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# A Facility-Integrated Flow Increment Problem for Emergency Evacuation under Budget Constraints

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**Abstract**

The designing of evacuation strategies is the process of making decisions concerning to the objectives that are multiple and, in most cases, conflicting to each other. This research work demonstrates a comprehensive framework of evacuation optimization that integrates multi-objective network flow concepts. The model put forward is the one that determines in a joint manner the most advantageous locations of the facilities as well as the flow assignments to realize not only the evacuation that is efficient but also balanced. To elevate network performance, a cost-aware contraflow strategy is presented which, by considering the switching and traffic management costs, along with the increasing capacity, reverses the arcs that are unused. The inclusion of these genuine constraints in the model provides a means to evaluate the compromises that exist between efficiency, cost, and the feasibility of operations during disaster response.

**Keywords:** *network flow, facility placement, flow maximization, resource limitation, reversal cost*

## 1. Introduction

Natural and human-induced disasters are basically changeable. Identifying suitable shelters and managing logistics during emergencies remains a demanding but complex task. Traffic dynamics within evacuation networks significantly influence the time span of evacuation. Thus, minimizing clearance time and flow loss has become a key concentration in evacuation research. Developing effective models and algorithms is challenging due to dynamic conditions and multiple constraints. The dynamic network flow theory of Ford and Fulkerson [10] provides the groundwork for time-dependent evacuation modeling, which has been extended and applied to address scenarios in buildings, urban areas, and large facilities (cf. [4]). A main challenge lies in mitigating congestion near shelters, where bottlenecks often delay movement. Hence, routing evacuees efficiently to safe destinations while considering capacity and operational limits is crucial.

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Fundamental flow models primarily aim to either maximize total flow or minimize overall cost. In two- terminal networks, the optimum dynamic flow can be efficiently computed in polynomial time using an equivalent static optimum-cost flow formulation given in [9]. Dynamic variants of network flow include the earliest arrival, quickest, lexicographically maximal, and quickest transshipment flows, each presenting different time-based and priority aspects. These time-dependent extensions support modern evacuation optimization, where continuous or discrete models govern feasible solutions, and continuous-time approximations are often based on discrete settings [8]. Most existing studies emphasize travel rather than evacuation costs (cf. [4]).

Location optimization is supported by mathematical models designed to balance coverage, approachability, and efficiency. Covering problem that selects sites to serve either all or maximum possible number of demand points. These concepts were applied by Jia et al. [14] in facility location models. Under uncertain demand, the pick-up location models [3, 18] determine optimal collection points and evacuee assignments to minimize system costs or optimum travel time. Rescue transfer location models [3] optimize the placement of intermediate rescue centers to reduce total travel cost. Shelter location models [16, 21, 23] apply hierarchical and multi-objective formulations to minimize vehicle hours and total evacuation time, linking upper-level shelter decisions with lower-level traffic assignment problems. FlowLoc models combine facility location and flow optimization, analyzing the impact of facility placement along road segments on the network [12, 13]. For a more realistic representation of evacuation scenarios, dynamic flow models account for time-variant as well as capacity limitations [20]. Network flow-based methods have established a strong capability in managing large-scale evacuation systems. Consequently, the allocation of facilities in evacuation networks directly influences flow optimality and the effectiveness of emergency responses.

Contraflow strategy reverses the direction of road lanes to enhance outbound capacity. The idea is introduced by Kim and Shekhar [15] as an integer programming problem. They have established the $\mathcal{NP}$-hardness of minimizing evacuation time. In symmetric two-terminal networks, contraflow can effectively double the flow value. Though the optimum dynamic contraflow problem can be solved in polynomial time for networks with two terminals. It becomes $\mathcal{NP}$-hard for a multi-terminal network [22].

The concepts of abstract flow and lane reversals can be integrated in an abstract contraflow setting to optimize evacuation operations. Efficient algorithms to maximize evacuees and reduce crossing conflicts have been discussed (cf. [5]). Facility locations may be treated as fixed or flexible; the former applies to predefined setups, while the latter allows adaptive placement. The two-terminal FlowLoc problem in dynamic networks, along with its static and dynamic contraflow extensions, is analyzed in [6]. Implementing all optimal arc reversals may be infeasible when resources are limited. In this context, contraflow problems under budget limitation have been studied. Only a fraction of arcs can be reversed. Practical algorithms have been designed to achieve optimum dynamic flow and contraflow enhancement in both static and time-dependent networks, [7]. For real-world emergency planning, they consider arc reversal expenses and total budget constraints, providing an applicable tool.

**Research Gap:** Network flow models with maximizing flow or minimizing cost have been widely studied in the literature. Contraflow strategy, which increases capacity by reversing underused arcs to increase the optimality. The facility location strategies, which reduce flow loss, have also been investigated in-

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dependently. However, allocating facilities in networks with reversible arcs while considering reversal costs remains uncovered. In evacuation optimization, determining optimal facility sites and maximizing evacuee throughput under resource constraints are critical. Yet a unified framework integrating facility placement, contraflow operations, and budget limits is still lacking.

**Research Contribution:** This research introduces a unified framework that combines facility location, contraflow management, and budget considerations within dynamic network flow models. It treats these components independently and simultaneously incorporates arc reversibility and switching costs to improve evacuation efficiency under resource limitations based on the theoretical formulations of the flow maximization and cost minimization. Based on several flow enhancements strategies, the multi-objective formulation was converted to single-objective budget-constrained problems within ContFlowLoc to optimize flow distribution and determine the suitable facility locations. We formulate and solve the optimum dynamic flow and contraflow enhancement with budget-limited arc reversals, presenting efficient algorithmic solutions applicable to both networks setting, static and dynamic. By optimizing facility placement alongside cost-aware contraflow strategies, the framework provides a practical and computationally effective tool for large-scale evacuation planning.

Section 2 introduces the key notations and the problem formulations on network flow, FlowLoc models, and contraflow issues. Section 3 examines the solution strategies on the flow increment and budget-constrained flow increment problems, including variants that incorporate switching costs. Section 4 introduces an integrated solution model that combines network flow, facility location, contraflow, and arc switching under budget-constrained switching costs. It also outlines the proposed algorithm and discusses its theoretical foundations. Finally, Section 5 concludes it.

