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Generic Networks of Votings

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Abstract

In this paper we propose the method for analysing the voting results of given kind of competition. Suppose that a competition is given, with the Borda Count used as a voting method. For every such competition there is a weighted network (called *voting network*) associated in a natural way. Namely, the nodes set corresponds to jurors and link weights correspond to correlation coefficients of voting results of adequate jurors. However, all main correlation coefficients, as well as related metrics considered as distance on rankings, do not distinguish between changes in high places in the competition and changes in low places. We propose a new distance on rankings that allows to observe such distinction. We analyse the results of 2016 International Henryk Wieniawski Violin Competition by comparing the properties of its voting network to the statistical properties of *generic networks of votes*. These generic networks are randomly chosen according to precisely given probability measure on the space of all possible votings of a single juror (i.e. the space of permutations of contestants set). We use these methods to confirm the hypothesis that jurors of 2016 International Henryk Wieniawski Violin Competition were far away from being consistent.

Keywords: *generic network, Lehmer norm, Borda count*

1. Introduction

There were many controversies concerning the results of the 15^{th} International Henryk Wieniawski Violin Competition (2016). Both Gazeta Wyborcza, one of the most popular Polish newspapers, see [6], and Ruch Muzyczny, the most significant Polish music journal, see [12], raised the possibility that the jurors of the most recent Wieniawski Competition formed cliques.

The results of the 2016 Wieniawski Competition were analysed in [26], where they were compared to the results of 16^{th} (2010) and 17^{th} (2015) International Chopin Piano Competitions. The metods of network theory, see for example [11], [18], [19] and [28], were applied to compare the voting results of the three aforementioned music competitions. For these three competitions weighted networks, see [10], W^{2016}, C^{2010} and C^{2015} were created and some numerical properties of these network were compared.

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Weighted networks are used in biology, see [10], in stock markets analysis, see [4] and [7], as well as in the studies on the structural and functional organisation of the human brain, see [22]. Usually, the weight of link l_{st} connecting nodes s and t are given by some kind of a correlation coefficient related to some rankings or processes asocciated to nodes. In [26], the weights w_{st} of links l_{st} were given by $w_{st} = \tau_{st}$, where τ_{st} were *Kendall's* τ *coefficients*, see [13], [14] and [1], of the voting results of jurors J_s and J_t. For stock market networks, this correlation was measured by the *Pearson correlation coefficient*, see [3].

In the case of rankings, many measures of disarray has been studied in the literature. In the case of the *Borda count* method of voting, see [9], [20] and [21] for a description of this method, votes can be regarded as elements of permutation groups S_k (k is the number of contestants). The best known measures of dissarray are *Kendall's* τ *correlation coefficient* and *Spearman's* ρ *correlation coefficient*, see [27], as well as metric measures such as *Kendall distance*, *Spearman distance*, *Hamming distance* and *Footrule distance*, see [5] and [24]. The weighted versions of the Kendall distance and the Footrule distance were considered in [15] and [23].

The classical measure of dissarray mentioned above has such a property that changing the first two positions in the Borda ranking has the same impact on the measure as changing the last two positions. On the other hand, the weighted generalisations of the Kendall distance and the Footrule distance proposed in [15] fail to be metrics (for some choices of weights).

This paper is inspired (partially) by some questions that arised during discussions on [26], where similar methods were used. There are two main differences comparing to [26]. First one - measures of disarray used. In [26] *Kendall's* τ *coefficient* is used, whereas in this paper we use the *Lehmer factorial norm* (see [29]). This is a symmetric, right–invariant norm on the permutation group S_n satisfying the triangle inequality and thus determining the metric on S_n. Additionally, this norm allows for distinguishing changes in the first positions and in the last positions of rankings. The second difference (and a new insight) is that in this paper we determine some probability measure on the space of all possible (Borda) votings (of 11 jurors and 7 contenstants) in such a way, that some statistic properties of networks of votes for such a probability measure fit best properties of network of votings related to the result of the 15^{th} International Henryk Wieniawski Violin Competition. The form of this probability measure in a certain sense "confirms" the controversies concerning the results of the 15^{th} International Henryk Wieniawski Violin Competition.

