design

A full-factorial experimental design based on Monte Carlo simulation was adopted to systematically examine the behaviour of the CA-WLD method under controlled conditions. This approach is an established method that provides robust and replicable evidence for the comparative evaluation of MCDM methods [33].

In each replication, a synthetic $m \times n$ contingency table was created. The profiles (the distribution of an alternative across criteria) were sampled from a Dirichlet distribution with concentration parameter α . The Dirichlet distribution is a multivariate continuous distribution defined over probability vectors whose components are non-negative and sum to one. With this feature, it is a natural choice for generating share (profile) data [11]. The probability density function of a K -dimensional Dirichlet distribution with concentration parameter $\alpha = (\alpha_1, \dots, \alpha_K)$ is defined on the simplex as

$$f(x_1, \dots, x_K; \alpha) = \frac{\Gamma\left(\sum_{i=1}^K \alpha_i\right)}{\prod_{i=1}^K \Gamma(\alpha_i)} \prod_{i=1}^K x_i^{\alpha_i-1}, \quad x_i \geq 0, \quad \sum_{i=1}^K x_i = 1 \quad (12)$$

where $\Gamma(\cdot)$ is the gamma function. In the symmetric case (all α_i equal), $\alpha > 1$ yields balanced (centred) vectors, $\alpha = 1$ yields a uniform distribution on the simplex, and $\alpha < 1$ yields extreme vectors concentrated on a single component. In this study, varying the α parameter from low to high ($\alpha = 0.3 \rightarrow 4.0$) enabled a systematic exploration of a spectrum ranging from balanced profiles to edge-expert profiles.

Row mass (the total usage frequency of the alternatives) was generated from a distribution controlled by the β parameter: high β represents equal popularity, while low β indicates strong popularity asymmetry.

In the study, two independent structural factors and one dimensional factor were examined. The profile structure (α) was defined at three levels: balanced ($\alpha = 4.0$), moderate ($\alpha = 1.0$), and edge-expert ($\alpha = 0.3$). Popularity asymmetry (β) was also studied at three levels: equal ($\beta = 15.0$), moderate ($\beta = 4.0$), and high asymmetry ($\beta = 1.5$). The problem dimension was considered at five and three levels as $m \in \{4, 6, 8, 10, 12\}$ alternatives and $n \in \{5, 8, 11\}$ criteria, respectively. For each factor combination, 50 independent replications were conducted. By crossing with three weighting schemes, a total of 20,250 scenarios were generated; after removing numerical invalidities, 20,242 valid tables were included in the analysis.

This total number was obtained as follows. When three profile levels, three popularity asymmetry levels, five alternative levels (m), and three criterion levels (n) are crossed, $3 \times 3 \times 5 \times 3 = 135$ factor combinations are produced. When each combination is repeated with 50 independent replications and crossed with three weighting schemes (entropy, CRITIC, external), $135 \times 50 \times 3 = 20,250$ scenarios are reached. After removing eight numerically invalid tables from these scenarios (tables from which a valid ranking could not be obtained with at least five MCDM methods due to non-discriminative columns or computational instability), the number of valid tables was reduced to 20,242.

Comparison protocol. For a fair comparison, all methods were run on the same data structure. In each replication, the generated contingency table was provided as a common decision matrix to both CA-WLD and the nine MCDM methods, all criteria were considered as benefit-type, and each weighting scheme was applied to all methods with the same weight vector. Each MCDM method used its own standard normalization. While the MCDM methods operate on raw frequencies, CA-WLD takes row and column masses into account. This distinction is part of the additional information produced by the method and does not constitute a flaw in the comparison. Thus, the differentiation observed among the methods arises not from different data inputs, but from processing the same data with different aggregation rules and geometries.

Generation and reproducibility. Each table was produced by the following mechanism. The total number of responses was set as $N = 50m$; the relative selection volumes of the alternatives (row masses) were sampled from a Dirichlet(β) distribution and scaled by N , then rounded to integers. Next, for each alternative, the profile on the criteria was drawn from a Dirichlet(α) distribution and cell frequencies were sampled from a multinomial distribution with this profile. In this way, the profile structure (α) and popularity asymmetry (β) become mathematically independent factors. Rows or columns that were entirely zero were smoothed by a minimal +1 correction. When all alternatives are equidistant on a criterion, a very small constant was added to the denominator so that min-max normalization does not become undefined. All generation was performed with a single fixed random seed (seed = 2026), making the results exactly reproducible. Of the 20,250 scenarios produced, eight degenerate tables were removed in which no valid ranking could be obtained from at least five MCDM methods. As these tables comprise 0.04% of the sample, they do not affect the results.

All methods were implemented in the Python environment (version 3.11). Method outputs that could

not be computed due to numerical instability or degenerate conditions were excluded from the relevant replication, and tables for which a valid ranking was obtained from at least five methods were included in the analysis.

3.6. Evaluation metrics

Three main metrics were used to quantify the agreement of the methods with each other and with CA-WLD. The first is the Spearman rank correlation coefficient. Spearman's ρ , which measures similarity between two rankings from -1 (perfectly inverse) to $+1$ (perfectly concordant), is widely used as a standard metric in rank comparisons [33]. In a replication, the mean ρ value calculated among the MCDM methods reflects the internal consistency (MCDM consensus) of the method set.

The second is the divergence metric. To determine the extent to which CA-WLD produces information different from established MCDM methods, an original metric was defined in this study. The difference between the average ρ among the MCDM methods (internal consistency) and the average ρ between CA-WLD and each of those methods (external agreement) in a replication is defined as divergence:

$$\text{Divergence} = \bar{\rho}_{\text{MCDM-internal}} - \bar{\rho}_{\text{CA-WLD-external}} \quad (13)$$

Here, $\bar{\rho}_{\text{MCDM-internal}}$ represents the internal consistency of the set of methods (the average Spearman correlation among the MCDM methods), while $\bar{\rho}_{\text{CA-WLD-external}}$ represents the average correlation between CA-WLD and these methods. When the divergence value is positive and high, CA-WLD is clearly distinguished from the consensus produced by the MCDM method set. Values close to zero indicate that CA-WLD behaves consistently with the MCDM set. This metric is used as a key indicator in interpreting simulation results.

Thirdly, there is the profile concentration index. The Herfindahl concentration index was used to measure the extent to which the performance of an alternative is concentrated in a single criterion among all criteria. Originally developed to measure market concentration, this index is a commonly used metric that summarizes how many components a distribution is spread across [24]. In this study, the index was calculated on the basis of each alternative's row profile (the relative frequency distribution across criteria):

$$H_i = \sum_{j=1}^n p_{ij}^2, \quad p_{ij} = \frac{N_{ij}}{\sum_{j=1}^n N_{ij}} \quad (14)$$

Here, p_{ij} represents the relative frequency of alternative i on criterion j (row profile), N_{ij} represents the raw frequency in the contingency table, and n represents the number of criteria. The index takes values between $1/n$ (a perfectly balanced profile where performance is distributed equally across all criteria) and 1 (a perfectly concentrated/specialized profile where performance is concentrated on a single criterion); a high H value indicates a specialized alternative profile, while a low H value indicates a balanced profile.

The index is the primary variable in the expert-penalty analysis used to test the method's sensitivity to imbalance. In each replication, the Herfindahl values of the alternatives were correlated with the positional difference between these alternatives' CA-WLD and MCDM rankings; the resulting correlation indicated the extent to which CA-WLD penalized concentrated profiles (see Section 4.2).

4. Findings

4.1. MCDM consensus and the divergence of CA-WLD (RQ1)

The simulation was conducted on the 20,242 valid contingency tables generated on the basis of factor combinations and three weighting schemes, and all findings were obtained from these tables. The first finding is that the established MCDM methods are highly consistent with each other. The average Spearman concordance among the methods was calculated as 0.92. This value indicates that the nine examined methods constitute, in practice, a tightly agreed set, meaning that they produced largely similar rankings.

In contrast, the average agreement of CA-WLD with the same consensus remained at only 0.53. The difference between the two values corresponds to a divergence of 0.40 on average (Figure 1). This result demonstrates that CA-WLD does not replicate the information produced by existing methods and occupies a distinct position in the ranking space.