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## 2. Problem Formulations on Network Flow

Consider a dynamic network $N = (V, A, u_e(\tau), \tau_e, T)$, where $V = V_+ \cup V_- \cup V_0$, with $V_+$, $V_-$, and $V_0$ representing sources, sinks, and transshipment nodes, respectively. In the two-terminal scenario, we set $V_+ = \{s\}$ and $V_- = \{t\}$ with $S = V_+ \cup V_-$. The evacuation period is $\mathcal{T} = \{0, 1, \dots, T\}$ and $\mathbf{T}_c = \{[0, 1), [1, 2), \dots, [T, T+1)\}$ for discrete-time and continuous-time models, respectively. Each arc $a \in A$ has an associated transit time $\tau : A \to \mathbb{R}_{\geq 0}$ and a capacity function $u : A \times \mathcal{T} \to \mathbb{R}_{\geq 0}$, limiting the optimum flow entering the arc at a given time. A dynamic flow is a function $h : A \times \mathcal{T} \to \mathbb{R}_{\geq 0}$, where $h_e(t)$ denotes the flow entering $e = (v, w)$ at time $t$, which arrives at node $w$ at time $t + \tau_e$. The resulting flow excess at a node induced by the arc flows is represented by $f : V \times \mathcal{T} \to \mathbb{R}_{\geq 0}$. A directed static network is obtained by ignoring the temporal aspects of the dynamic network. For notional consistency, the symbols $f$ and $h$ are used in both dynamic and static contexts, with their meaning dynamic or static, determined by the context with or without time.

## 2.1. Flow Optimization

In the discrete-time framework, the dynamic flow problem is characterized by Constraints (1)-(4).

$$\sum_{e\in A_+^v,\, t\geq 0} h_e(t) - \sum_{e\in A_-^v,\, t-\tau_e\geq 0} h_e(t-\tau_e) = \begin{cases} f_v(t), & v\in V_+ \\ 0, & v\in V_0, \quad \forall t\in\mathcal{T} \\ -f_v(t), & v\in V_- \end{cases} \tag{1}$$

$$f_v(t) \geq 0 \quad \forall v\in V,\; t\in\mathcal{T} \tag{2}$$

$$0\leq h_e(t) \leq u_e(t) \quad \forall a\in A,\; t\in\mathcal{T} \tag{3}$$

$$h_e(t) = 0, \quad \forall e\in A,\; t\in\{T-\tau_e+1,\ldots,T\} \tag{4}$$

where $\forall v\in V$, $A_+^v = \{(v,w)\in A \mid w\in V\}$ and $A_-^v = \{(u,v)\in A \mid u\in V\}$ denote the set of arcs leaving and entering $v$, respectively. Constraint (1) enforces flow conservation, (2) ensures non-negativity of supplies, (3) bounds flows by arc capacities, and (4) restricts infeasible late departures. An optimum dynamic flow problem is to optimize the total flow reaching the sink nodes by the end of the planning horizon $T$,

$$\max \sum_{t=0}^{T} \sum_{v\in V_+} f_v(t). \tag{5}$$

To handle a multi-terminal single-commodity network, one may construct an equivalent two-terminal formulation by appending a virtual source and sink. The virtual source is connected to all original sources, and all sinks are connected to the virtual sink, via zero-transit arcs constrained by the terminal capacities. In a static network, $\tau_e = 0$ for every $e\in A$ and $T = 0$.

The time-expanded network is a static representation derived from a dynamic network. It is obtained by creating time-indexed copies of the original network topology and connecting them according to travel times and capacities. Let $N = (V, A, u(\tau), \tau_e, T)$ be a dynamic network over the time horizon $\mathcal{T} = \{0, 1, \ldots, T\}$. The corresponding time-expanded network is constructed in the discrete-time setting as follows. The vertex set is defined by $V_T := \{\, v(\tau) : v\in V,\ \tau\in\mathcal{T}\,\}$. The arc set consists of two types of arcs. The first type represents movement along original arcs:

$$A_T := \{\, (v(\theta), w(\theta+\tau_e)) : e = (v,w)\in A,\ 0\leq\theta\leq T-\tau_e \,\}.$$

The second type represents holdover (waiting) arcs:

$$A_T^H := \{\, (v(\theta), v(\theta+1)) : v\in V,\ 0\leq\theta\leq T-1 \,\}.$$

The capacity of each movement arc $(v(\theta), w(\theta+\tau_e))\in A_T$ is given by the time-dependent capacity $u_e(\tau)$ of the corresponding dynamic arc. Each holdover arc in $A_T^H$ is assigned a sufficiently large (typically infinite) capacity to avoid restricting flow that remains at a node. A similar construction applies in the continuous-time setting by introducing time-indexed node copies over continuous intervals. In practice, a discrete-time time-expanded network can be obtained by selecting representative time points,

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such as the lower bounds of the corresponding time intervals. This transformation provides a static representation that is equivalent to the underlying dynamic network flow. In the maximum flow problem, the objective is to determine the greatest amount of flow that can be transmitted from the source to the sink within a prescribed time horizon, subject to the capacity constraints of the arcs. To incorporate temporal considerations, the transit times of arcs may be interpreted as costs, [9].

In contrast, the minimum cost flow problem assumes that the amount of flow to be sent is specified a priori, and seeks to minimize the total transportation cost while satisfying capacity and flow conservation constraints. This correspondence illustrates the fundamental relationship between the maximum flow and minimum cost flow problems. A formal presentation of the minimum cost flow problem is provided in the subsequent section.

## 2.2. Cost-optimized Flow

Let $d : V \to \mathbb{Z}$ be the supply-demand assignment, $d_v > 0$ for supply nodes and $d_v < 0$ for demand nodes, and let $c : A \to \mathbb{Z}$ denote arc costs. The cost-optimized (minimum-cost) flow problem [9] is:

$$\min \sum_{a \in A} c_e h_e \tag{6}$$

$$\text{s.t.} \sum_{e \in A_+^v} h_e - \sum_{e \in A_-^v} h_e = d_v, \quad \forall v \in V \tag{7}$$

$$0 \leq h_e \leq u_e, \quad \forall e \in A \tag{8}$$

$$\sum_{v \in V} d_v = 0. \tag{9}$$

Feasibility can be checked using a optimize flow approach [2, 9]: a super-source $s^*$ is added, connecting to each $v \in V_+$ with arcs of capacity $d_v$, while a super-sink $t^*$ is introduced, receiving arcs of capacity $-d_v$ from every $v \in V_-$. Given that $\sum_{v \in V_+} d_v = -\sum_{v \in V_-} d_v$, the network is feasible exactly when every arc out of $s^*$ is saturated by the optimize flow.

## 2.3. FlowLoc Problem

Let $\mathbb{L} \subseteq A$ be the collection of eligible arcs for facility placement, and let $\mathbb{P}$ denote the set of available facilities. Each facility $p \in \mathbb{P}$ has a specified size given by the function $r : \mathbb{P} \to \mathbb{N}$, while $\text{nol} : \mathbb{L} \to \mathbb{N}$ defines the maximum number of facilities that can be assigned to each location. The FlowLoc model determines an assignment $loc : \mathbb{P} \to \mathbb{L}$ that positions facilities on arcs to achieve the greatest possible $s$-$t$ flow in the modified network $N^{\mathbb{L}} = (V, A, u', s, t)$, where the updated capacity on each arc becomes, $u'_e = u_e - \max\{r_p : loc(p) = e\}$. Alternative formulations handling multiple facilities on a single arc can be found in [12] and is applicable in evacuation planning for emergency shelters. The multi-facility FlowLoc variant focuses on distributing $q$ facilities from $\mathbb{P}$, each with size $r_p$, across the feasible arcs to minimize the total loss in the optimal network flow, while ensuring that no arc $l \in \mathbb{L}$ hosts more than $\text{nol}(l)$ facilities. The Single-FlowLoc case, where $q = 1$, involves locating a single facility from the available set.