This article is organised in the following way. In Section 2, we present basic definitions concernig permutation groups and we set the notations used in this paper. We also recall the definition and basic properties of the Lehmer factorial norm. In Section 3, we analyse the results of the 15^{th} International Henryk Wieniawski Violin Competition using the network approach. In Section 4, we describe the procedure of generating random networks of votings. These *generic networks* are used later in Section 5 for determinig the model of generating random networks with properties best fitting the properties of the network related to the results of the 15^{th} International Henryk Wieniawski Violin Competition. Section 6 contains conclusions and open questions related to the subject of these studies.

2. Basic definitions and notations, the Lehmer norm

In this section, we present some basic definitions used in this paper, and we set some notations. We also refer to the definition and basic properties of the Lehmer factorial norm.

For a natural number $n > 0$, by $[n]$ we denote the set $\{1, 2, \dots, n\}$ and by S_n – the group of all permutations of $[n]$. Permutation $\sigma \in S_n$ is denoted by

$$\sigma = (\sigma(1), \sigma(2), \dots, \sigma(n)).$$

In particular $\varepsilon_n = (1, 2, \dots, n)$ denotes the identity permutation.

By σ^{-1}, we denote the inverse permutation to σ, and by $\sigma\tau$ – the composition of σ and τ, defined by $(\sigma\tau)(i) = \sigma(\tau(i))$ for $i = 1, 2, \dots, n$. By $\bar{\sigma}$, we denote the reverse permutation to σ given by $\bar{\sigma}(i) = \sigma(n+1-i)$ for $i = 1, 2, \dots, n$.

For $s = 1, 2, \dots, n-1$ let

$$\alpha_n^s = (1, 2, \dots, s-1, s+1, s, s+2, \dots, n),$$

so α_n^s is adjacent transposition, $(s, s+1)$ in the cycle notation.

For permutation $\sigma \in S_n$, its *Lehmer code* $\mathrm{lc}(\sigma)$, see [16], [17] and [8], is defined by

$$\mathrm{lc}(\sigma) = [c_1(\sigma), c_2(\sigma), \dots, c_n(\sigma)]$$

where numbers $c_i(\sigma)$ are given by

$$c_i(\sigma) = |\{j \in [n] : j > i \text{ and } \sigma(j) < \sigma(i)\}|$$

for $i = 1, 2, \dots, n$.

Definition 1 (Definition 3.4 in [29]). Let $\sigma \in S_n$ be a permutation with the Lehmer code

$$\mathrm{lc}(\sigma) = [c_1(\sigma), c_2(\sigma), \dots, c_n(\sigma)].$$

Lehmer factorial norm $\mathcal{LF}_2 : S_n \to \mathbb{N}$ is given by

$$\mathcal{LF}_2(\sigma) = \sum_{i=1}^{n} \left[2^{n-i} - 2^{n-i-c_i(\sigma)}\right].$$

In the next theorem, we refer to some basic properties of the Lehmer norm.

Theorem 1 (Theorem 3.6 in [29]). Norm $\mathcal{LF}_2$ satisfies the following:

(i) $\mathcal{LF}_2(\varepsilon_n) = 0$ is minimal and ε_n is the only permutation with this property.

(ii) $\mathcal{LF}_2(\bar{\varepsilon}_n) = 2^n - (n+1)$ is maximal and $\bar{\varepsilon}_n$ is the only permutation with this property.

(iii) $\mathcal{LF}_2(\alpha_n^s) = 2^{n-1-s}$ for $s = 1, 2, \dots, n-1$, and therefore

$$\mathcal{LF}_2(\alpha_n^1) > \mathcal{LF}_2(\alpha_n^2) > \dots > \mathcal{LF}_2(\alpha_n^{n-1}).$$

(iv) $\mathcal{LF}_2(\sigma) = \mathcal{LF}_2(\sigma^{-1})$ for all $\sigma \in S_n$.

(v) $\mathcal{LF}_2(\sigma\tau) \leq \mathcal{LF}_2(\sigma) + \mathcal{LF}_2(\tau)$ for all $\sigma, \tau \in S_n$.

Note that properties (i), (iv) and (v) imply that $\mathcal{LF}_2$ determines the metric on S_n. Indeed, the function $d_{\mathrm{L}} : S_n \times S_n \to \mathbb{N}$ given by

$$d_{\mathrm{L}}(\sigma, \tau) = \mathcal{LF}_2(\sigma\tau^{-1})$$

is a metric. We call it the *Lehmer distance*.

Note also that d_{L}, considered as a distance on rankings, distinguishes changes in high places in the competitions from changes in low places.