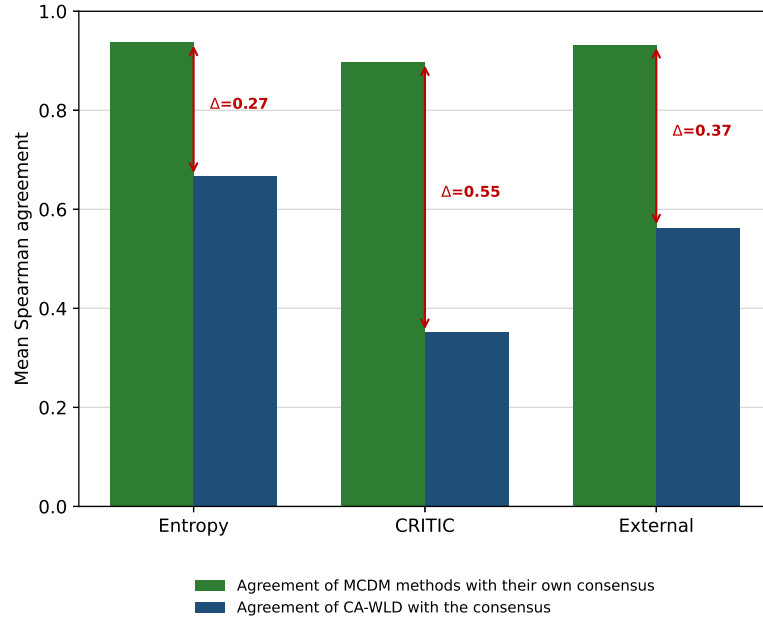


Figure 1. The distance of CA-WLD from the MCDM consensus by weighting scheme (Δ = divergence).

This divergence becomes even more pronounced upon detailed examination of the Spearman correlations between method pairs (Table 2). While the nine MCDM methods exhibit high intercorrelations ranging from $\rho = 0.75$ to 0.98, the correlation of CA-WLD with each MCDM method remains within the range of 0.46 to 0.58 (mean 0.51). In other words, CA-WLD does not behave like any member of the MCDM set, but instead consistently occupies a distinct position outside the set. This finding confirms that the proposed method generates different information from conventional MCDM techniques (RQ1). A concrete example of this statistical divergence is presented in Section 4.5: an alternative ranked first by all nine MCDM methods can fall to the very last position according to CA-WLD.

4.2. Data structure and non-compensatory behaviour (RQ2)

The level of divergence varies according to the profile structure of the data. When the data profile structure is considered alone, divergence is highest for balanced profiles (0.47) and lowest for edge-expert

Table 2. Pairwise Spearman rank correlation between methods (mean over 20,242 conditions)

	TOPSIS	VIKOR	PROM. II	WASPAS	ARAS	EDAS	MARCOS	CoCoSo	MABAC	CA-WLD
TOPSIS	1.00	0.81	0.76	0.84	0.90	0.87	0.91	0.80	0.91	0.47
VIKOR	0.81	1.00	0.76	0.77	0.78	0.78	0.80	0.75	0.80	0.53
PROMETHEE II	0.76	0.76	1.00	0.82	0.83	0.83	0.87	0.88	0.85	0.58
WASPAS	0.84	0.77	0.82	1.00	0.89	0.88	0.90	0.87	0.91	0.53
ARAS	0.90	0.78	0.83	0.89	1.00	0.97	0.96	0.86	0.95	0.46
EDAS	0.87	0.78	0.83	0.88	0.97	1.00	0.94	0.86	0.92	0.47
MARCOS	0.91	0.80	0.87	0.90	0.96	0.94	1.00	0.89	0.98	0.48
CoCoSo	0.80	0.75	0.88	0.87	0.86	0.86	0.89	1.00	0.89	0.55
MABAC	0.91	0.80	0.85	0.91	0.95	0.92	0.98	0.89	1.00	0.51
CA-WLD	0.47	0.53	0.58	0.53	0.46	0.47	0.48	0.55	0.51	1.00

Note. Bold cells indicate the correlations between CA-WLD and the other methods. Values are means across 20,242 conditions.

profiles (0.32) (RQ2).

The mechanism underlying this divergence is the imbalance-sensitive and relatively non-compensatory behaviour of CA-WLD. When the difference in position between an alternative's CA-WLD and MCDM rankings is correlated with the Herfindahl index, which measures the concentration of an alternative's profile, a consistently negative correlation is found (mean $r = -0.25$; Figure 2). The negative correlation indicates that alternatives whose performance is concentrated in a single criterion, that is, with a high Herfindahl value, are ranked lower by CA-WLD compared to compensatory MCDM methods.

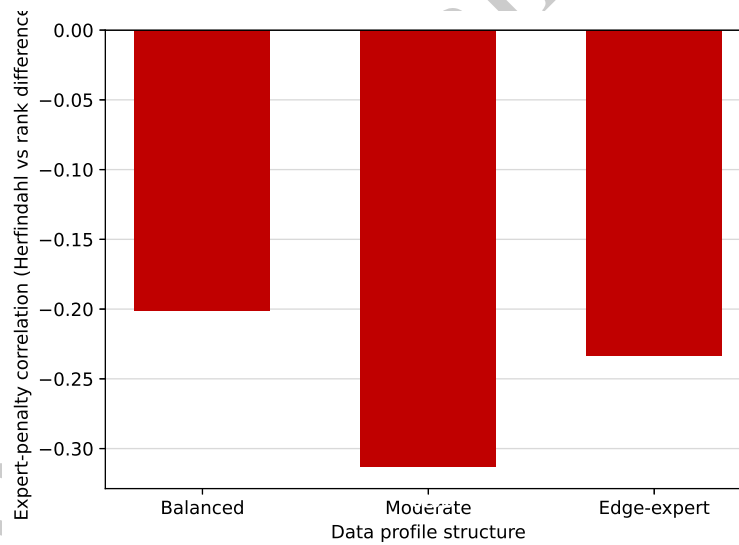


Figure 2. Imbalance-sensitive behaviour of CA-WLD: correlation between profile concentration (Herfindahl) and rank difference, by data profile level.

This behaviour is the empirical counterpart of the non-compensatory assessment rationale discussed in Section 2.5: CA-WLD distinctly retracts imbalanced profiles. The expert penalty was observed at all profile levels, with the intermediate profile structure showing the strongest effect ($r = -0.31$).

However, the effect of the profile structure on divergence is not solely determinative. Its effect is conditional on the weighting method. This interaction is addressed in Section 4.3 together with the weighting findings.

4.3. The determinative role of weighting (RQ3)

The weighting method is the factor with the strongest effect on divergence. The highest divergence was obtained with CRITIC weights (0.55), followed by external weighting (0.37) and entropy (0.27). In other words, CA-WLD diverges most strongly from the MCDM set under CRITIC weights, while it is closest to the set under entropy weights (RQ3). The same pattern is observed in top-rank overlap as well (Table 3): the rate of sharing the first rank between CA-WLD and the MCDM consensus is highest under entropy (0.63) and lowest under CRITIC (0.27).

Table 3. Top-rank overlap of CA-WLD with the MCDM consensus, by weighting method (20,242 simulation scenarios)

Weighting method	Divergence (avg.)	First-rank overlap (ratio)	Top-3 overlap (avg.)
Entropy	0.271	0.625	2.128
CRITIC	0.546	0.269	1.713
External	0.370	0.439	1.979

Note. First-rank overlap = rate at which the first rank of CA-WLD coincides with the first rank of the MCDM consensus. Top-3 overlap = mean number of common alternatives in the top three (0–3).

However, the effect of weighting is not independent of data structure. The main determinant is the interaction of the two (Figure 3a). The highest divergence occurs when the balanced profile is combined with CRITIC weighting (0.70), and the lowest when the same balanced profile is combined with entropy weighting (0.24). That both extremes arise within a single profile level underlines the dominant role of the weighting scheme. The popularity asymmetry factor exhibits a similar interaction, and the effect of popularity asymmetry on divergence again depends on the weighting method (Figure 3b). This finding indicates that when CA-WLD will produce different information is determined not by data structure alone or weighting choice alone, but by the combination of both.

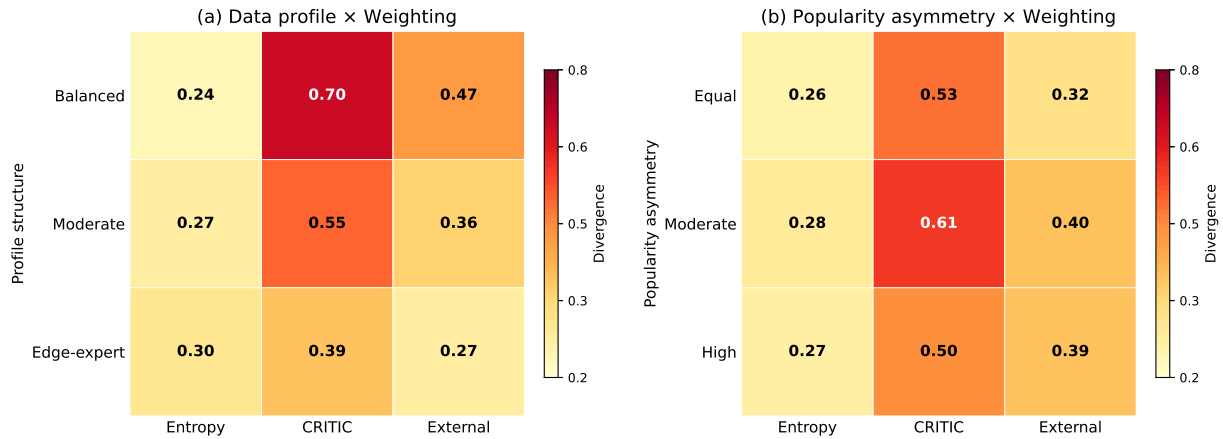


Figure 3. Divergence as a function of the interaction between the weighting method and the data structure: (a) data profile × weighting, (b) popularity asymmetry × weighting.

