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# Bounded rationality and multicriteria decision aiding

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## Abstract

Compared to deciding under a single criterion, in Multicriteria Decision Aiding the role of the decision-maker is much more predominant. This is because in the former case, once the decision model is formulated, the notion of optimal decision is well defined, whereas in the latter, it is not. The behavioral biases influencing the model construction can be in both cases the same; however, in Multicriteria Decision Aiding decisions are selected from a range of incomparable Pareto optimal candidates. Thus, in Multicriteria Decision Aiding there is more space where behavioral biases can emerge.

In this study we identify, within the Multicriteria Decision Aiding setting, the most important decision process drivers that cause the decisions to be satisficing but rarely the best possible. To tame the impact of those drivers on the decision aiding process, we propose a generic Multicriteria Decision Aiding tool, a sort of *what-if analysis*, and show how to make it operational.

**Keywords:** *Multicriteria Decision Aiding, behavioral biases, decision process drivers, satisficing decisions, what-if analysis*

## 1. Introduction

In his Nobel Memorial Lecture, Herbert Simon summarized his *bounded rationality* theory against classical and neoclassical theories of economics and its subdomain, decision-making (notably in firms), which assume pure rationality-entailing maximization of utilities [23]. <sup>1</sup> The following passage from this lecture best relates his theory to Multicriteria Decision Aiding.

*When the goals of an organization cannot be connected operationally with actions (when the production function can't be formulated in concrete terms), then decisions will be judged against subordinate goals that can be so connected. There is no unique determination of these subordinate goals. Their formulation will depend on the knowledge, experience, and organizational environment of the decision maker. In the face of this*

<sup>1</sup> For a concise discussion of the concept of rationality in decision theory and philosophy, see [32]

*ambiguity, the formulation can also be influenced in subtle, and not so subtle, ways by his self-interest and power drives.*

This is exactly what constitutes the realm of Multicriteria Decision Aiding (MCDA).<sup>2</sup> Its springboard is the assumption (backed by a considerable amount of practical evidence) that: “the goals of an organization cannot be connected operationally with actions”, and this can be only made by identifying “subordinate goals” – *criteria* – and selecting a compromise of their value, that suits best “the goals of an organization” (or an individual).

Below, we identify the most important decision process drivers that cause the process, as a rule, to end up with a satisficing decision and not with the best possible. We investigate decision processes in the Multicriteria Decision Aiding setting. To limit the impact of those drivers we propose a generic Multicriteria Decision Aiding tool, a sort of ‘what-if analysis’. We also show how to operationalize that tool with elementary and intuitive notions. With that we aim to enable the decision maker to make, presumably better, satisficing or even the best possible, decisions under lesser cognitive pressure than the existing tools demand.

It needs to be clarified that the notion of a *satisficing* decision is used by Simon to mean decisions that fail to maximize a utility function (in the sense of classical and neoclassical economics). It seems that in none of his works does he allude directly to whether the decision should be or should be not Pareto optimal in terms of subordinate goals, or in the MCDA vocabulary, in terms of attributes.<sup>3</sup> However, his idea of using target values for subordinate goals and considering decisions that attain such targets as satisficing ones suggests that the question of Pareto optimality of decisions escaped him. But there is no reason why one should not directly search, whenever technically possible, for Pareto-optimal decisions and that is regarded as the basic requirement in MCDA.<sup>4</sup> It is worth noticing that the reference point method developed by [30, 33], intended to operationalize Simon’s concept of satisficing decisions [31], always yields Pareto-optimal decisions.