3. Results of the 15^{th} Wieniawski Competition. A network approach

In this section, we analyse the results of the 15^{th} International Henryk Wieniawski Violin Competition using the network approach.

In this paper, we consider simply undirected networks. A *(simply undirected) network* is a pair $N = (N(N), L(N))$ consisting of set $N(N)$ of *nodes*, usually finite, and set $L(N)$ of *links*, where every link $l \in L(N)$ is a subset of $N(N)$ consisting of two different elements. Networks are often called *graphs* in the literature, nodes and links - *vertices* and *edges*, *sites* and *bonds*, or *actors* and *ties*, respectively. Let N be a network, and suppose that there is a map $w : L(N) \to \mathbb{R}$. Triple $(N(N), L(N), w)$ is called a *weighted network*.

A good introduction to the concept of networks can be found in [18] and [19], whereas [28] contains the same ideas described in the languange of graphs. Methods of weighted networks can be found in [10].

There were 11 jurors and 7 contestants in the final stage of the 15^{th} Wieniawski Competition. The jurors rated the contestants in the final according to the Reverse Borda count: 1 point for the best and 7 points for the worst. The winner was the contenstant with the lower sum of points. The results are presented in Table 1.

Table 1. Final results of the 15^{th} International Henryk Wieniawski Violin Competition

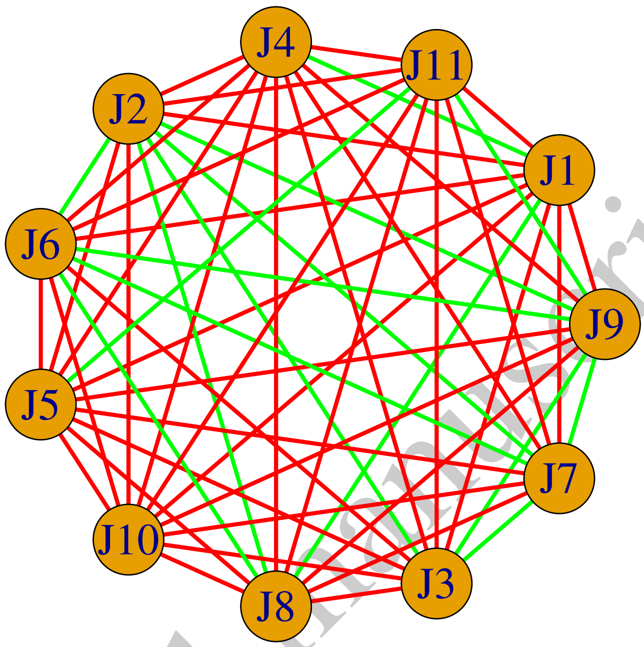
	J1	J2	J3	J4	J5	J6	J7	J8	J9	J10	J11
A	7	3	2	7	7	4	3	7	7	7	7
B	4	7	7	2	2	7	7	2	5	6	5
C	5	5	5	3	6	6	5	5	6	1	6
D	3	6	4	5	1	5	4	4	3	5	1
E	1	4	6	1	3	3	6	3	4	3	4
F	6	2	1	6	4	2	1	6	1	2	2
G	2	1	3	4	5	1	2	1	2	4	3

We define the weighted network $N(\mathbf{W})$ in the following way. The nodes set of $N(\mathbf{W})$ corresponds to the jurors set $\{J_i : i \in [11]\}$, whereas the links set consists of all links $\{J_s, J_t : s \neq t \in [11]\}$. For link l_{st} connecting J_s with J_t, we assign weight $w(l_{st}) = w_{st}$, where

$$w_{st} = \mathcal{LF}_2(\alpha_s \alpha_t^{-1}).$$

Here α_s and α_t denote the votes of jurors J_s and J_t, respectively. In particular, for $i = 1, 2, \ldots, 7$, $\alpha_s(i)$ is the number of points given to the i-th contestant by juror J_s. The similar holds for juror J_t. Note that α_s and α_t can be considered as elements of S_7. Network $N(\mathbf{W})$ is presented in Figure 1.