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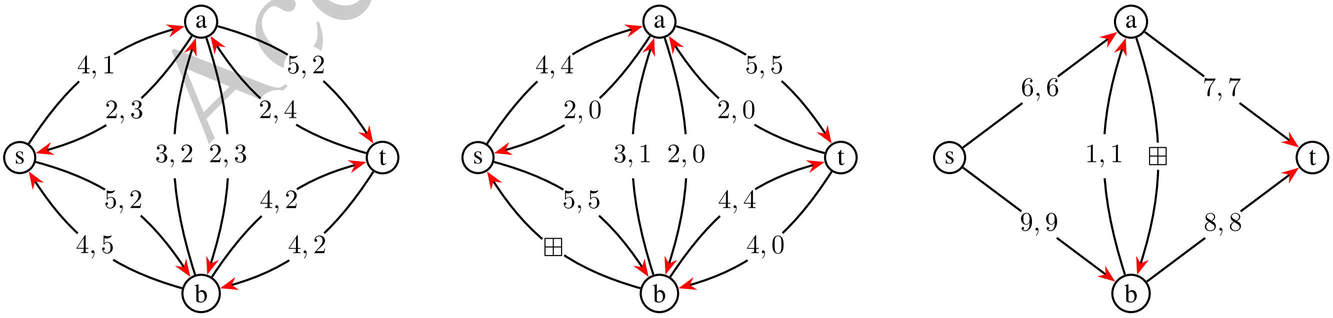
The placement of emergency units or supporting facilities significantly affects network performance, influencing both optimum flow and evacuation duration. The multi-terminal $q$-FlowLoc problem is $\mathcal{NP}$-complete [13], heuristic solutions have been proposed. Recent extensions of FlowLoc include the quickest FlowLoc, solvable in polynomial time for a single facility, while mixed-integer programming formulations and heuristics have been developed for multi-facility scenarios. Integration with contraflow strategies has led to the development of the ContFlowLoc models, which have been successfully implemented and solved [20].

**Problem 1.** The optimum FlowLoc problem for a static network $N = (V, A, u, s, t)$ involves placing facilities at feasible locations within the network in a way that maximizes the static flow in the modified network $N^{\mathbb{L}} = (V, A, u', s, t)$.

```
Require: $N = (V, A, u, s, t)$, locations $\mathbb{L}$, size $r_p$ of facility $p$
Ensure: opt_flow, loc$(p)$
Set: opt_flow $:= -1$
for all $l \in \mathbb{L}$ do
    if $u_l \geq r_p$ then
        $u_l = u_l - r_p$
        opt_flow_cu $= v($opt_flow$(N = (V, A, u, s, t)))$
        $u_l = u_l + r_p$
        if opt_flow $<$ opt_flow_cu then
            opt_flow $=$ opt_flow_cu
            loc$(p) = l$
        end if
    end if
end for
return Optimum flow value opt_flow, location loc$(p)$ in $N$.
```

**Algorithm 1.** Optimum Static FlowLoc

**Theorem 1.** The optimal solution to the FlowLoc problem can be obtained in $\mathcal{O}(|\mathbb{L}|\, nm)$ time.



(i) (ii) (iii)

**Figure 1.** (i) Static network with (u, c), (ii) optimized FlowLoc solution with (u,h), (iii) ContFlowLoc without cost.

**Example 1.** Consider the static network Figure 1 (i), where each arc is annotated with its capacity and switching cost. Let nol $= 1$ and $r_p = 4$; then the feasible location set determined by capacity is $\mathbb{L} = \{e \in A : u_e \geq 4\} = \{(s, a), (a, s), (s, b), (a, t), (b, t), (t, b)\}$. Before placing any facility, the network can deliver a total of 9 units of flow through $s \to a \to t$, $s \to b \to a \to t$, and $s \to b \to t$, carrying $4$, $1$, and $4$ units, respectively. Choosing either $(b, s)$ or $(t, b)$ as the facility location does not decrease the current flow, so the total flow remains the same. It should be noted, however, that this invariance is specific to this network; in general, placing a facility on other feasible arcs may decrease the total flow. The optimal FlowLoc solution for this instance corresponds to placing the facility on $(b, s)$ (or $(t, b)$), achieving the optimum flow of $9$ units, as depicted in Figure 1 (iii).

# 3. Solution Strategies for the Flow Increment

## 3.1. Optimum Flow Increment

Let each arc $e \in A$ be associated with an additional non-negative upper capacity $U'_e$, allowing its base capacity $u_e$ to be increased at a non-negative unit cost $c_e$, subject to $U'_e \geq u_e$. The capacity enhancement becomes $I : A \to \mathbb{Q}_{\geq 0}$. The goal is to maximize the $s - t$ flow by increasing arc capacities subject to a budget constraint, where the cost is proportional to the added capacity. The flow enhancement problem has been introduced and discussed the status of problem in [17].

The optimum static flow enhancement under a continuous augmentation scheme, where each arc enhancement $I(a)$ may assume any rational value up to $U_e$, is solvable in polynomial time. When enhancements are restricted to integers, the problem reduces to a budget-constrained minimum-cost flow formulation, which is also polynomially tractable. Let $b_e \geq 0$ denote the unit cost of enlarging the capacity of arc $e \in A$. The optimal increase on arc $e$ is then $I^*_e = \max\{0,\, h^*_e - u_e\}$, $h^*$ represents the optimal flow after augmentation; any other assignment would lead to excess cost. This behavior is captured by the flow-dependent cost function $c_e(h)$ defined as

$$c_e(h) = \begin{cases} 0, & 0 \leq h \leq u_e, \\ b_e \cdot (h - u_e), & u_e < h \leq U'_e, \end{cases} \qquad \forall e \in A. \tag{10}$$

By definition, $c_e$ is piecewise-linear and convex. To linearize it, each $e \in A$ is replaced by two parallel arcs, $e_0$ and $e_1$, with capacities and costs

$$\begin{aligned} \overline{u}_{e_0} &:= u_e, & \overline{c}_{e_0} &:= 0, \\ \overline{u}_{e_1} &:= U'_e - u_e, & \overline{c}_{e_1} &:= b_e. \end{aligned} \tag{11}$$

This formulation arises from the convex nature of $c_e$. Therefore, the enhanced optimal flow value $H^*_{st}$ is constrained by a total cost not exceeding $B$, using a binary search over the range $[0, nU'_{\max}]$.