So, when we speak about behavioral issues in MCDA, it means, exactly in line with Simon’s concept of bounded rationality, that we admit - and rightly so - that there exist some *power drivers* of decision processes that are external to the adopted models (which can be verbal, but it is better if they are formal), and thus not under the direct control of the decision-maker (whether individual or collective).<sup>5</sup> Some of those drivers are identifiable (e.g., price fluctuations), some can be quantified (e.g., price volatility), all are considered under the general term *uncertainties*. The identification of such power drivers takes place as the decision process proceeds, as framed in [22]: from the *formulation* phase to the *modeling*, *choice*, and *overview* phases, reiterating them in the form of a *learning loop* or rather, accounting for time flow, *learning helix*, as shown in Figure 1 [21, 22]. The decision maker’s interest in finding a more preferred decision than the incumbent fades away over time. This observation is in full accordance with Simon’s

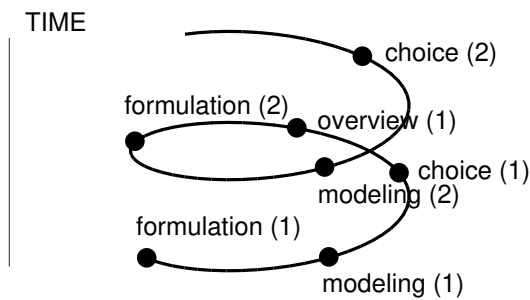
<sup>2</sup> In this study we use the generic term Multicriteria Decision Aiding that subsumes Multiple Criteria Decision Making and, to some extent, Multiobjective Optimization.

<sup>3</sup> An admissible decision is Pareto optimal if there does not exist another admissible decision with all attributes (goals) not worse and at least one attribute better.

<sup>4</sup> Note that Simon formulated his opinions well before researchers got access to enormous computing capabilities via their desktop and later laptop computers.

<sup>5</sup> We speak here about *novel* problems, i.e. such ones that are not solved routinely.

concept of satisficing decision-aiding as well as with every person's experience in daily life.



**Figure 1.** The learning helix. *Source:* original drawing.

It is interesting to note where Simon, the true economist, locates the decision theory against the economic science. For him, decision theory is a branch of economic science concerned with normative, mostly formal, and, whenever possible, quantitative framing of economic activities. However, the research in the MCDA domain has broadened that perspective greatly. Because MCDA encompasses the concept of bounded rationality (in the sense in which Simon perceived it), applicable not only to the economic science but to any domain of human activities, be it economics, social life, or technology, MCDA can claim a high, if not the highest, position among sciences involving any sort of deciding on action-taking.

The mainstream research in MCDA concentrates on the third phase of decision processes, namely the choice phase. Publications on that subject outnumber publications about decision processes as a whole, all four phases together. As a rule, publications concerning the choice phase assume that the model is the *ready-made* output of the second phase, i.e. the modeling phase.<sup>6</sup> Procedures designed for that phase to guide the decision-maker (DM) towards the most preferred decision (that is, the decision that the DM prefers most), are formal schemes that have no, or only limited, means to accommodate and account for the behavioral biases of the DM. Instead, such biases are rather *indirectly* accommodated in the DM preference for the most favorable combination of attribute values.

The outline of this study is as follows. In Section 2 we recall some of the previous contributions to the behavioral aspects of MCDA. In Section 3 we discuss two decision process drivers - in our opinion: those two that are the most important of all - which control the distribution of available resources (the project *budget*) among the decision process phases. Section 4 presents a case study illustrating the content of the previous section. Section 5 outlines a generic MCDA tool – the MCDA *WHAT-IF* analysis, whereas Section 6 collects arguments for its viability. The MCDA *WHAT-IF* analysis is made operational in Section 7. Section 8 concludes the study.

<sup>6</sup> In soft forms, all the process biases can be, at least in theory, accommodated in the first and the second phase where the decision problem is *structured*, cf., eg., [2].

## 2. Related works

The topic of behavioral aspects in decision aiding (but not in multicriteria decision aiding), has been investigated in many works, see Winterfeldt [28] for the state-of-the-art presentation of research results until the mid-1980s. These works are continued to the present, cf., e.g., Winterfeldt [27] and Montibeller and Winterfeldt [20].

In the mid of ninetieth of the previous century, the need to account for behavioral aspects in, until that time rather formal, mathematically skewed research on MCDA, was pronounced by Korhonen and Wallenius [17].