Figure 1. Network $N(\mathbf{W})$, green – small weights, red – large weights

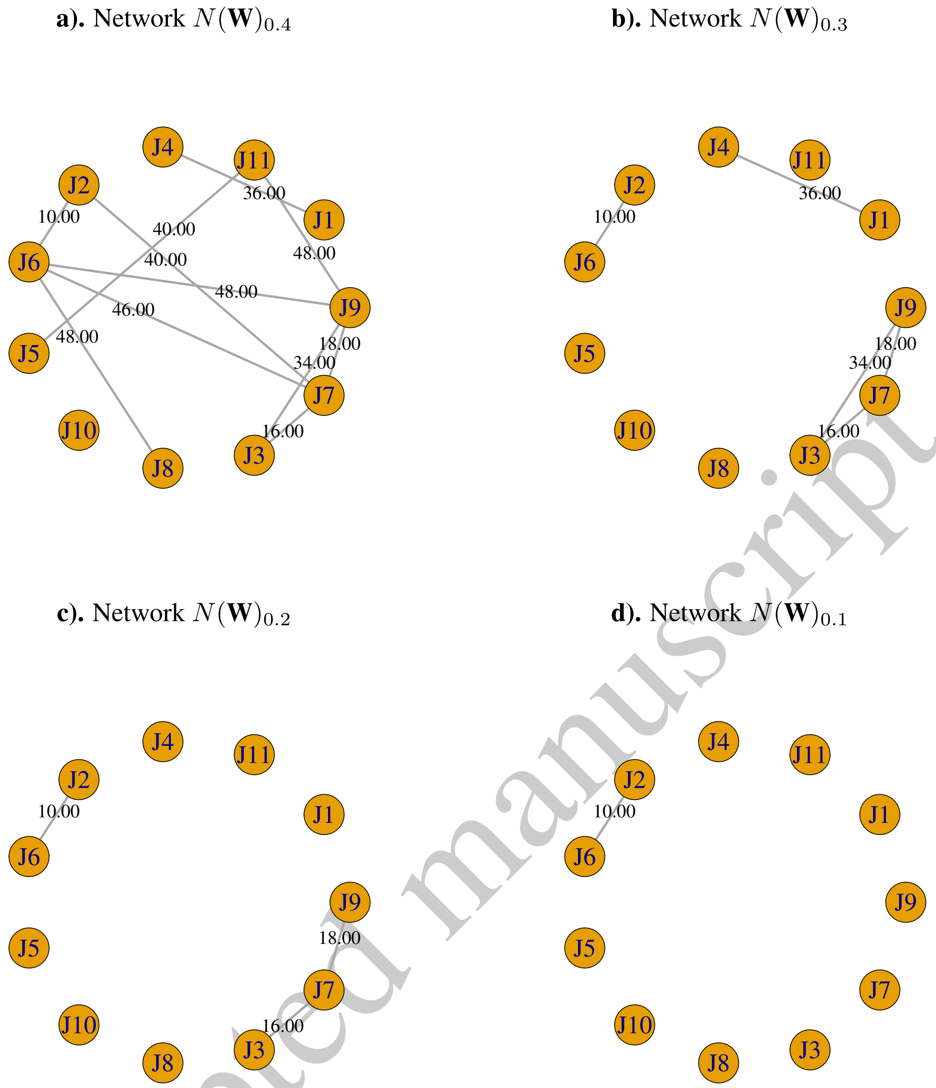


For $p = 1, 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1, 0.0$, we create networks $N(\mathbf{W})_p$ removing from $N(\mathbf{W})$ links l_{st} with weights satisfying the condition

$$w(l_{st}) > p \cdot \max\{\mathcal{LF}_2(\sigma) : \sigma \in S_7\} = p \cdot 120.$$

Note that for p decreasing, $N(\mathbf{W})_p$ contains links connecting jurors voting more and more consistently. They will be used later in Section 5. Networks $N(\mathbf{W})_{0.4}$, $N(\mathbf{W})_{0.3}$, $N(\mathbf{W})_{0.2}$ and $N(\mathbf{W})_{0.1}$ are presented in Figure 2.