The relative effect of the factors on divergence was tested by a factorial analysis of variance (ANOVA) involving four factors (profile structure, weighting, popularity asymmetry, problem size) and the profile × weighting interaction (Table 4). The weighting method is by far the most powerful factor. Alone, it explains 10.8% of the total variance ($F(2, 20230) = 1325$; $p < 0.001$), and this share is higher than the sum of all other main effects. It is followed by the profile × weighting interaction (3.3%) and profile structure (3.0%). The statistical significance of the interaction confirms that weighting and data structure

need to be considered together. Popularity asymmetry (0.5%) and problem size (0.02%) are relatively weak effects. Divergence means and 95% confidence intervals by weighting scheme do not overlap (entropy 0.27 [0.27–0.28], CRITIC 0.55 [0.54–0.56], external 0.37 [0.36–0.38]). This indicates that the differences are statistically robust. When the same analysis is repeated using Kendall's τ instead of Spearman's ρ , the weighting rank is preserved (correlation between the two concordance criteria $r = 0.98$).

Table 4. Relative effect of factors on divergence: factorial ANOVA (SS = sum of squares, η^2 = proportion of explained variance)

Source	SS	df	F	p	η^2 (%)
Profile structure (RQ2)	72.30	2	364.36	< 0.001	2.97
Weighting (RQ3)	262.91	2	1325.02	< 0.001	10.79
Popularity asymmetry	12.58	2	63.42	< 0.001	0.52
Profile $\times$ Weighting	81.45	4	205.25	< 0.001	3.34
Problem size	0.60	1	6.00	0.014	0.02
Residual (replication)	2007.05	20230	—	—	82.36

Note. The dependent variable is divergence. Model $R^2 = 0.18$; most of the remaining variance arises from random table variability at the replication level.

4.4. Conditional effect of problem size (RQ4)

The effect of problem size on divergence also depends on the weighting method (Figure 4). As the size increases, divergence shows a slight upward trend across all weighting schemes. However, the magnitude of the difference is determined by the weighting method, and with the CRITIC method, divergence is higher at every size compared to the other schemes. This result indicates that the problem size does not have a strong and direct effect on divergence by itself, and that its effect is conditioned by the weighting scheme (RQ4).

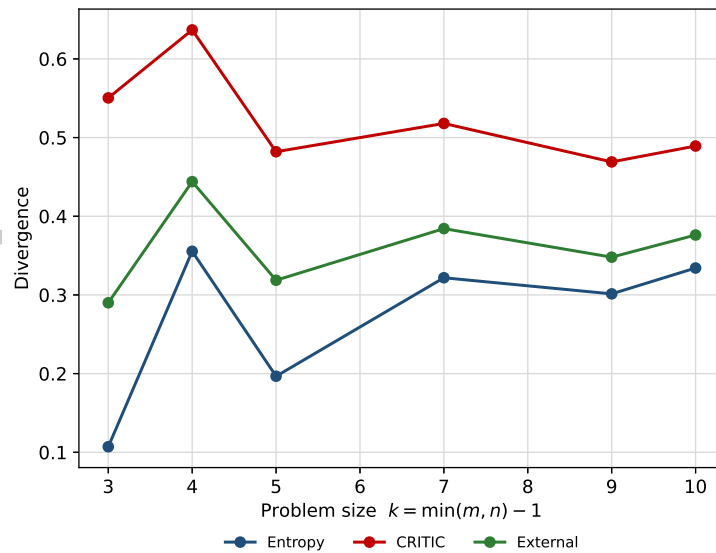


Figure 4. Variation of the effect of problem size on divergence by weighting scheme.

4.5. An illustrative example based on expert profile and weight dependence

The statistically observed divergence was analyzed on a set of 20,242 contingency tables encompassing different problem dimensions ($m \in \{4, 6, 8, 10, 12\}$, $n \in \{5, 8, 11\}$), profile structures, and weighting schemes. To illustrate this divergence with a concrete case, a single, small contingency table ($m = 5$ alternatives, $n = 4$ criteria) was also examined under three weighting schemes. The contingency table is presented in Table 5, and the method-based rankings obtained with entropy, CRITIC, and external weights are shown in Table 6.

Table 5. Illustrative example: contingency table and profile concentration index ($m = 5$, $n = 4$)

Alternative	C1	C2	C3	C4	Total	Herfindahl (H)
a_1	1	9	10	12	32	0.318
a_2	11	12	3	37	63	0.414
a_3	15	15	10	1	41	0.328
a_4	2	19	18	4	43	0.381
a_5	4	4	9	3	20	0.305

Note. H = Herfindahl concentration index (based on the row profile). The highest value is in bold.

Alternative a_2 displays a profile concentrated on the C4 criterion (Herfindahl $H = 0.414$, the highest value). On average, the MCDM methods rank a_2 first under entropy and CRITIC weights and second under external weights. Individual methods deviate from this average: PROMETHEE II places a_2 second under CRITIC weights, and under external weights the MCDM ranks of a_2 range from first to third. The behaviour of CA-WLD distinctly varies depending on the weighting scheme. With CRITIC weights, a_2 falls to fifth (last) place, and with external weights it also drops to fifth. However, with entropy weights, CA-WLD ranks a_2 first, the same as the MCDM methods. This concretizes the simulation finding in Table 3.

Step-by-step computation. To make the procedure concrete, the computation is followed through for a_2 . The table totals 199 responses. Dividing every cell by this total gives the correspondence matrix P , whose row and column sums are the masses $\mathbf{r} = (0.161, 0.317, 0.206, 0.216, 0.101)$ and $\mathbf{c} = (0.166, 0.296, 0.251, 0.286)$. The singular value decomposition of the standardized residual matrix yields principal inertias of 0.279, 0.108 and 0.017, that is 69.0%, 26.9% and 4.2% of total inertia; all three dimensions are retained.

In the resulting asymmetric map the Euclidean distances from a_2 to the four criteria are 2.333 (C1), 1.899 (C2), 2.263 (C3) and 0.954 (C4). Normalizing each criterion column to the $[0, 1]$ range and converting the normalized distances into proximities gives

$$\tilde{d}(a_2) = (0.664, 1.000, 1.000, 0.000), \quad 1 - \tilde{d}(a_2) = (0.336, 0.000, 0.000, 1.000).$$

The profile of a_2 is thus placed nearest to C4 among all alternatives and farthest from both C2 and C3.

The decision score follows from the weights. Entropy assigns $\mathbf{w} = (0.303, 0.088, 0.103, 0.505)$, concentrating half of the total weight on C4, which is the criterion with the greatest spread. The score of a_2 is therefore $0.336 \times 0.303 + 0 \times 0.088 + 0 \times 0.103 + 1 \times 0.505 = 0.607$, the highest in the set, and a_2 is ranked first. CRITIC assigns the far more even vector $\mathbf{w} = (0.267, 0.176, 0.263, 0.294)$. The same

proximities now give $0.336 \times 0.267 + 0 \times 0.176 + 0 \times 0.263 + 1 \times 0.294 = 0.384$, the lowest score in the set, and a_2 falls to fifth place.

The two zero proximities are what drive the reversal. Under entropy weighting they are almost cost-less, because C2 and C3 together carry only 19% of the weight. Under CRITIC weighting they carry 44%, and no advantage on C4 can offset them. This is the non-compensatory behaviour of the method in its most explicit form, and it shows why the weighting scheme, rather than the data alone, determines whether CA-WLD converges with or diverges from the compensatory methods.

Table 6. Ranking comparison by method under three weighting schemes ($m = 5, n = 4$)

Method	a_1	a_2	a_3	a_4	a_5
<i>Entropy weights, $\mathbf{w} = [0.303, 0.089, 0.103, 0.505]$</i>					
TOPSIS	3	1	2	4	5
VIKOR	2	1	3	4	5
PROMETHEE II	3	1	4	2	5
WASPAS	3	1	2	4	5
ARAS	3	1	2	4	5
EDAS	2	1	3	4	5
MARCOS	4	1	2	3	5
CoCoSo	4	1	2	3	5
MABAC	4	1	2	3	5
MCDM consensus	3	1	2	4	5
CA-WLD (A)	2	1	3	5	4
CA-WLD (B)	2	1	4	5	3
<i>CRITIC weights, $\mathbf{w} = [0.267, 0.176, 0.263, 0.294]$</i>					
TOPSIS	4	1	2	3	5
VIKOR	3	1	4	2	5
PROMETHEE II	4	2	3	1	5
WASPAS	4	1	2	3	5
ARAS	4	1	2	3	5
EDAS	4	1	2	3	5
MARCOS	4	1	2	3	5
CoCoSo	4	1	3	2	5
MABAC	4	1	2	3	5
MCDM consensus	4	1	2	3	5
CA-WLD (A)	3	5	2	4	1
CA-WLD (B)	2	4	3	5	1
<i>External weights, $\mathbf{w} = [0.250, 0.319, 0.225, 0.206]$</i>					
TOPSIS	4	3	1	2	5
VIKOR	4	2	1	3	5
PROMETHEE II	4	3	2	1	5
WASPAS	4	1	2	3	5
ARAS	4	1	2	3	5
EDAS	4	1	2	3	5
MARCOS	4	2	1	3	5
CoCoSo	4	3	2	1	5
MABAC	4	3	1	2	5
MCDM consensus	4	2	1	3	5
CA-WLD (A)	4	5	1	2	3
CA-WLD (B)	3	5	1	4	2

Note. External weights were generated with Dirichlet($\alpha = 3$, seed = 42). The CA coordinates are shared across the three weighting schemes; only the scoring formulas (Formulations A and B) use the weights.