*We think that we can do a much better job in supporting DMs if our decision tools are based on a solid behavioural foundation. Hence the underlying assumptions should be behaviorally plausible. Also, the information requested from DMs should be something that they can reliably furnish.*

A query in Google Scholar with the phrase "Behavioral Multicriteria Decision Aiding" presently yields well over 20,000 results. Below, we limit ourselves only to discussing briefly the ability of the MCDA general concepts to absorb behavioral issues into formal decision-aiding models. However, the existence of numerous papers that concentrate on graphic and analytical tools to ease holistic evaluations of explicitly given (and not too numerous) decisions is notable, cf., e.g. [7], [29]. Likewise, numerous papers report and discuss cases in which a complete formal model is not formulated, mainly because of decision process stakeholders' inability to provide enough structural information, but a decision was taken anyhow, cf., e.g. [4], [1]. However, these topics fall outside the scope of this study.

The two basic concepts in MCDA are the atomistic (in the sense of relating to elements) versus holistic decision evaluations.

The first concept is represented by *weighting methods*, the prototype being the method proposed in [35]<sup>7</sup>. According to this concept, decisions are evaluated indirectly, by assigning relative importance (weights) to (numerical) attributes and aggregating them by a scalarizing (scoring) function. Quite understandably, the DM providing weights, e.g., 1, 2, 3, intuitively expects that the attributes of decisions selected with those weights replicate, even approximately, those proportions. However, there is no direct relationship between weights and attributes. Depending on the nature of the decision problem and the scalarizing function, the result may be very far from that expectation.

The second concept consists in providing a holistic expectation of proportions of decision attributes that the DM prefers. A scalarizing function (the Chebyshev function or one of its variants) ensures that the selected decision follows those proportions as close as possible. A range of methods based on this concept are proposed, the prototypes being the method of displaced ideals (*attractive* reference points located outside the set of *outcomes*, i.e., vectors of attributes of individual decisions) by Zeleny [34] and the method of reference points by Wierzbicki [30, 33].

The weighting concept does not offer much room (if any) for mechanisms capable to accommodate behavioral issues of the decision processes. On the other hand, the reference point concept, heavily

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<sup>7</sup> It is worth noting that Khun and Tucker [16] formulated the necessary and sufficient conditions for optimality in terms of weights of multiple attribute (criteria) functions.

relying on holistic decision evaluations, absorbs such issues indirectly in the act of selecting a reference point. To our knowledge, the only work that directly involves a behavioral issue in the MCDA formal model is that by Kaliszewski and Michalowski [13]. They argued that decisions that offer high trade-offs are *psychologically unstable* and proposed a mechanism to avoid selecting them, based on their earlier work on generating decisions with a priori bounded trade-offs [12].

Stewart [25] pointed to different contexts (ranging from individuals to corporations) in which decisions are made, and the need to invent a "meta-approach" to MCDA that would serve all the cases. We claim that MCDA *WHAT-IF* analysis presented in Section 5 responds to this need.

### 3. Decision process drivers

We should first note that there are two opposite decision-aiding settings of decision processes with a continuum of settings between them. The first one is where the DM is the sole stakeholder, the second involves a complex network of different stakeholders' interests, relative powers, and mutual relationships. The former setting is self-contained and therefore not observable, thus hardly any general conclusions or rules or recommendations can be offered. In the remaining part of this paper, we shall be concerned with the latter type of settings or settings closely akin to them.

Decision process drivers can be problem-specific or decision process-specific. Having no particular decision problem in mind, here we are concerned with the latter ones. Two drivers influencing decision processes most, as a rule not taken into models (built in the modeling phase), are:

- time pressure,
- operational costs.