The optimal static flow increment problem produces several important algorithmic and computational outcomes. Its integral form can be resolved in $O(\log(nU'_{\max}))$ where $U'_{\max} = \max\{U'_e : e \in A\}$. For any precision level $\epsilon > 0$, a $(1+\epsilon)$-approximate solution can be obtained through $O(\log\log_{1+\epsilon}(nU'_{\max}))$, each applied to a network with $O(gm)$ arcs, where $g$ represents the optimal number of breakpoints in the piece wise-linear cost functions. Megiddo's parametric search method [19] provides strongly polynomial algorithms, which extend naturally to the rational version of the problem. In contrast, the optimal static flow increment problem, where each arc is either maintained at its initial capacity or upgraded to its

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upper limit, is $\mathcal{NP}$-hard. Its decision form is $\mathcal{NP}$-complete even for series-parallel and bipartite graphs. Nevertheless, for series-parallel networks, both the fixed-cost optimal flow problem and the optimal static flow increment problem admit the approximation scheme at polynomial-time based on scaling techniques. The resulting optimal enhanced flow is denoted by $h^*$, with corresponding arc increments given by $I_e^* = \max\{0, h_e^* - u_e\}\ \forall\, e \in A$.

The optimum static flow enhancement problem extends to the optimum dynamic flow enhancement by introducing the time dimension. Let $I(t)$ and $b(t)$ represent the enhancement function and the corresponding unit cost at time $t$, respectively, then such flow increment can be expressed as

$$\max \sum_{t=0}^{T} \sum_{v \in V_+} f_v(t) \tag{12}$$

$$\text{s.t.} \sum_{e \in A_v^+} h_e(t) - \sum_{\substack{e \in A_v^- \\ t-\tau_e \geq 0}} h_e(t-\tau_e) = \begin{cases} f_v(t), & v \in V_+ \\ 0, & v \in V_0,\ \forall t \in \mathcal{T} \\ -f_v(t), & v \in V_- \end{cases} \tag{13}$$

$$f_v(t) \geq 0,\ \forall v \in V,\ \forall t \in \mathcal{T} \tag{14}$$

$$I_e(t) \geq 0,\ \forall e \in A,\ \forall t \in \mathcal{T} \tag{15}$$

$$h_e(t) = 0,\ \forall e \in A,\ t \in \{T - \tau_e + 1, \dots, T\} \tag{16}$$

$$u_e(t) + I_e(t) \geq h_e(t) \geq 0,\ \forall e \in A,\ \forall t \in \mathcal{T} \tag{17}$$

$$U_e'(t) \geq u_e(t) + I_e(t) \geq 0,\ \forall e \in A,\ \forall t \in \mathcal{T} \tag{18}$$

$$\sum_{t=0}^{T} \sum_{e \in A} I_e(t) b_e(t) \leq B \tag{19}$$

Here, Constraint (19) bounds the total enhancement cost, Constraint (18) ensures that the capacity increase does not exceed the optimum permissible enhancement, and Constraint (17) guarantees flow feasibility with the improved capacities.

**Theorem 2.** The optimization of continuous as well as integral flow augmentation in dynamic networks is achievable within pseudo-polynomial time [5, 7].

In the model (12)-(19), dynamic enhancements and their associated costs are transformed into static terms by using time-expanded network. When both the augmentation values and costs remain time-invariant, each replica of an arc in the transformed network preserves the same augmented capacity, while the cost associated with its enhancement is considered only once.

## 3.2. Flow Increment with Switching Costs on Arcs

The static contraflow optimization problem, originally examined in [22], focuses on maximizing the total $s - t$ flow by reversing certain arcs, neglecting reversal costs. Later, [7] extended this framework by introducing a budget constraint and proposing a specialized method for two-terminal networks. The switching-cost contraflow problem involves $N = (V, A, u_e, s, t)$ in which arc reversals are permitted but incur specific switching costs. This formulation closely relates to the minimum concave-cost network flow model [11], where the cost on each arc comprises a fixed component triggered by activation and a

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variable component depending on the flow magnitude. If the variable cost term is omitted, the problem reduces to the minimum-cost fixed-flow. Moreover, capacity enhancement is modeled as a binary decision indicating whether additional capacity is applied to a given arc. In this work, we focus exclusively on integral capacity augmentation strategies.

**Theorem 3.** When all reversed arc capacities are integral, the contraflow with switching costs problem admits an optimal solution computable in polynomial time [7].

The algorithm addressing the contraflow problem with switching costs operates on arcs with integral capacities, ensuring that any resulting flow enhancements are also integral. As a result, the optimal flow solution for the problem is guaranteed to be integral, which allows the search procedure to consider only integer flow values within the relevant range. Rather than using a conventional binary search, one can employ Megiddo's parametric search method, [19].

Specifically, the optimum feasible value $H'_{st}$ under the budget constraint is selected from the set $\{1,\, 1+\epsilon,\, \ldots,\, (1+\epsilon)^k\}, \quad k = \lceil \log_{1+\epsilon}(nU_{\max}) \rceil$, where $\epsilon > 0$ is a predetermined accuracy parameter, and $U_{\max} = \max\{U_e : e \in \tilde{A}\}$. This adapted search guarantees a solution satisfying $H'_{st} \geq \frac{H^*_{st}}{1+\epsilon}$. Incorporating this modification produces a $(1+\epsilon)$-approximation algorithm for the budget-constrained optimum contraflow problem, requiring $\big(\log \log_{1+\epsilon}(nU_{\max})\big)$ minimum-cost flow computations.

**Theorem 4.** When all input parameters are positive integers, the contraflow problem with a fixed switching cost is equivalent to the $0/1$ optimum static flow increment problem [22].

The contraflow with fixed switching costs problem is $\mathcal{NP}$-hard, as it is like $0/1$ optimum flow enhancement problem. Even in static networks, fixed costs for arc reversals render the problem computationally intractable, and conventional static contraflow algorithms are unable to make informed reversal decisions, limiting their effectiveness. Incorporating a temporal dimension further increases the complexity, while contraflow with fixed switching costs remains $\mathcal{NP}$-hard even on series-parallel setting. Nonetheless, an approximation solution exists at polynomial-time for this class of networks. Approximate solutions can be obtained using a cost-per-unit-capacity approach, assigning each arc a capacity $u_e$ and a unit cost $c_e/u_e$, thereby enabling standard approximation techniques [2, 17].

The notion of optimal dynamic contraflow was first explored in [22], where arc reversals were permitted at the initial time without any associated cost. When reversal expenses are introduced, however, the problem shifts to maximizing the total dynamic contraflow under a specified budget limit. This formulation serves as a natural extension of the optimal static contraflow problem to time-dependent networks [7]. The dynamic contraflow problem with switching costs can thus be viewed as a modified form of the optimal dynamic flow problem, allowing arc reversals that incur individual costs while satisfying an overall budget constraint.