4.6. Divergence from a non-compensatory baseline

A natural objection to the results reported so far is that they may simply restate a known fact: a non-compensatory rule will of course rank unbalanced alternatives differently from compensatory ones. If that were the whole story, a genuinely non-compensatory reference should diverge from the compensatory consensus in the same way as CA-WLD and should produce rankings close to it. The first expectation is partly met, the second is not, and the difference between the two is what identifies the contribution of the proposed method.

Averaged over the 20,242 valid tables, the max-min rule attains a mean Spearman agreement of 0.551 with the compensatory consensus, a divergence of 0.371, which is indeed close to the 0.395 recorded for CA-WLD (Table 7). Taken alone, this figure would suggest that the two approaches occupy the same position. They do not. Their mutual agreement is only $\rho = 0.405$, against $\rho = 0.922$ within the compensatory family itself. In other words, the compensatory methods form a tight cluster, while CA-WLD and max-min lie outside it in *different directions* rather than together. The region outside the compensatory consensus is not a single place, and being non-compensatory does not by itself determine a ranking.

Table 7. Divergence from the compensatory consensus: CA-WLD versus non-compensatory baselines (20,242 tables)

	Agreement with consensus	Divergence	ρ with CA-WLD
Compensatory family (internal)	0.922	—	0.527
CA-WLD (Formulation A)	0.527	0.395	—
Max-min	0.551	0.371	0.405
Weighted max-min	0.528	0.394	0.356
Augmented max-min	0.775	0.148	0.467
<i>By data profile</i>			
	Divergence CA-WLD	Divergence max-min	$\rho(\text{CA-WLD, max-min})$
Balanced	0.469	0.287	0.427
Moderate	0.394	0.446	0.394
Edge-expert	0.323	0.514	0.357
<i>By weighting scheme</i>			
	Divergence CA-WLD	Divergence max-min	
Entropy	0.271	0.353	
CRITIC	0.546	0.341	
External	0.370	0.420	

The two approaches also respond to the data structure in opposite directions (Table 7 and Figure 5). The divergence of CA-WLD falls monotonically from 0.469 for balanced profiles to 0.323 for edge-expert profiles, whereas the divergence of max-min rises over the same range, from 0.287 to 0.514. The reason is structural. Max-min evaluates each alternative by its single worst criterion, so it departs from the compensatory ordering precisely when profiles are extreme and one criterion collapses. CA-WLD evaluates the *shape* of a row profile relative to the average profile, together with the row masses, so it adds most information when profiles are comparatively balanced and the compensatory methods have little left to separate. Imbalance-sensitivity is therefore a consequence of the geometry of CA-WLD, not its defining mechanism.

A third difference concerns responsiveness to the weighting scheme. The divergence of CA-WLD ranges from 0.271 under entropy weights to 0.546 under CRITIC weights, whereas that of max-min

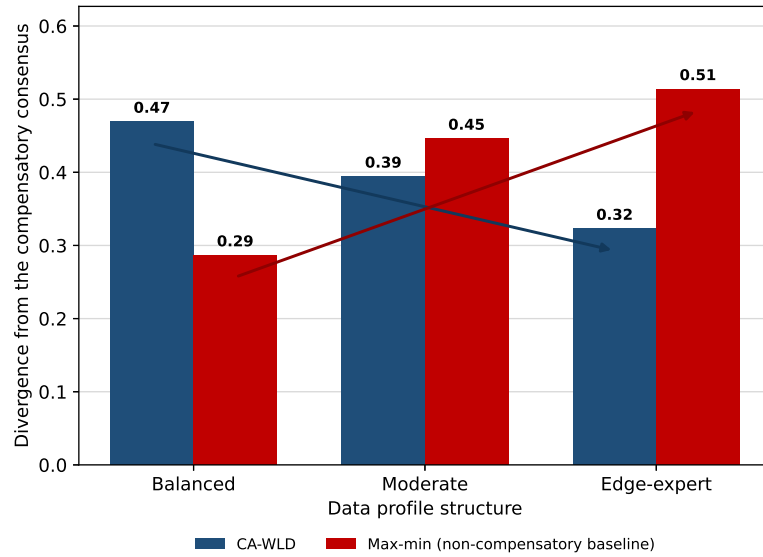


Figure 5. Divergence from the compensatory consensus by data profile structure: CA-WLD versus the max-min non-compensatory baseline. The two trends run in opposite directions.

stays within a narrow band (0.341 to 0.420) and its ranking does not use the weights at all. From the standpoint of decision support this matters: CA-WLD can be steered by the analyst's weighting choice, and its output is therefore interpretable in relation to that choice, whereas max-min cannot.

Finally, the augmented max-min variant, which adds a small multiple of the mean performance to the minimum in order to reward balance, moves sharply back toward the compensatory cluster (agreement 0.775, divergence 0.148). Even a modest compensatory component is thus enough to collapse a non-compensatory rule into the consensus, which further indicates that the position occupied by CA-WLD is not obtained simply by refusing compensation.

4.7. Rank-reversal behaviour

A ranking method exhibits rank reversal when the relative order of two alternatives changes after another alternative is added to or removed from the set. The phenomenon is well documented for established techniques, from the analytic hierarchy process [2] to TOPSIS [12]. Because CA-WLD obtains its coordinates from a singular value decomposition of the entire contingency table, its geometry depends in principle on the composition of the alternative set. This behaviour was therefore examined explicitly.

A leave-one-out experiment was carried out on 810 tables drawn from the same factorial grid, covering all profile, popularity, size, and weighting levels. In each table every alternative was removed in turn, the reduced $(m - 1) \times n$ table was re-analyzed, and the relative order of the surviving alternatives was compared with their order in the full table. The criterion weights were held fixed, so that the measured effect reflects the ranking procedure itself rather than the re-estimation of data-driven weights. The design yielded 6,392 removals and 86,945 method-level comparisons. Two indicators are reported: the proportion of removals producing at least one pairwise reversal among the survivors, and the mean proportion of surviving pairs whose order is reversed.

Table 8 reports the results. Rank reversal is not a peculiarity of the proposed method. All nine established techniques display it, with mean pairwise reversal rates ranging from 0.023 for WASPAS to 0.039 for TOPSIS. CA-WLD-B, at 0.036, is indistinguishable from EDAS and ARAS (both 0.036) and

marginally more stable than TOPSIS. CA-WLD-A is the most sensitive procedure in the set, at 0.067. The non-compensatory max-min rule is the most stable of all (0.006), which is expected, since its score depends on a single criterion value rather than on an aggregate of the whole matrix.

Table 8. Rank-reversal behaviour under leave-one-out removal (810 tables, 6,392 removals, 86,945 comparisons)

Method	Any reversal	Pairwise reversal	Top-1 change	Spearman ρ
CA-WLD-B (fixed geometry)	0.000	0.0003	0.000	1.000
Max-min	0.062	0.006	0.025	0.990
WASPAS	0.249	0.023	0.032	0.974
MARCOS	0.244	0.026	0.036	0.969
PROMETHEE II	0.482	0.030	0.060	0.971
VIKOR	0.333	0.031	0.036	0.963
COCOSO	0.378	0.032	0.049	0.966
MABAC	0.330	0.033	0.056	0.961
EDAS	0.459	0.036	0.070	0.961
CA-WLD-B	0.476	0.036	0.069	0.962
ARAS	0.451	0.036	0.063	0.961
TOPSIS	0.338	0.039	0.050	0.950
CA-WLD-A (fixed geometry)	0.439	0.047	0.126	0.943
CA-WLD-A	0.691	0.067	0.161	0.922

Note. Lower values indicate greater stability. The “fixed geometry” rows retain the coordinates estimated from the full table, which is the computational equivalent of supplementary-point projection.

To locate the source of this sensitivity, the experiment was repeated with the correspondence analysis geometry held fixed: the coordinates estimated from the full table were retained and the removed alternative was simply dropped from the score computation. This is the computational counterpart of treating an alternative as a supplementary (passive) point projected onto existing axes, a standard device in correspondence analysis. Under fixed geometry the pairwise reversal rate of CA-WLD-B falls from 0.036 to 0.0003 and its rank correlation with the original ordering becomes 1.000, whereas the rate of CA-WLD-A falls from 0.067 to 0.047 without vanishing.