#### Time pressure

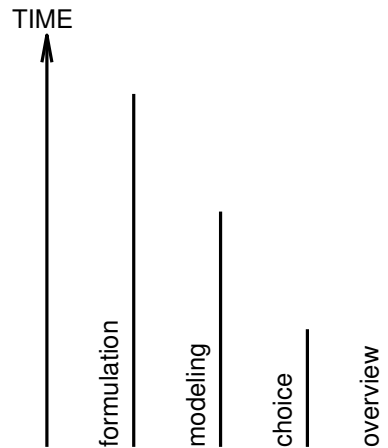
Decision processes are always under time pressure. This is evident if there is a fixed date for the final decision to be taken. It is less evident if there is no fixed date; but still the time is never an unlimited resource. From the start to the end of a decision process, time constitutes its main resource that is unevenly distributed among the four phases of the decision process.

Figure 2 presents a hypothetical and qualitative distribution of disponible time to the phases of decision processes. We are not aware of any reported quantitative research on that topic; the graph is based solely on the Author's experiences, one of which is reported in the next section.

#### Operational costs

Every single effort in the decision process incurs some cost. Some manpower must be designated to tasks at every phase of the decision process. In the simplest case, the cost is just the time spent on the process. In more complex cases, costs are loosely related to elapsed time, e.g. a short but costly consultancy. In the formulation phase, this may be the costs of gathering information from paid repositories, in the modeling phase, the cost of hiring analysts, in the choice phase, the cost of computing, in the overview phase, the cost of organizing meetings with stakeholders. Depending on the case, some of those costs can be negligible, some can be limiting. Like time, the costs of decision-making have some bidding limit.

Besides these two generic process drivers, there might be other, more case specific ones. Considering them requires some attention and/or effort that consume overall budget of the decision process, consisting



**Figure 2.** The hypothetical, qualitative distribution of disponible time to the phases of decision processes. *Source:* original drawing.

at least of time and operational cost issues. The faster the decision process consumes the budget, the more the DM is inclined to accept the incumbent decision as the satisficing one.

## 4. A case study

In the late 1990s and early 2000s, banks were busy implementing the European Union Capital Adequacy Directive (the original 93/6/EEC – *CAD1* – directive [5] and its 98/31/EEC amendment – *CAD2* [6]). The Author advised the executive board of a medium-sized bank on selecting software to gather and aggregate data on bank exposures to risks and calculate the obligatory capital reserve. Such software was offered by vendors at prices in the range of several millions of US\$. The four phases of Simon’s decision framework could be easily identified as the software selection process evolved.

### Formulation

A number of bank employees was assigned to the implementation project, with a Deputy CEO as the project leader. This phase consisted of learning the underlying regulations, mapping them onto the bank structure and internal procedures, and identifying implementation tasks, required skills and manpower.

*Duration: 6 months.*

### Modeling

#### A. Before the call for software bids.

Two principal criteria were selected: cost and technical viability. The latter criterion was an aggregate of specific subcriteria, technical (e.g., computer and network requirements), economic (e.g., servicing costs, servicing timeliness, the guarantee period and duration), and human (e.g., the interface user-friendliness), all expressed in selected numerical scales. It was decided that the relative importance of the two principal criteria would be expressed by assigning weights to them.

The bulk of work in this phase consisted of deciding on scales of subcriteria and verification of the overall subcriteria consistency.

*Duration: 3 months.*

B. After the call for software bids.

Three offers submitted were validated; question and answer sessions with the representatives of the bidding vendors were held.

*Duration: 2 months.*

### Choice

The results of the modeling phase was presented to the Executive Board and top bank officers at a meeting. A toy animation was set to present the effect of changing the relative importance of criteria. On an Excel graph, the arrow pointing to the cost and the technical viability of the most preferred bid (that with the highest value of the weighted sum of criteria values)<sup>8</sup> jumped each time a change of weights caused the change of the *winning offer*. All three offers were nondominated.

After a short discussion, the members of the Executive Board asked have the presentation installed on their laptops.

*Duration: 4 hours.*

### Overview

The course of the overview phase, which was carried out by the members of the Executive Board only, was not disclosed.

*Duration: unspecified.*

The final decision was made at the discretion of the Executive Board, the sole owner of the process. A software was selected eventually.