**Theorem 5.** A pseudo-polynomial time algorithm exists for computing an optimal solution to the optimum dynamic contraflow problem when switching costs are considered [7].

## 4. Solution Strategies on ContFlowLoc Problems

The contraflow paradigm improves overall network throughput by redirecting arcs toward sink nodes, as flows leading back to sources are ineffective in emergency contexts. Initially proposed without consider-

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ing reversal costs in [15] and subsequently studied in [22], the standard formulation presumes that each arc is either completely reversed with its original capacity or left intact. In real-world applications, however, reversing an arc may involve switching expenses. Different contraflow variants and corresponding solution techniques that incorporate such costs are examined in this work. Figure 1 (i) (summed capacity of the parallel arcs) shows the contraflow procedure in a network setting where no reversal costs on arcs are incorporated. For $N = (V, A, u_e, s, t)$, transformation constructs its auxiliary $\tilde{N} = (V, \tilde{A}, \tilde{u}_e, s, t)$, where the arc set $\tilde{A}$ contains an element $\tilde{e}$ whenever either $(i, j) \in A$ or $(j, i) \in A$. The corresponding capacity and travel time functions are defined in a symmetric manner as $\tilde{u}_{\tilde{ij}} := u_{ij} + u_{ji}$. All remaining parameters of the original network are preserved in the auxiliary construction. This section presents the formulation ContFlowLoc problem and provides efficient algorithmic approach for its solution. The developed optimum ContFlowLoc algorithm achieve the same computational complexity as their corresponding optimum FlowLoc algorithms, although the achievable flow value may increase after applying contraflow reconfiguration.

## 4.1. ContFlowLoc Problems without Switching Costs

Here, FlowLoc problems with predefined feasible facility locations are presented. A variant of the FlowLoc problem in which a location is considered feasible only if its available size meets or exceeds the corresponding arc capacity is addressed. The model is analyzed under two settings, with and without the allowance of lane reversals and efficient algorithms are developed for both. In this formulation, applying contraflow adjustments expands the set of feasible facility locations by altering the arc capacities.

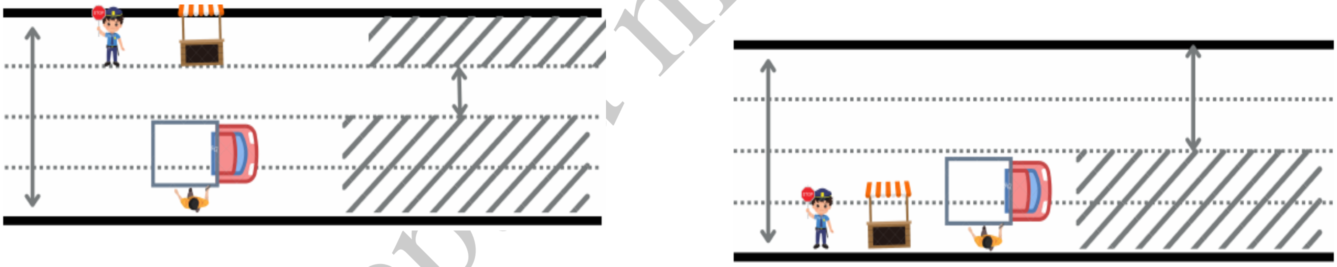

**Figure 2.** Comparison of facility arrangement strategies for Opposite-side placement and Same-side placement, respectively

If a facility is placed on $(i, j)$ and the optimum contraflow exceeds $u'(i, j)$, the reverse arc $(j, i)$ is to reoriented. When the facility's capacity is greater than those of both arcs, the problem simplifies to a single-arc case, allowing placement on either side. If only one arc can accommodate the facility while its reverse cannot, the configuration becomes asymmetric, corresponding to either a forward or backward placement depending on the feasible direction [6]. Figure 2 illustrates an example of facility placement, showing an incorrect and its corresponding correct location, respectively.

**Problem 2.** Given $N = (V, A, u, s, t)$ and a set of feasible locations $\mathbb{L} = \{e \in A \mid u_e \geq r_p\}$, where $r_p$ represents the size of a facility $p$, the objective of the optimum static ContFlowLoc problem is to determine the maximum achievable flow in its modified network $N^{\mathbb{L}} = (V, A, u', s, t)$, allowing arc reversals where beneficial without any reversal costs.