The decomposition isolates two distinct sources. The first is the re-estimation of the geometry: removing an alternative changes the row masses and the principal axes, so all coordinates shift. This source accounts for essentially the whole instability of Formulation B. The second is the criterion-wise min–max normalization employed by Formulation A, which rescales distances relative to the alternatives currently present. That mechanism is shared by every range-normalized aggregation procedure, TOPSIS included, and it accounts for the residual instability of Formulation A.

Two practical implications follow. When the alternative set is fixed, both formulations may be used, and their stability is comparable to that of established techniques. When alternatives are expected to be added or withdrawn, Formulation B together with supplementary-point projection is preferable, because it preserves the relative order of the incumbent alternatives almost exactly.

4.8. Controlled perturbation analysis: when does CA-WLD align with the classical methods?

The simulation results establish how often, and under which conditions, CA-WLD diverges from the compensatory consensus, but they do not show directly how a particular disagreement should be read. To make that reading explicit, a controlled perturbation analysis was carried out on artificial data. Its starting

point is the situation that most often produces a disagreement: a specialist alternative S performs very well on a single criterion, is ranked first by the classical methods, and is not ranked first by CA-WLD. S , or its competitive environment, is then modified step by step, and the step at which CA-WLD begins to rank S first, and thereby aligns with the classical methods, is recorded.

Design. Each base table contained S and $m - 1$ competitors. Competitor profiles were drawn from a symmetric Dirichlet distribution with concentration $\alpha_c \in \{0.5, 2, 8\}$, ranging from specialized to balanced competitors, and their totals from a uniform distribution on $[80, 120]$. S concentrated a share $\pi \sim U(0.5, 0.8)$ of its responses on a randomly chosen peak criterion j^* , the remainder being spread evenly over the other criteria. Its volume was set to 1.25 times the smallest volume at which the MCDM consensus ranked it first, so that the classical methods preferred S clearly rather than marginally. Configurations in which no such volume existed, or in which CA-WLD already ranked S first, were redrawn; this was necessary for 2.7% of the base tables. Equal criterion weights were used except where the weights themselves were varied. Crossing α_c with $n \in \{4, 6, 8\}$ criteria and $m \in \{5, 8, 12\}$ alternatives gave 27 cells, each replicated 60 times, for 1,620 base tables. Five paths were then traced in 21 equal steps of a path parameter $s \in [0, 1]$:

- (P1) *Rebalancing*: the profile of S moves linearly from its initial shape to a uniform profile, with its volume held constant.
- (P2) *Strengthening the weak criteria*: the count of S on its peak criterion is held fixed and its other counts grow until they equal it, so that its volume increases.
- (P3) *Volume only*: the profile of S is held fixed and its volume rises to five times the initial value.
- (P4) *Weighting*: the table is held fixed and the weight of the peak criterion rises from $1/n$ to 0.90, the remaining weight being shared equally.
- (P5) *Competitive environment*: S is held fixed and every competitor moves toward a profile concentrated (share 0.85) on a randomly chosen criterion.

Alignment at a step is defined as CA-WLD (Formulation A) and the MCDM consensus both ranking S first. The analysis comprised 170,100 evaluations, each involving CA-WLD and all nine MCDM techniques. It used a vectorized implementation of CA-WLD that reproduced the rankings of the implementation used in Section 3.5 exactly on 1,620 tables drawn from the simulation generator, and it was run with the fixed seed 2026.

Results. Table 9 and Figure 6 report the outcome. The MCDM consensus ranks S first along virtually every path, so the changes described below take place on the side of CA-WLD.

Rebalancing (P1) aligns the two in 76.5% of the base tables. The alignment point is best described relative to the competitors: at the median, CA-WLD begins to rank S first when the concentration of S has fallen to 0.82 times the mean concentration of its competitors [IQR 0.70–0.92]. Alignment is therefore nearly universal when the competitors are themselves specialized (98.1%), less frequent against moderately balanced competitors (80.6%), and reached in only half of the cases against strongly balanced competitors (50.9%), where S must become almost perfectly uniform. In the remaining 23.5% of cases Formulation A lifts S to second place (59.7% of these) or third place (22.6%) but not to first, whereas

Table 9. Controlled perturbation analysis: share of base tables in which CA-WLD aligns with the MCDM consensus, and the point at which alignment occurs (1,620 base tables)

Path	Aligned, Formulation A (%)				Aligned, Form. B (%)	MCDM ranks S first throughout (%)	Alignment point, median [IQR]
	All	$\alpha_c = 0.5$	$\alpha_c = 2$	$\alpha_c = 8$			
P1 Rebalancing	76.5	98.1	80.6	50.9	100.0	100.0	$H_S/H_c = 0.82$ [0.70–0.92]
P2 Strengthening weak crit.	79.2	98.3	83.0	56.3	100.0	100.0	volume $\times 1.87$ [1.47–2.49]
P3 Volume only	12.8	36.5	2.0	0.0	4.5	100.0	—
P4 Weighting	99.9	99.8	99.8	100.0	100.0	99.6	$w_{j^*} = 0.41$ [0.35–0.45]
P5 Competitive environment	97.8	96.3	98.1	98.9	99.6	93.2	$\bar{H}_c/H_S = 1.08$ [1.00–1.19]

Note. H_S is the Herfindahl concentration of S and $\bar{H}_c$ the mean Herfindahl concentration of its competitors, both at the step of alignment. α_c is the Dirichlet concentration of the competitor profiles (0.5 specialized, 8 balanced). “Aligned” means that there is a step at which both CA-WLD and the MCDM consensus rank S first.

Formulation B ranks S first in every case once its profile is balanced. The difference reflects the construction of Formulation A. Its criterion-wise min–max normalization gives full proximity to whichever alternative lies closest to a criterion, so an alternative that becomes perfectly average is not rewarded for averageness as such; what Formulation A rewards is closeness to the weighted criteria. Strengthening the weak criteria (P2) produces the same pattern (79.2%), with alignment reached when the volume of S has grown by a median factor of 1.87.

Volume alone (P3) rarely produces alignment. Although the MCDM consensus ranks S first throughout, CA-WLD follows it in only 12.8% of the cases (Formulation B 4.5%), and almost all of these occur in the specialized-competitor condition; against moderately balanced and balanced competitors the rates fall to 2.0% and 0%. The residual effect arises because a growing row changes the column masses, and with them the geometry of the map, which can matter when the competitors are themselves concentrated. CA-WLD is therefore largely, although not strictly, insensitive to volume.

Weighting (P4) aligns the methods in virtually every case (99.9%). The median threshold weight on the peak criterion is 0.41 [0.35–0.45], and it rises from 0.32 against specialized competitors to 0.46 against balanced ones. In other words, CA-WLD endorses the specialist once the decision maker declares its peak criterion sufficiently important. The competitive environment (P5) yields alignment in 97.8% of cases, and the threshold again has a relative interpretation: CA-WLD begins to rank S first when the mean concentration of the competitors reaches 1.08 times that of S [IQR 1.00–1.19], that is, once S is no longer more specialized than the alternatives around it. This result provides direct experimental support for the explanation, given in Section 4.9, of why divergence is smaller when all profiles are concentrated.

Decision map. Figure 7 shows the same mechanisms for a single representative table ($n = 5$, $m = 6$, $\alpha_c = 2$, mean competitor concentration 0.27), varying two characteristics of S simultaneously. In panel (b) the boundary of the region in which CA-WLD ranks S first lies at a peak share between 0.36 and 0.38 for every volume between 0.5 and 4 times that of a competitor, whereas the boundary of the region in which the MCDM consensus ranks S first moves from a peak share of 0.42 at equal volume to 0.74, 0.82 and 0.86 at two, three and four times that volume. The region between the two boundaries is the popularity trap observed in the Çorum application (Section 5): alternatives whose first place under the classical methods rests on volume combined with concentration. In panel (a) the weight on the peak criterion that CA-WLD requires before endorsing S rises with its concentration, from 0.26 at a peak share of 0.40 to 0.40 at 0.50 and 0.50 at shares of 0.80 and above.