The presented case can be regarded as representative for the dynamic of the relative distribution of the decision process budget (consisting, as has been said, of time and operational cost issues at the least) to the phases of the decision process. Irrespective of differences between various decision-aiding settings and environments, during prolonged searches for the most preferred decisions the choice phase is handicapped in claims for reasonable resources from the process budget. If so, this phase has to do *its best* under confining time restrictions.

## 5. MCDA *WHAT-IF* analysis for the *choice* phase

The existence of decision process drivers discussed in Section 3 imposes stringent requirements on decision-supporting tools in the choice phase. Operating under a limited process budget, this phase can only be effective when supported by formal tools (consisting of a mixture of mathematics and IT) that are cognitively simple, so that the DM can work with them relatively fast. Prolonged elicitation of the DM's preferences would quickly reach the time limit that the DM is willing to allot to this phase. On top of this, one has to account for the obvious reluctance of, outside-the-lab, flesh-and-blood, business-oriented DMs to be examined about their preferences. It is sensible then to provide the DM with a versatile, simple, and effective tool to perform a form of what-if analysis. In MCDA, what-if analysis reduces to *probing* the set of outcomes of Pareto optimal decisions. The working of the probing tool has to be fully

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<sup>8</sup> The idea to use the weighted Chebyshev function as a scalarizing tool, in the form as presented in the next section, was dismissed without even a chance to present it.

explainable and comprehensible to the DM. The following probing tool (here we refer to it as the MCDA *WHAT-IF* analysis) has been proposed in [11]; cf. also [14], [15].

The MCDA *WHAT-IF* analysis starts with calculating the ideal point. Assuming all  $k$  attributes are in the form of the more, the better, components  $\hat{y}_\ell$ ,  $\ell = 1, \dots, k$ , of the ideal point  $\hat{y}$  are maxima of the respective attribute values.<sup>9</sup>

The ideal point provides a decision problem framing for the DM. No decision can have a better attribute than that represented by the corresponding component of the ideal point. Attributes of admissible decisions represent compromises on the ideal point. Which compromise is the least regrettable, i.e., is the most preferred?

In the space of attributes, a compromise is represented by a half line (*the compromise half line*) starting from the ideal point and directed towards outcomes.

#### Remark

A compromise half line can be defined directly by its directional vector (*the vector of concessions*  $\tau$ ), or indirectly by selecting a point (in particular, an outcome)  $y$  that is strictly dominated by the ideal point  $\hat{y}$ , i.e.  $y_\ell < \hat{y}_\ell$ ,  $\ell = 1, \dots, k$ . In the latter case,  $\tau = \hat{y} - y$ ,  $\tau_\ell > 0$ ,  $\ell = 1, \dots, k$ , (cf. [14], [15]). In the latter case, the MCDA *WHAT-IF* analysis becomes akin to the classes of *reference point* methods.

In short, the MCDA *WHAT-IF* analysis consists in a series of answers to the question: “Which Pareto-optimal decision would be selected if the DM were willing to accept concessions on the ideal point made *along* a compromise half line?” Such questions are repeatedly asked at each iteration of the four-phased decision process.

Since a compromise half line can contain no Pareto-optimal outcomes or even no outcomes (quite obvious if the set of outcomes is discrete), precautions should be made that in any case a Pareto-optimal decision is derived. For that purpose, one can harness the *preference cone*, i.e., the nonnegative orthant  $R_+^k$ , where  $k$  denotes the number of attributes. Then, assuming that all attributes are of “the more, the better” type, a decision is Pareto optimal if its outcome  $y$  satisfies

$$\{y + R_+^k\} \cap Z = y,$$

where  $Z$  is the set of outcomes.