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Require: $N = (V, A, u, s, t)$; facility size $r_p$
Ensure: Optimum contraflow value opt_cont and facility location loc($p$)
Initialize feasible location set: $\mathbb{L} \leftarrow \{e \in A \mid u(e) \geq r_p\}$
Transform $N$ into $\tilde{N} = (V, \tilde{A}, \tilde{u}, s, t)$
$\tilde{V} = V, \tilde{A} = \phi$
for all adjacent node pair $\{i, j\} \in N$ do
    $\tilde{A} \leftarrow \tilde{A} \cup \{(i, j), (j, i)\}$
    $\tilde{u}_{ij} = \tilde{u}_{ji} \leftarrow u_{ij} + u_{ji}$ {Treat missing arcs as having 0 capacity}
end for
Define $\tilde{\mathbb{L}} \leftarrow \{\tilde{e} \in \tilde{A} \mid \tilde{u}(\tilde{e}) \geq r_p\}$
Execute Algorithm 1 on $\tilde{N}$ considering $\tilde{\mathbb{L}}$
Decompose the resulting optimum flow into chain/cycle flows
Ignore all cycle flows
for all $(i, j) \in A$ do
    if $h_{ij} > u_{ij}$ or ($h_{ij} \geq 0$ and $(i, j) \notin A$) then
        Reverse arc $(j, i)$
    end if
end for
if facility is located on $(i, j) \in \tilde{A}$ and $h_{ij} > 0$ then
    Allocate facility along $(j, i)$
end if
return (opt_cont, loc($p$))
```

**Algorithm 2. Optimum ContFlowLoc without Switching Costs**

**Theorem 6.** The extended ContFlowLoc problem can be solved to optimality in $\mathcal{O}(|\tilde{\mathbb{L}}|nm)$ time, as established in [6].

**Example 2.** Let $r_p = 4$ in Figure 1(i) Then, $\tilde{\mathbb{L}} = \{e \in \tilde{A} : r(e) \geq 4\} = \tilde{A}$.The optimal solution of the ContFlowLoc problem, derived using Algorithm 2, achieves a total maximum flow of 15 units via the routes $s-a-t$, $s-b-t$, and $s-b-a-t$, carrying 6, 8, and 1 units of flow, respectively, with the facility positioned at $(a, b)$. Alternatively, the location $(b, a)$ can also serve as a feasible site if arc $(a, b)$ is reversed with capacity 2, while capacity 1 remains in the original direction. In this configuration, the facility would be positioned on the same direction of flow on arc. Hence, the location $(a, b)$ is adopted, aligning with the opposite direction of the optimized flow [6] on that adjacency pair. In this instance, the total flow increases by approximately 67% compared to the case without contraflow implementation. The optimal solution is shown in Figure 1 (iii).

## 4.2. Optimum ContFlowLoc with Switching Costs

The optimal ContFlowLoc problem under capacity-constrained locations unifies the objectives of maximum flow, contraflow reconfiguration, and facility location optimization within a single framework. In practical applications, however, reversing arcs is often costly such as in traffic management and related systems and therefore cannot be neglected. Since the problem already involves flow maximization, optimal facility allocation, and contraflow decisions, it constitutes a multicriteria optimization problem. Incorporating the cost of flow increment further places it within the $\mathcal{NP}$ class. In this research, the cost function is embedded within a budget constraint to determine the optimal facility location in a given net-

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work, thereby transforming the model into a single-objective optimization problem. This section presents the corresponding model, algorithm, theoretical justification, and illustrative example. The formulation integrates the budget-constrained arc-switching framework from [7] with the ContFlowLoc problem described in [6].

**Problem 3.** For $N = (V, A, u, s, t)$ having arc capacities $u_e$ and switching (reversal) costs $c_e$, consider a set of feasible locations $\mathbb{L} = \{e \in A \mid u_e \geq r_p \text{ and } b_e \leq B\}$, $r_p$ represents the size of facility $p$. The objective of the optimum ContFlowLoc problem with capacity-driven locations and budget-limited switching is to identify the optimal placement of facility $p$ and the corresponding set of arc reversals that maximize the total flow value in $N^{\mathbb{L}} = (V, A, u', s, t)$, subject to the total reversal cost not exceeding a given budget constraint.

**Theorem 7.** Consider $N = (V, A, u, s, t)$ with capacities $u_e \geq 0$, switching costs $b_e$, total budget $B > 0$, and a facility of size $r_p > 0$. Algorithm 3 yields a feasible and budget-admissible solution that is optimal for the optimum ContFlowLoc problem under the given budget constraint.

**Proof.** Feasibility follows through the transformed auxiliary network $\tilde{N} = (V, \tilde{A}, \tilde{u}, s, t)$, where every arc $\tilde{e} \in \tilde{A}$ satisfies $\tilde{u}_{\tilde{e}} > 0$ and the updated capacity $\tilde{u}'_{\tilde{e}} \leq \tilde{u}_{\tilde{e}}$ after facility placement. For any location $\tilde{e} \in \tilde{\mathbb{L}}$, the algorithm extracts a feasible flow in $\tilde{N}'$ using a bisection search on the interval $[0, |V| \cdot U_{\text{max}}]$, ensuring that the total switching cost $C_{\text{total}} = \sum_{e \in \tilde{A}} b_e h_e$ never exceeds $B$. Cycle flows are removed without affecting feasibility, since they do not contribute to net flow at the terminals.

Optimality holds because, for each fixed location $\tilde{e}$, the algorithm performs an exact minimum-cost flow computation at each candidate flow level, and the bisection method ensures that the optimum feasible flow respecting $C_{\text{total}} \leq B$ is found. By iterating over all feasible locations $\tilde{\mathbb{L}}$, the algorithm identifies the placement $\text{loc}(p)$ and the corresponding flow $h^*$ that maximize the total contraflow value opt_cont. Since no further feasible reorientation of arcs can yield a higher flow without violating the budget constraint, the obtained solution is globally optimal for the given instance. □

**Theorem 8.** An optimal solution to the problem ContFlowLoc incorporating switching costs under budget constraint can be achieved by Algorithm 3 within $O\big(m^3(\log n)\log(nU_{\text{max}})\big)$ time.

**Proof.** The auxiliary network is formed by scanning each adjacent node pair $\{i, j\}$ in the given network and inserting both $(i, j)$ and $(j, i)$ into $\tilde{A}$. Since each pair of adjacent nodes is examined exactly once, the total number of processed arcs is at most $2m$. Assigning the corresponding combined capacities $\tilde{u}_{ij} = \tilde{u}_{ji} = u_{ij} + u_{ji}$ and initializing missing arcs with zero capacity both require constant time per pair. Thus, the total work is linear in the size of the input network, $O(n + m)$.

The algorithm applies bisection on the flow level in the interval $[0, nU_{\text{max}}]$ which needs at most $O\big(\log(nU_{\text{max}})\big)$ iterations [17]. Each bisection iteration tests a single flow value and invokes one minimum cost flow (MCF) solve on the graph $\tilde{N}'$. For a fixed candidate location $\tilde{e}$ and a single bisection step we perform: one MCF solve at $T_{\text{MCF}}(n, m) = O(m^2 \log n)$, [1] and update of switching costs and simple feasibility checks in $O(m)$. Thus the time per iteration is $O\big(T_{\text{MCF}}(n, m) + m\big) = O\big(m^2 \log n + m\big) = O(m^2 \log n)$, since $m^2 \log n$ dominates $m$ for $m \geq 2$.

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Require: $N = (V, A, u, s, t)$; size of facility $r_p$; switching cost $b_e$ for each $e \in A$; total budget $B$
Ensure: Optimum contraflow value opt_cont and optimal facility location loc$(p)$
Initialize feasible location set $\mathbb{L} \leftarrow \{e \in A \mid u_e \geq r_p\}$
Transform $N$ into $\tilde{N} = (V, \tilde{A}, \tilde{u}, s, t)$:
$\tilde{V} = V, \tilde{A} = \phi$
for each adjacent node pair $\{i, j\}$ in $N$ do
    $\tilde{A} \leftarrow \tilde{A} \cup \{(i, j), (j, i)\}$
    $\tilde{u}_{ij} = \tilde{u}_{ji} \leftarrow u_{ij} + u_{ji}$ {Treat missing arcs as zero-capacity}
end for
Compute $U_{\max} \leftarrow \max\{\tilde{u}_e \mid e \in \tilde{A}\}$
Consider switching cost function of Equation1 and linearized it (as in Equation 11) for reversed arcs
Define feasible location set $\tilde{\mathbb{L}} \leftarrow \{e \in \tilde{A} \mid \tilde{u}(e) \geq r_p \;\&\; c_e \leq B\}$
Initialize $C_{\text{total}} \leftarrow 0$, opt_cont $\leftarrow 0$, loc$(p) \leftarrow \emptyset$
for each location $e \in \tilde{\mathbb{L}}$ do
    Place facility on $\tilde{e}$ and define updated capacities:
        $$\tilde{u}'_e = \begin{cases} \tilde{u}_e - r_p, & \text{if } e = \tilde{e}, \\ \tilde{u}_e, & \text{otherwise.} \end{cases}$$
    Define $\tilde{N}' = (V, \tilde{A}, \tilde{u}', s, t)$
    Apply bisection search in interval $[0, |V| \cdot U_{\max}]$ to determine feasible flow levels
    while $C_{\text{total}} \leq B$ do
        Solve a optimum-cost flow problem in $\tilde{N}'$ for the given flow level
        Update $C_{\text{total}} \leftarrow \sum_{e \in \tilde{A}} c_e \cdot x_e$
        if $C_{\text{total}} \leq B$ and obtained flow $f$ increases total flow then
            opt_cont $\leftarrow |f|$
            loc$(p) \leftarrow \tilde{e}$
            Store $h^* \leftarrow h$ as current optimal flow
        else
            Exit inner loop when no further feasible flow increment is possible
        end if
    end while
end for
Decompose the resulting flow cycle/chain flows, ignore cycle flows
for each arc $(i, j) \in A$ do
    if $h^*_{ij} > u_{ij}$ or $h^*_{ij} > 0$ and $(i, j) \notin A$ then
        Reverse arc $(j, i)$ to $(i, j)$ and set $u_{ij} \leftarrow h^*_{ij}$ and $u_{ji} = u_{ji} - (h^*_{ij} - u_{ij})$
    end if
end for
if facility is located on $(i, j) \in \tilde{A}$ and $h_{ij} > 0$ then
    Allocate facility along $(j, i)$
end if
Allocate facility on location loc$(p)$ in $N$
return (opt_cont, loc$(p)$)
```