Reading a disagreement. Taken together, the perturbation analysis gives a practical reading of the case

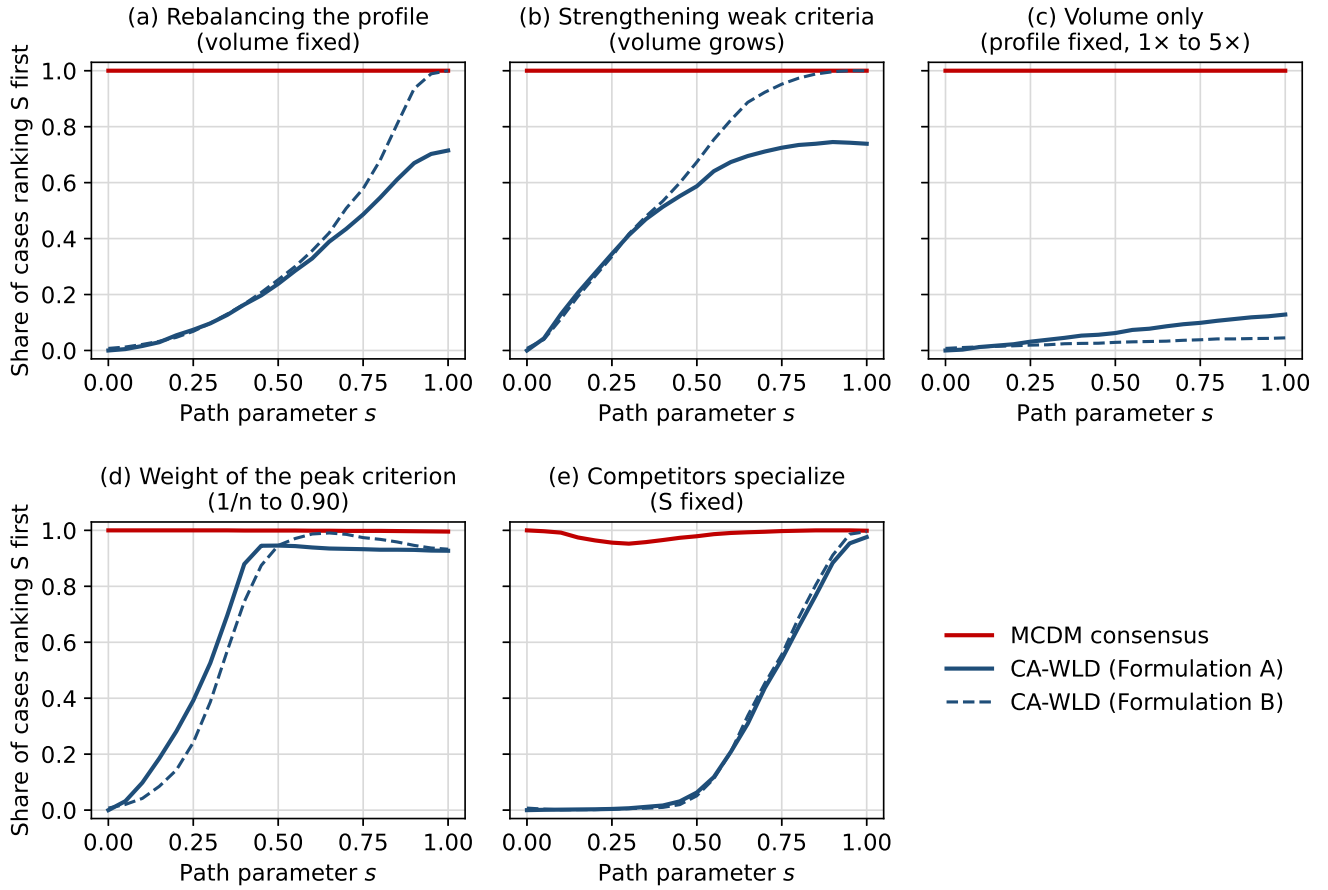


Figure 6. Controlled perturbation analysis: share of the 1,620 base tables in which each method ranks the specialist alternative S first, along the five perturbation paths.

in which the classical methods rank an alternative first and CA-WLD does not. Such a disagreement signals that the leading position rests on one or both of two sources that CA-WLD does not reward: volume, to which it is largely insensitive, and a concentration that exceeds that of the competing alternatives. The disagreement disappears in three ways: when the profile of the leader becomes no more concentrated than those of its competitors, when the decision maker assigns its peak criterion a weight commensurate with that concentration (about 0.4 in the median case), or when the competitors themselves become comparably specialized. It rarely disappears through volume alone (12.8% of the base tables, almost all with specialized competitors). An analyst can therefore interpret a disagreement by asking three questions: whether the advantage of the leader is a matter of volume, whether its profile is more concentrated than those of its competitors, and whether the decision maker regards its peak criterion as important enough to justify that concentration.

4.9. Conditions of use and recommendations (RQ5)

When the findings are considered as a whole, it appears that CA-WLD is not a competitor replacing established MCDM methods, but rather an analytical tool that complements them. This method produces distinct ranking information not repeated by the MCDM set (RQ1); this additional information is particularly valuable in cases where a non-compensatory evaluation logic is desired.

The likelihood of CA-WLD generating distinct information increases under certain conditions: (i)

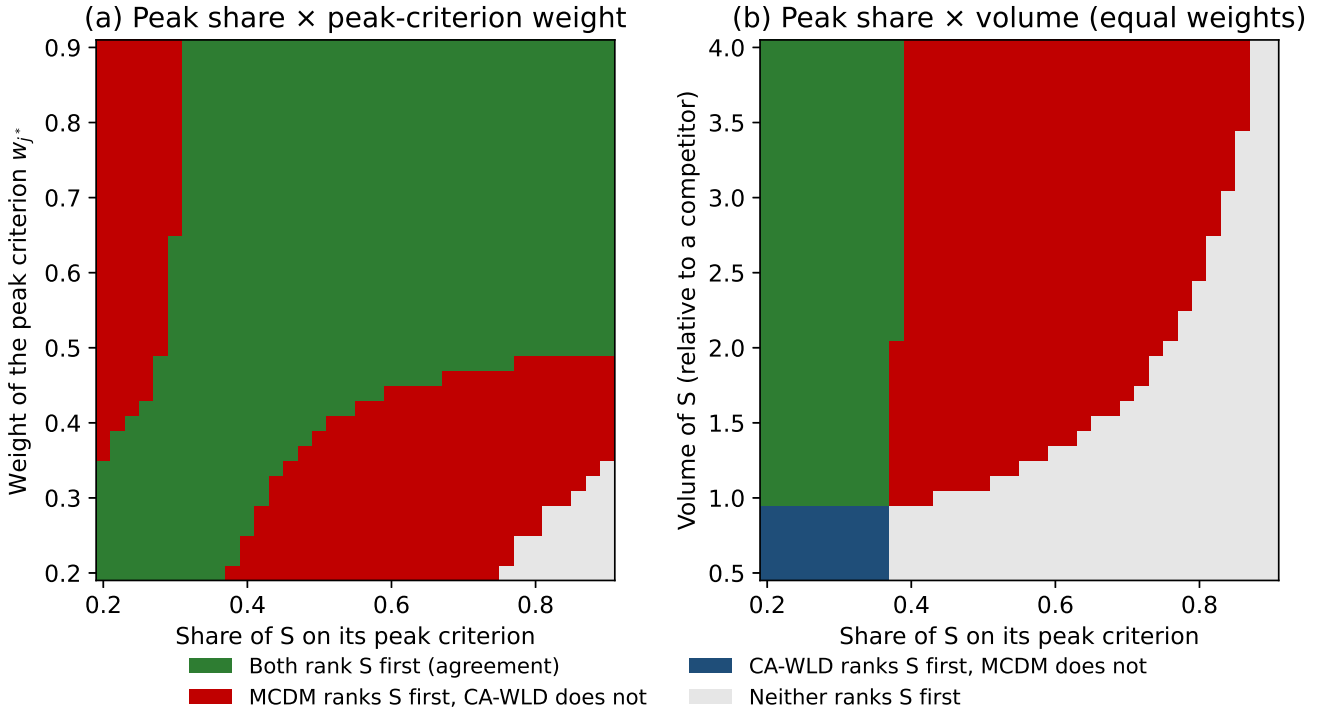


Figure 7. Decision map for a representative table: whether the MCDM consensus and CA-WLD rank the specialist alternative S first, as functions of (a) its peak share and the weight of its peak criterion, and (b) its peak share and its volume.

when criterion weights are determined using an objective and contradiction-based method (CRITIC), (ii) when alternative profiles are relatively balanced, and (iii) when the problem size increases. In contrast, when weights are determined by entropy or when profiles are highly concentrated on a single criterion, CA-WLD approaches the MCDM consensus and its additional informational contribution decreases. This pattern may appear to contradict the imbalance-sensitivity mechanism reported in Section 4.2, but it does not. When every alternative is strongly concentrated (edge-expert profiles), the imbalance penalty applies almost uniformly and therefore separates the alternatives very little, so CA-WLD moves closer to the MCDM ordering. The controlled perturbation analysis confirms this explanation directly: a specialist that the classical methods rank first is endorsed by CA-WLD once its competitors become as concentrated as it is (Section 4.8, path P5). Divergence is largest when profiles are comparatively balanced, because in that regime the remaining differences in concentration are what distinguish the alternatives, and CA-WLD is the only method in the comparison set that responds to them.

Therefore, it is recommended that CA-WLD be used as a diagnostic and cross-validation tool alongside established MCDM methods in decision problems where unbalanced or extreme profiles are considered a design flaw and compensatory behaviour is not desired, for example in cases where alternatives that excel in a single dimension but are inadequate in others should be eliminated. Evaluating both perspectives, compensatory MCDM and non-compensatory CA-WLD, together rather than relying on a single ranking will provide a richer understanding of the robustness of the results. When the two disagree about the leading alternative, the three questions set out at the end of Section 4.8 indicate what the disagreement means.