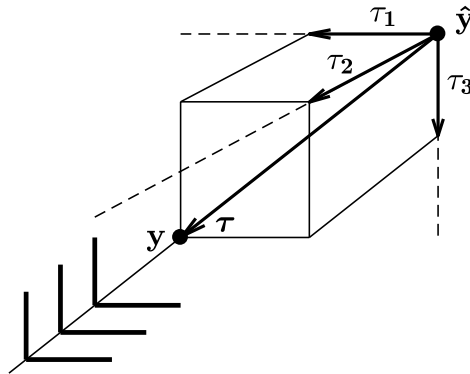
This definition is operationalized by the Chebyshev function; its contours have shapes of the preference cone with its apex gliding on the compromise half lines. Figure 3 provides a graphical explanation to this, whereas the next section presents formally the relation between the compromise half line and the parameters of the Chebyshev function (every compromise half line defines an instance of the Chebyshev function). However, the DM, if supported by an analyst or facilitator, does not need to resort to, or even know, such details.

The concept of the compromise half line is behind many MCDA methods. In fact, any method based on projecting a (reference) point onto the set of attributes of Pareto-optimal decisions defines a specific compromise half line. Perhaps the most close to the MCDA *WHAT-IF* analysis proposed here is the Inter-

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<sup>9</sup> The problem is not trivial if there is no decision corresponding to the ideal point; otherwise any such decision is clearly the most preferred one.

active Weighted Tchebycheff Procedure by Steuer and Choo [24], and the Light Beam Search method by Jaskiewicz and Słowiński [8]. The main difference between point projecting methods and the MCDA *WHAT-IF* analysis is in the full cognitive command the DM has in the latter over projections of the unique reference point (i.e., ideal point) in terms of the compromise half lines, see Section 7.



**Figure 3.** Vector  $\tau$  defines the half line to search for the outcome of a Pareto-optimal decision. The probe is the preference cone (here: the nonnegative orthant in three-dimensional space) with its apex gliding on that half line towards the set of outcomes. *Source:* original drawing.

## 6. MCDA *WHAT-IF* analysis – pros and cons

Below we list pros and cons of the MCDA *WHAT-IF* analysis presented in the previous section.

### 1. Feature: *Simplicity*.

*Pros.* Hardly any simpler decision-aiding method than MCDA *WHAT-IF* analysis can be offered. The only terms it uses that may sound new for the DM are *ideal point* and *the compromise half line*.

*Cons.* With MCDA *WHAT-IF* analysis, the decision process and the DM escape any formal characterization.

### 2. Feature: *Adequacy*.

*Pros.* A satisficing Pareto-optimal decision can be determined in a few MCDA *WHAT-IF* iterations.

*Cons.* There is no guarantee that the whole set of outcomes of Pareto-optimal decisions will be fairly represented in the analysis.

### 3. Feature: *Preference encapsulation*.

*Pros.* There is no attempt to elicit the DM's preferences. A majority of the DMs in the business world would be content with that.

*Cons.* As the DM is not prompted to give any feedback, there is no way to reveal and structure the DM's preferences.

### 4. Feature: *Single reference point*.

*Pros.* Dealing with the single reference point, namely the ideal point, is at the bottom of the MCDA *WHAT-IF* analysis simplicity. This frees the DM from a cognitive burden to provide multiple reference

points or even, like in reference point methods methods, multiple pairs of (*positive* and *negative* or *aspiration* and *reservation*) points. In MCDA *WHAT-IF* analysis, point  $y$ , which can be optionally specified whenever the DM cannot define the compromise half line directly by providing vector  $\tau$ , need not have any particular interpretation. Placing the single reference point at the ideal point guarantees that each outcome of the Pareto-optimal decision can be intercepted by some compromise half line. There exists a formal proof of that fact (see Appendix). Moreover, the ideal point is objective, i.e., given an MCDA model, it is unambiguously defined.

*Cons.* None.

5. Feature: *The anchoring effect.*

The anchoring effect occurs when the DM heavily relies on the first decision determined, called "the anchor", even when subsequent decisions are determined [18, 26].

*Pros.* With the single reference point, fixed and independent of the DM's preferences, and the simple MCDA *WHAT-IF* analysis framework, the DM can be much more willing to make "broad sweeps" over the set of Pareto-optimal outcomes than when reference points (or pairs of such points) have to be selected.