**Algorithm 3. Optimum ContFlowLoc with Switching Costs under Budget Constraint**

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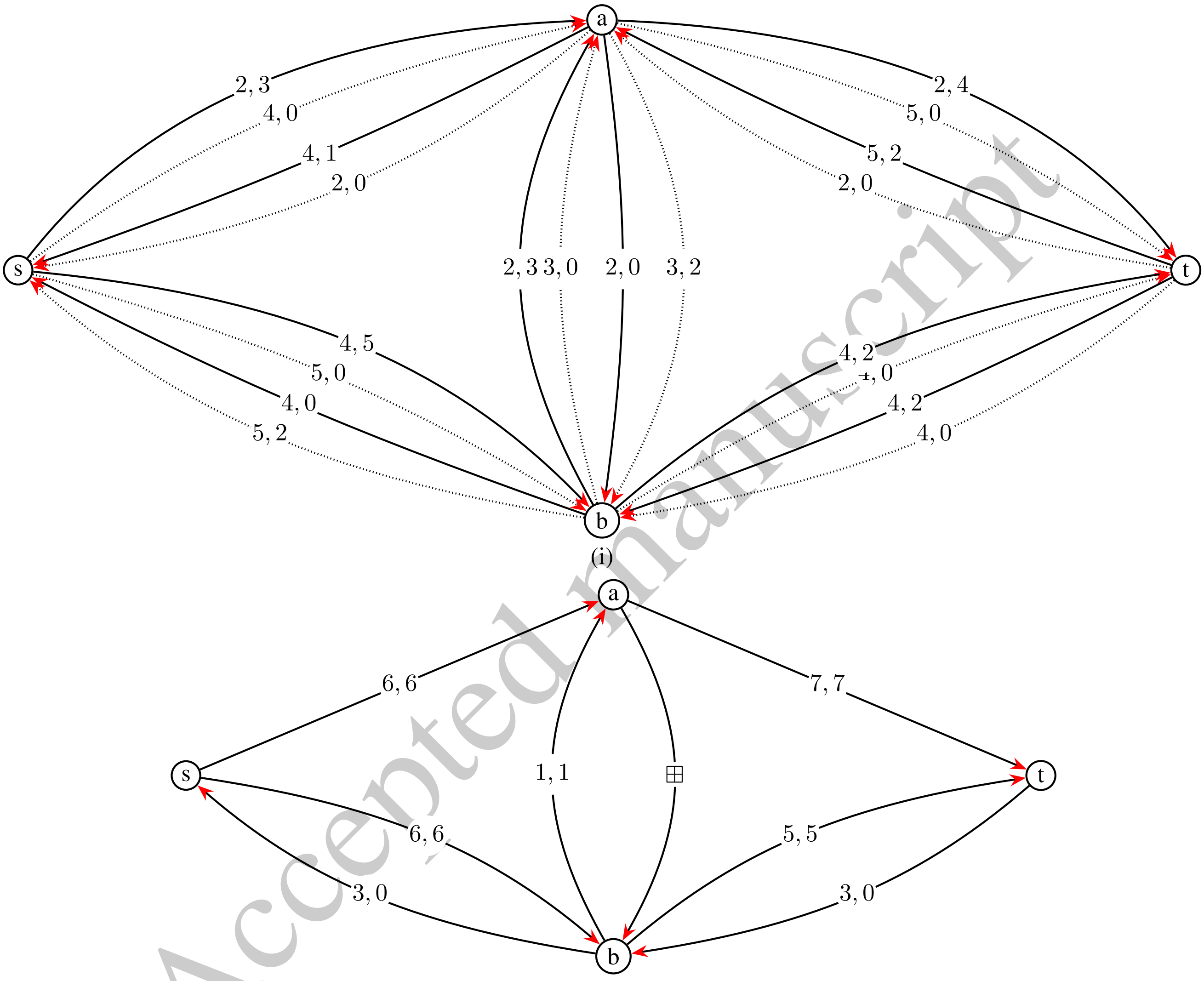
Running the $k$ bisection iterations for one candidate $\tilde{e}$ costs $O\big(k\cdot(T_{\mathrm{MCF}}(n,m)+m)\big) = O\big(\log(nU_{\max})\cdot m^2\log n\big)$. There are $|\tilde{\mathbb{L}}|$ candidates; substituting $|\tilde{\mathbb{L}}| = O(m)$ yields $T(n,m) = O\big(|\tilde{\mathbb{L}}|\cdot\log(nU_{\max})\cdot m^2\log n\big) = O\big(m\cdot\log(nU_{\max})\cdot m^2\log n\big)$. Simplifying gives the stated bound $O\big(m^3(\log n)\log(nU_{\max})\big)$. Final flow decomposition and arc-reversal add only $O(n+m)$ and $O(m)$ time which do not affect the dominant asymptotic term above. Therefore, under the polynomial MCF assumption, the final time complexity is $O\big(m^3(\log n)\log(nU_{\max})\big)$. ☐



(i)

(ii)

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**Figure 3.** Auxiliary network of Figure 1(i) and ContFlowLoc solution for $B = 30$, respectively

**Example 3.** For the parameter setting $r_p = 4$ and budget $B = 30$, applied to the network Figure 1 (i), and its auxiliary network in Figure 3(i), the set of feasible augmented arcs is determined as $\tilde{\mathbb{L}} = \{e \in \tilde{A} : u(e) \geq r(e) = 4 \text{ and } B \leq 30\} = \tilde{A}$. The solution derived using Algorithm 3 for the *ContFlowLoc* problem yields an optimal flow value of 12, distributed along $s-a-t$, $s-b-t$, and $s-b-a-t$, with corresponding flow amounts of 6, 5, and 1, respectively, as illustrated in Figure 3(i). The optimal facility placement occurs on the arc pair $(a,b)$, utilizing a total budget of 27. Additionally, for budget levels $B = 20, 40$, and 50, the respective maximum flow values are 11, 13, and 14, with total incurred costs of 14, 34, and 48, while retaining the same facility configuration. A comparative assessment of flow enhancement strategies relative to the baseline flow and associated budget utilization is presented in Figure 4.