5. Real data application: shopping mall location selection in Çorum

To illustrate the simulation findings with a real decision problem, a dataset regarding the selection of a shopping mall site in the province of Çorum was used. The problem consists of eight candidate locations (a_1 – a_8) and ten evaluation criteria. Data were collected from 241 participants who would directly use the service. Each participant indicated the location they found most suitable and the criterion they considered most important; these responses were converted into a site $\times$ criterion contingency table, reported in full in Appendix A. In this respect, the study is based on a participatory design that draws decision input directly from end users rather than experts or investors. Criterion weights were obtained through an AHP analysis based on pairwise comparisons conducted with 40 participants who would again use the service [25, 30]. The consistency ratio of the pairwise comparison matrix was found to be below the acceptable threshold ($CR = 0.04 < 0.10$). Site a_1 represents the currently most selected existing location. The rankings produced by the methods are presented in Table 10.

Table 10. Çorum shopping mall application: candidate site rankings by method (1 = best)

Alternative	CA-WLD-A	CA-WLD-B	AHP	MCDM consensus
a_1	5	4	4	1
a_2	8	8	8	5
a_3	2	2	1	4
a_4	7	7	2	8
a_5	1	1	3	2
a_6	4	3	7	7
a_7	6	6	5	6
a_8	3	5	6	3

Note. The MCDM consensus is the mean rank of the nine count-based methods. The current and most popular location a_1 is shown in bold. While all count-based methods rank a_1 first, CA-WLD and AHP push it back, which is the popularity trap.

The most striking finding concerns a_1 , which is both the current and most frequently selected site. While all nine count-based MCDM methods place a_1 in first position, CA-WLD ranks a_1 fifth (Formulation A) or fourth (Formulation B), and AHP ranks it fourth (Figure 8). Whereas count-based methods are directly influenced by the raw selection volume and therefore highlight a_1 , CA-WLD, by considering profile geometry and row masses, does not succumb to this popularity bias. This result demonstrates that the sensitivity to imbalance observed in the simulation also applies to the real data.

The second finding is the concordance of CA-WLD with the AHP ranking. While the Spearman correlation of CA-WLD with AHP is 0.40 (Formulation A) and 0.43 (Formulation B), the mean correlation of the nine count-based methods with AHP is only 0.09 (ranging from -0.10 to 0.21). In other words, CA-WLD is distinctly closer to the AHP ranking, which is obtained more rigorously through pairwise comparisons, than the count-based methods are. The high concordance between the two CA-WLD formulations ($\rho = 0.93$) was maintained in the real data as well.

Robustness check. Count-based methods were also run on the row-profile normalized decision matrix. Profile normalization increased the agreement between CA-WLD and the AHP consensus (Spearman ρ from 0.52 to 0.71), but did not achieve complete concordance. This result indicates that the divergence of CA-WLD cannot be attributed solely to raw frequency magnitude, and that the method produces different

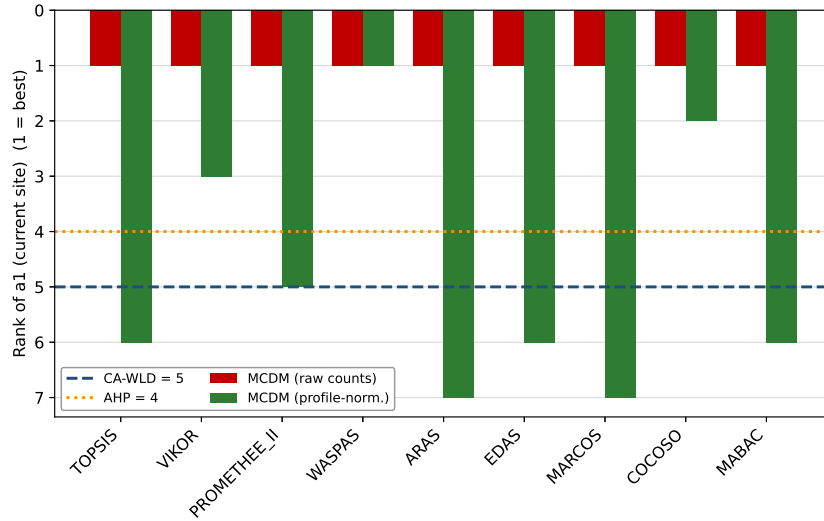


Figure 8. Popularity trap: the rank assigned to the existing and most selected location a_1 by each method.

information even on profile-normalized data.

At the time of the survey, residents stated that the proper establishment sites were a_3 , a_4 and a_5 along the ring road. CA-WLD places a_5 and a_3 from this cluster in the top two positions, while pushing back the current location a_1 , which count-based methods placed at the top. However, CA-WLD ranks a_4 low (seventh). This limitation arises from the fact that a_4 was selected by few participants in the survey, and it affects all methods based on the contingency table in a similar manner.

How the output enters a decision. To make the practical use of the method explicit, it is worth following how these results would enter an actual decision. The nine count-based techniques agree that a_1 , the existing site, should be ranked first. Taken on its own, that ordering would support concentrating further investment at the current location. CA-WLD places a_1 fifth and the AHP ranking places it fourth. The disagreement is not noise, and it is interpretable. Site a_1 attracts the largest number of responses, 81 of 241, a third of the sample, and its profile is dominated by a single criterion: 29 of those 81 responses concern proximity to the city centre, while vehicle access, noise and pollution, and size each receive a single response. Count-based methods reward the volume. CA-WLD reads the profile and finds that the advantage of a_1 rests on one attribute. In the terms of Section 4.8, a_1 combines both sources of a leading position that CA-WLD does not reward: its volume is 3.5 times that of the average competing site, and its profile is more concentrated than theirs (Herfindahl 0.21 against a mean of 0.18, a ratio of 1.16, well above the median alignment point of 0.82).

The practical conclusion is therefore not that a_1 is the wrong choice, but that its leading position is attributable to incumbency and to one dimension rather than to balanced performance. For a decision maker this changes the question from which site ranks first to whether the advantage of the leading site is broad enough to justify the investment. The two rankings can then be used jointly. The count-based ordering identifies the option with the widest public support, and the CA-WLD ordering identifies the options whose support is most evenly distributed across criteria, here a_5 and a_3 . A decision maker who values robustness across criteria would shortlist those two and would ask what would have to change at a_1 , in parking capacity or environmental mitigation, for its lead to become substantive. This is the sense in which the method is complementary: it does not replace the ranking produced by established

techniques, it qualifies it and makes the basis of a leading position explicit.

6. Discussion

6.1. Interpretation of the main findings

The central finding of the study is that while established MCDM methods produce highly consistent rankings with each other (mean Spearman $\rho = 0.92$), the proposed CA-WLD method is positioned outside this consensus set (mean agreement 0.53). This result, indicating that established methods converge with one another, is consistent with observations in the literature. It has previously been reported that different MCDM methods yield similar rankings under most conditions [6, 33]. The contribution of this study is to reveal the existence of a ranking source that consistently stands outside the MCDM set by adding a relation-geometry-based method to the same comparison framework.

The divergence of CA-WLD is not random noise, but a natural consequence of the method's structure, which is sensitive to imbalance. The findings show that alternatives with performance concentrated on a single criterion (high Herfindahl value) are ranked lower by CA-WLD. Recent studies have also emphasized that the choice between compensatory and non-compensatory evaluation is decisive for ranking results [26]. The penalization of unbalanced profiles by non-compensatory composite indicators is a known feature of this family of methods [19]. The behaviour of CA-WLD shares the same rationale with this family: the method implicitly rewards balance among criteria. It is not, however, reducible to that family, as Section 4.6 demonstrates.

Perhaps the most important aspect of the findings is that divergence depends not on a single factor, but on the interaction between the weighting method and the data structure. Weighting emerged as the dominant factor, with the highest divergence under CRITIC weights, but the effect of the data profile was conditioned by the weighting scheme. This result rules out a simple generalization that CA-WLD always produces a different outcome in every case. Whether the method yields additional information is determined in conjunction with the weighting structure and profile balance of the problem.

A useful way to summarize the behaviour observed across the simulation is that CA-WLD is condition-responsive where the compensatory family is comparatively invariant. The nine established techniques agree with one another at 0.92 irrespective of the profile structure, the popularity asymmetry, or the weighting scheme. They move together, and in the direction one would predict from the magnitudes recorded in the decision matrix. CA-WLD does not. Its divergence from that cluster ranges from 0.271 to 0.546 according to the weighting scheme and from 0.323 to 0.469 according to the profile structure. The max-min baseline is invariant in a third way, since its ranking ignores the weights altogether. The practical implication is that CA-WLD carries information about the *conditions* of a decision problem and not only about its alternatives: the magnitude of the divergence is itself diagnostic, indicating how far the ordering produced by the established methods depends on magnitude rather than on the structure of association.