*Cons.* None.

6. Feature: *Absence of weights.*

*Pros.* MCDA *WHAT-IF* is weights-free. As in general there is no clear relationship between weights and attribute weights, despite their ubiquitous use in MCDA analysis, their use is problematic. From the perspective of the MCDA *WHAT-IF* analysis, the need to specify weights seems an unnecessary burden.

*Cons.* People perceive weights as an intuitive notion<sup>10</sup>.

7. Feature: *Intuitiveness.*

Intuitiveness is essential for the DM's acceptance of decision supporting tools.

*Pros.* The MCDM *WHAT-IF* analysis is to the fullest extent intuitive.

*Cons.* None.

8. Feature: *Objectiveness.*

*Pros.* Decision theory would qualify the MCDA *WHAT-IF* analysis as the DM's subjective. On the contrary, every flesh-and-blood decision-maker (as opposed to laboratory decision-maker avatars) would classify it as objective, because it gives her/him the full command of the decision process.

*Cons.* The MCDA *WHAT-IF* analysis offers only satisficing Pareto-optimal decisions without guarantee that they are the most preferred (one can imagine a situation that each subsequent decision derived is more preferred than the previous one, and the set of efficient decisions is too numerous to evaluate each).

9. Feature: *Explainability.*

*Pros.* To explain the working the MCDA *WHAT-IF* analysis, a minimal set of notions is required. In

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<sup>10</sup> For example, what can be more intuitive than relating candy weights to their prices when calculating the cost of a basket of them?

$$\text{cost} = \text{weight of candy } 1 \times \text{price of candy } 1 + \dots$$

But this intuition is misleading when applied to decision aiding. Basket *attributes* are physical weights of candies in that basket, not prices, and prices are attribute *weights*.

complex cases, the MCDA *WHAT-IF* analysis can serve as a ramp towards more elaborate decision-aiding tools.

*Cons.* None.

10. Feature: *Rank reversal*.

*Pros.* As long as the preference structure remains unchanged, rank reversals cannot happen. In MCDA *WHAT-IF* analysis the preference structure remains unchanged if the ideal point does not change. Decisions are quantitatively evaluated independently of each other by the Chebyshev function defined by a compromise half line.

*Cons.* None.

11. Feature: *Convergence*.

*Pros.* No sort of DM's consistency in probing the set of Pareto-optimal decisions is expected.

*Cons.* No type of convergence is offered.

The list of the features can be continued. In fact, the MCDA *WHAT-IF* analysis deprives any single MCDA method of its distinctive feature(s). We are quite positive that, taking into account all the behavioral issues listed above, this is in its favor.

## 7. MCDA *WHAT-IF* analysis made operational

The explanation of how to operationalize the MCDA *WHAT-IF* analysis takes just a few lines.

Let  $x$  denote decisions (alternatives, solutions),  $X_0$  the set of admissible decisions, and  $y(x)$  outcomes of  $x$ , each outcome composed of  $k$  attributes. For the definition of  $\tau$  cf. Section 5.

Given  $\tau = (\tau_1, \dots, \tau_k)$ ,  $\tau_\ell > 0$ ,  $\ell = 1, \dots, k$ , one can derive Pareto-optimal decision  $x$  by solving

$$P_{lin} : \min t$$

$$t \geq (1 \setminus \tau_\ell) ((\hat{y}_\ell - y_\ell(x)) + \rho \sum_{i=1}^k (\hat{y}_i - y_i(x))), \ell = 1, \dots, k,$$

$$x \in X_0,$$

where  $\rho$  is a small positive number [10], [14],[15].

MCDA *WHAT-IF* analysis is represented by the following pseudocode.