**Figure 4.** Comparison of FlowLoc and contraflow variants with budget usage

The ContFlowLoc problem incorporating switching costs can be generalized to various network configurations addressing different objectives like prioritized flow, optimal dynamic flow within a specified time horizon, or optimal dynamic flow at each time step. The ContFlowLoc framework can be expanded to include scenarios with $q$ facilities and adapted to accommodate alternative cost formulations.

## 5. Conclusions

This study develops a unified framework integrating network flow optimization, contraflow reconfiguration, and facility location under budget constraints including switching costs. Based on theoretical foundations of flow maximization and cost minimization, the work investigates optimal flow enhancement and contraflow problems considering integral, rational, and all-or-nothing capacity modifications. Given the importance of improving network topology within resource constraints, several flow enhancement strategies were examined, and the multi-objective formulation was converted to single-objective budget-constrained problems within ContFlowLoc to optimize flow distribution and determine suitable facility locations. An efficient algorithmic approach was designed, and a full-fledged complexity analysis for different model variants has been conducted, ensuring computational feasibility. This provided an appropriate and systematic framework for improving network performance under financial constraints, merging flow reconfiguration, contraflow operation, and facility location planning together, and providing useful insights for decision-making in a constrained network environment. Such a situation is highly applicable in real life scenarios where the evacuation planning strategies are to be performed within different constraints like the limited budgets provided.

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## References

[1] Ahuja, R. K., Magnanti, T. L., and Orlin, J. B. *Network Flows: Theory, Algorithms, and Applications*. Prentice Hall, Englewood Cliffs, NJ, USA, 1993.

[2] Ahuja, R. K., Magnanti, T. L., Orlin, J. B., and Weihe, K. Network flows: theory, algorithms and applications. *ZOR-Methods and Models of Operations Research 41*, 3 (1995), 252–254.

[3] An, S., Cui, N., Li, X., and Ouyang, Y. Location planning for transit-based evacuation under the risk of service disruptions. *Transportation Research Part B: Methodological 54* (2013), 1–16.

[4] Dhamala, T. N., Pyakurel, U., and Dempe, S. A critical survey on the network optimization algorithms for evacuation planning problems. *International Journal of Operations Research 15*, 3 (2018), 101–133.

[5] Dhungana, R. C. *Efficient Dynamic Flow Algorithms for Evacuation Planning*. PhD thesis, Tribhuvan University, 2021.

[6] Dhungana, R. C., and Dhamala, T. N. Maximum flowloc problems with network reconfiguration. *International Journal of Operations Research 16*, 1 (2019), 13–26.

[7] Dhungana, R. C., and Dhamala, T. N. Flow improvement in evacuation planning with budget constrained switching costs. *International Journal of Mathematics and Mathematical Sciences 2020*, 1 (2020), 1605806.

[8] Fleischer, L., and Tardos, É. Efficient continuous-time dynamic network flow algorithms. *Operations Research Letters 23*, 3-5 (1998), 71–80.

[9] Ford, L. R., and Fulkerson, D. R. *Flows in networks*. Princeton University Press, Princeton, New Jersey, Princeton, New Jersey, 1962.

[10] Ford Jr, L. R., and Fulkerson, D. R. Constructing maximal dynamic flows from static flows. *Operations Research 6*, 3 (1958), 419–433.

[11] Guisewite, G. M., and Pardalos, P. M. Minimum concave-cost network flow problems: Applications, complexity, and algorithms. *Annals of Operations Research 25*, 1 (1990), 75–99.

[12] Hamacher, H. W., Heller, S., and Rupp, B. Flow location (flowloc) problems: dynamic network flows and location models for evacuation planning. *Annals of Operations Research 207*, 1 (2013), 161–180.

[13] Heller, S., and Hamacher, H. W. The multi terminal q-flowloc problem: A heuristic. In *International conference on network optimization* (2011), Springer, pp. 523–528.

[14] Jia, H., Ordóñez, F., and Dessouky, M. A modeling framework for facility location of medical services for large-scale emergencies. *IIE transactions 39*, 1 (2007), 41–55.

[15] Kim, S., and Shekhar, S. Contraflow network reconfiguration for evacuation planning: a summary of results. In *Proceedings of the 13th annual ACM international workshop on Geographic information systems* (2005), pp. 250–259.

[16] Kongsomsaksakul, S., Yang, C., and Chen, A. Shelter location-allocation model for flood evacuation planning. *Journal of the eastern Asia society for transportation studies 6* (2005), 4237–4252.

[17] Krumke, S. O., Noltemeier, H., Schwarz, S., Wirth, H.-C., and Ravi, R. Flow improvement and network flows with fixed costs. In *Operations Research Proceedings 1998* (1999), Springer, pp. 158–167.

[18] Kulshrestha, A., Lou, Y., and Yin, Y. Pick-up locations and bus allocation for transit-based evacuation planning with demand uncertainty. *Journal of Advanced Transportation 48*, 7 (2014), 721–733.

[19] Megiddo, N. Combinatorial optimization with rational objective functions. *Mathematics of Operations Research 4*, 4 (1979), 414–424.

[20] Nath, H. N., Pyakurel, U., Dhamala, T. N., and Dempe, S. Dynamic network flow location models and algorithms for quickest evacuation planning. *Journal of Industrial and Management Optimization 17*, 5 (2021), 2943–2970.

[21] Ng, M., Park, J., and Waller, S. T. A hybrid bilevel model for the optimal shelter assignment in emergency evacuations. *Computer-Aided Civil and Infrastructure Engineering 25*, 8 (2010), 547–556.

[22] Rebennack, S., Arulselvan, A., Elefteriadou, L., and Pardalos, P. M. Complexity analysis for maximum flow problems with arc reversals. *Journal of Combinatorial Optimization 19*, 2 (2010), 200–216.

[23] Sherali, H. D., Carter, T. B., and Hobeika, A. G. A location-allocation model and algorithm for evacuation planning under hurricane/flood conditions. *Transportation Research Part B: Methodological 25*, 6 (1991), 439–452.