6.2. Theoretical and methodological contributions

The first theoretical contribution of the study is the proposal of explicit scores derived from the geometry of correspondence analysis, which can be used in ranking and decision-making. Although the descriptive

use of correspondence analysis in decision contexts has previously been suggested [29], to our knowledge studies that directly and systematically treat the technique as a ranking method are limited. By proposing an explicit formulation that links alternative–criterion distances in the correspondence analysis space to a decision score, and by testing this with an extensive simulation design, this study contributes to filling this gap.

The second contribution is methodological. The two proposed formulations associate correspondence analysis with established MCDM families. Proximity-based Formulation A corresponds to the weighted sum (SAW) approach, while Formulation B, based on weighted ideal-point distance, corresponds to the TOPSIS approach [30]. Both formulations rely on the direct interpretability of the distances in the asymmetric correspondence analysis map [14]. Thus, the method integrates the relational geometry of correspondence analysis and the ranking logic of MCDM within a single framework.

A further way to state the contribution is that CA-WLD couples a descriptive multivariate technique with a normative decision framework. Correspondence analysis is a statistical instrument: it describes the association structure of a contingency table without any reference to preference. Multi-criteria techniques are analytical instruments: they impose a preference model and return a prescription. CA-WLD joins the two by using the relational geometry recovered by the statistical technique as the input to a weighted decision score. The additional information the method supplies originates in precisely that coupling. The compensatory techniques operate on the magnitudes recorded in the decision matrix, whereas CA-WLD operates on the position of each alternative within an association space estimated from the same matrix. This is also why its divergence cannot be reproduced by simply refusing compensation: a non-compensatory rule applied to the same magnitudes remains a magnitude-based rule, while CA-WLD changes the space in which the alternatives are compared.

The third contribution concerns positioning. The findings support the view that CA-WLD is not a competitor to established MCDM methods intended to replace them, but a tool that complements them. Since the method does not duplicate the information produced by the MCDM set, it offers the decision maker an additional and distinct perspective.

6.3. Implications for practice

In terms of application, the main value of CA-WLD lies in its diagnostic nature. Compensatory MCDM methods can promote an alternative that excels in a single dominant criterion to top ranks despite its weaknesses in other criteria. Since CA-WLD evaluates such unbalanced profiles differently, it allows the decision maker to distinguish whether a high ranking of an alternative arises from a truly balanced superiority or from a concentration in a single dimension. Using both methods together provides a richer understanding of the robustness of results compared to a single ranking, as the Çorum application in Section 5 illustrates. The perturbation analysis in Section 4.8 turns this into a simple check: when the classical methods and CA-WLD disagree about the leader, the analyst asks whether its advantage is a matter of volume, whether its profile is more concentrated than those of its competitors, and whether its peak criterion carries a weight that would justify that concentration.

The method can be naturally applied to decision problems where the input is a contingency (frequency or preference) table. Problems such as site selection, supplier evaluation, and service positioning based on survey-based preference data, usage frequencies, or categorical count data fall within this scope. A

usage guide on the conditions that increase divergence is presented in Section 4.9.

6.4. Limitations and future research

The study has some limitations. First, in any given decision problem, the magnitude of the difference between CA-WLD and the MCDM methods cannot be reliably predicted in advance from correspondence analysis indicators such as the number of dimensions or explained inertia. It can be determined only after both approaches are computed and compared. This shows that, although the divergence is patterned at the group level, it remains difficult to predict at the individual-problem level.

Secondly, the low-dimensional representation of the correspondence analysis map can account for only a portion of the total inertia in large problems. Although in this study distances were calculated across all meaningful dimensions, this limitation should be taken into consideration when interpreting the two-dimensional visualization.

Third, the findings are based on a simulation design constructed upon specific factor levels (Dirichlet-based profile structures, three weighting schemes, and the selected dimensionality range). Generalization of the results to all real datasets should be considered with caution.

Fourth, CA-WLD can only be naturally applied to non-negative frequency or profile data. Cost-benefit differentiation, decision matrices containing mixed-scale criteria, or negative values require additional transformation. In addition, in tables with low frequencies or many zero cells, the correspondence analysis coordinates may become more sensitive.

Fifth, the rank-reversal analysis in Section 4.7 identifies a genuine limitation of Formulation A: its criterion-wise min-max normalization makes scores depend on the alternatives currently under consideration. Formulation B does not share this property, and when the correspondence analysis geometry is fixed by supplementary-point projection it is almost completely stable. Applications in which the alternative set is revised frequently should therefore rely on Formulation B.

Future research may proceed in several directions. Applications using real datasets from different fields can test the external validity of the method. Rank stability under weight changes can be examined systematically, complementing the alternative-set analysis reported here. Extending the method to fuzzy or interval-valued data, and comparing it directly with non-compensatory composite-indicator approaches [19], are also fruitful research directions.

7. Conclusions

This study has demonstrated that correspondence analysis is not merely a method for describing relationships between variables but can also be used for ranking and decision-making purposes through weighted proximity- and distance-based scoring formulations (CA-WLD) derived from the geometry of correspondence analysis. The proposed method is based on two complementary formulations, proximity-based and ideal-point-based, that transform alternative-criterion distances in the correspondence analysis space into a weighted decision score. The behaviour of the method was compared with nine established MCDM techniques within a full-factorial simulation design encompassing 20,242 valid contingency tables.

The findings showed that MCDM methods produced highly consistent rankings with each other, whereas CA-WLD provided a distinct ranking outside this consensus set (RQ1). The extent of this diver-

gence does not depend on a single factor, but rather on the interaction between the weighting method and the data structure. Weighting emerges as the most decisive factor (RQ2, RQ3). The effect of problem size is weak on its own, being conditional on the weighting scheme (RQ4). The underlying mechanism of the divergence is the method's sensitivity to imbalance and its relatively non-compensatory structure: unbalanced profiles, which concentrate performance in a single criterion, tend to be ranked lower. A non-compensatory max-min baseline, however, diverges from the compensatory consensus in the opposite direction and agrees with CA-WLD at only $\rho = 0.405$, which shows that the position occupied by the proposed method is not obtained by refusing compensation. A controlled perturbation analysis on 1,620 artificial base tables further shows that CA-WLD withholds first place from a classical leader whose lead rests on volume or on a concentration above that of its competitors, and aligns with the classical methods once that concentration falls, once the weight of its peak criterion rises to about 0.4, or once its competitors become comparably specialized; volume alone rarely produces alignment.

The main contribution of the study is that it proposes a clear formulation linking correspondence analysis to established MCDM families, and systematically maps, with evidence, the conditions under which this approach produces distinct information. CA-WLD is not a competitor replacing established techniques, but a complementary diagnostic tool that offers a non-compensatory, relation-geometry-based perspective. Decision makers, by evaluating compensatory MCDM rankings alongside CA-WLD rankings, can distinguish whether an option's superiority stems from balanced performance or from concentration in a single dimension. Testing the method with real datasets from different fields and extending it to fuzzy or interval-valued data present promising directions for future research.

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A. Primary data for the Çorum application

Table 11 reports the complete contingency table used in Section 5. Each cell gives the number of respondents who selected the corresponding site as the most suitable and the corresponding criterion as the most important ($n = 241$).

The AHP criterion weights used in Section 5 are, for C1 to C10 respectively, 0.054, 0.050, 0.079, 0.075, 0.110, 0.124, 0.126, 0.133, 0.135 and 0.115, with a consistency ratio of $CR = 0.04$. The AHP alternative priorities are a_1 0.124, a_2 0.102, a_3 0.148, a_4 0.141, a_5 0.133, a_6 0.114, a_7 0.122 and a_8 0.116.

Data availability. The contingency table analysed in the real-data application is reported in full above,

Table 11. Contingency table of candidate sites against evaluation criteria (8×10 , $n = 241$)

Site	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	Total
a_1	29	1	6	3	5	16	1	13	6	1	81
a_2	14	3	15	1	1	3	0	3	1	1	42
a_3	5	4	2	1	2	6	2	2	0	1	25
a_4	4	3	0	1	1	1	0	2	0	2	14
a_5	0	1	2	3	3	1	3	6	4	3	26
a_6	4	3	1	1	1	4	1	1	2	0	18
a_7	0	3	0	0	0	1	2	1	3	2	12
a_8	3	0	0	4	4	4	2	6	0	0	23
Total	59	18	26	14	17	36	11	34	16	10	241

Criteria. C1 proximity to the city centre; C2 vehicle access; C3 pedestrian access; C4 low traffic congestion; C5 parking capacity; C6 contribution to social and communal activities; C7 absence of noise and air pollution; C8 positive contribution to regional development; C9 protection of natural and cultural assets; C10 adequate size for a shopping mall.

together with the AHP criterion weights and alternative priorities. The simulation is fully reproducible from the fixed random seed (2026) reported in Section 3.5, and the analysis code is available from the author upon reasonable request.