### INPUT:

$X_0$  - set of admissible decisions  $x$

$y(X_0)$  - set of outcomes of  $x \in X_0$

### DYNAMIC INPUT:

$\tau$  - vectors of concessions

*continue* - 'YES', 'NO'

### OUTPUT:

$\bar{x}$  - the satisficing decision

### BEGIN

1. *continue* := 'YES'

```

2. Calculate  $\hat{y}$ 
3. while  $continue = \text{'YES'}$ 
    Read  $\tau$ ; if  $\tau$  not available, read  $y, \tau := \hat{y} - y$ 
    Solve  $P_{lin}$ , output the optimal solution
    Read  $continue$ ; if  $continue = \text{'NO'}$ , STOP
END

```

As said in Section 5, vector  $\tau$  can be provided directly or by selecting any point  $y$ , irrespective whether admissible or not, satisfying  $y_\ell < \hat{y}_\ell$ ,  $\ell = 1, \dots, k$ .

Note the generality of this form of deriving a Pareto-optimal decision, as it requires no assumptions except the existence of element  $\hat{y}$ . That in practice is not an issue at all.

Problem  $P_{lin}$  is the equivalent linearized version of the optimization problem with the Chebyshev function (see Appendix).

Problem  $P_{lin}$  is at the core of the following Pareto optimality test.

### Pareto Efficiency Test

- For decision  $\bar{x}$  tested for Pareto optimality, set

$$\bar{\tau}_\ell = (\hat{y}_\ell - y_\ell(\bar{x}))^{-1}, \quad \ell = 1, \dots, k.$$

- Solve problem  $P_{lin}$  with  $\bar{\tau} = (\bar{\tau}_1, \dots, \bar{\tau}_k)$  and  $\rho = 0$ .

If  $\bar{x}$  solves problem  $P_{lin}$  with  $\bar{\tau}$  and  $\rho = 0$ , it is Pareto optimal, otherwise it is not.

**End of Test**

## 8. Concluding remarks

In a casual discussion on MCDA issues, the following, rather bitter opinion was voiced: “If you guys have well over one hundred methods to support decision aiding, it only means that you have none”. At the bottom, it probably should be understood that MCDA does not offer an easy start for decision-makers from outside the academia. So why not offer something elementary first and by this pave a road to more involved MCDA methodologies and methods? Approaching MCDA stepwise, in an *essentials first* manner, can be only beneficial to all parties involved. In our opinion, the proposed MCDA *WHAT-IF* analysis is well suited to serve for such a venture. The notion of bounded rationality may sound at first pejoratively, but it adequately encapsulates the reality of decision making in practice, and the MCDA *WHAT-IF* analysis truly follows that framing.

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## Appendix

Let  $\bar{X}_0, \bar{X}_0 \subset X_0$ , be set of decisions such that for each  $x \in \bar{X}_0$ ,  $y(x)_\ell < \hat{y}_\ell$ ,  $l = 1, \dots, k$  and  $\hat{y}$  is the ideal point (cf. Section 5).

**Theorem.** *Decision  $\bar{x} \in \bar{X}_0$  is properly Pareto optimal if and only if there exists  $\lambda = (\lambda_1, \dots, \lambda_k)$ ,  $\lambda_\ell > 0$ ,  $l = 1, \dots, k$ , and  $\rho > 0$ , such that  $\bar{y}(x)$  solves*

$$P : \min_{x \in \bar{X}_0} \max_{1 \leq \ell \leq k} \lambda_\ell ((\hat{y}_\ell - y_\ell(x)) + \rho \sum_{i=1}^k (\hat{y}_i - y_i(x))).$$

□

This theorem is an immediate consequence of Theorem 4.2 in Kaliszewski (1994).

The notion of proper Pareto optimality emerges only whenever infinitesimal changes between attribute values of Pareto-optimal decisions can take place. Otherwise (e.g. in the case of finite number of decisions), the notions of proper Pareto optimality and Pareto optimality coincide. A positive value of  $\rho$  ensures that the solution is indeed Pareto optimal (and not weakly Pareto optimal) [9], [19], [3].

The proof of the necessity part of Theorem 4.2 in [9] (cf. also [15]) provides the methodological base for the Pareto optimality test presented in Section 7.