Figure 2. Voting networks based on the final results of the 15th International Henryk Wieniawski Violin Competition



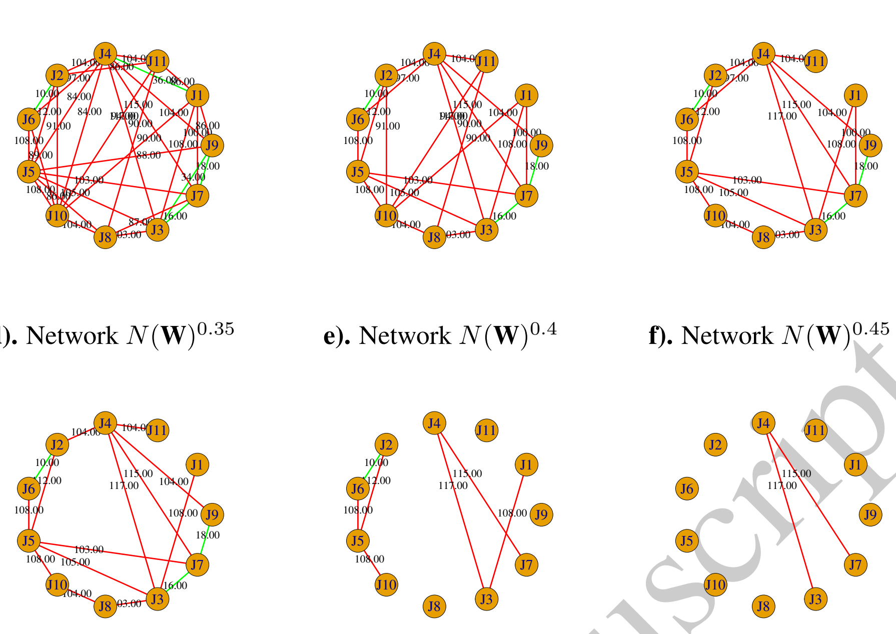
For better understanding of the consistency of jurors' voting, for $s = 0, 0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50$, we create networks $N(\mathbf{W})^s$ removing from $N(\mathbf{W})$ links l_{st}, with weights satisfying the condition

$$|w(l_{st}) - 0.5 \cdot \max\{\mathcal{LF}_2(\sigma) : \sigma \in S_7\}| < s \cdot \max\{\mathcal{LF}_2(\sigma) : \sigma \in S_7\}, \text{ i.e.}$$

$$|w(l_{st}) - 60| < s \cdot 120.$$

Note that when s is increasing, then $N(\mathbf{W})^s$ contains only the links connecting jurors that vote more and more consistently and more and more inconsistently, since we remove the links connecting jurors voting independently. Networks $N(\mathbf{W})^{0.2}$, $N(\mathbf{W})^{0.25}$, $N(\mathbf{W})^{0.3}$, $N(\mathbf{W})^{0.35}$, $N(\mathbf{W})^{0.4}$ and $N(\mathbf{W})^{0.45}$ are presented in Figure 3.

Figure 3. Voting networks based on the final results of the 15th International Henryk Wieniawski Violin Competition



a). Network $N(\mathbf{W})^{0.2}$ **b).** Network $N(\mathbf{W})^{0.25}$ **c).** Network $N(\mathbf{W})^{0.3}$

d). Network $N(\mathbf{W})^{0.35}$ **e).** Network $N(\mathbf{W})^{0.4}$ **f).** Network $N(\mathbf{W})^{0.45}$

Note, that the method of removing links used here is very similar to the method of selecting decision variables in econometrics (Bartosiewicz method), see [2]. On the other hand, we are not able to use Bartosiewicz method strictly, since we analyse the votes of all jurors, which are equally important.

4. Generic networks of votes

In this section, we describe the procedure of generating random networks of votings. These *generic networks* will be used later in Section 5 for determining the model of generating random networks with properties best fitting the properties of network $N(\mathbf{W})$. We determine this best fitting model to check the hypothesis that jurors of 15^{th} International Henryk Wieniawski Violin Competition voted controversially. Namely, their votings were neither consistent nor random.

Consider the space of all possible votings of a single juror in $\mathbf{W}$ - the final stage of the 15^{th} Wieniawski Competition. This space can be seen as S_7. Let $\mathbb{P}$ denote the probability measure on S_7. We consider the measures of the form

$$\mathbb{P} = \mathbb{P}(d, \alpha, \beta) = \alpha \mathbb{P}_{\sigma_1} + \beta \mathbb{P}_{\sigma_2} + (1 - \alpha - \beta)\mathbb{P}_{\text{uniform}}$$

where:

- $\alpha, \beta \in [0, 1]$ satisfy the condition $\alpha + \beta \leq 1$,
- $d \in [0, 1]$,
- $\mathbb{P}_{\text{uniform}}$ is the uniform probability measure on space S_7,

- $\mathbb{P}_{\sigma_1}$ and $\mathbb{P}_{\sigma_2}$ are the Dirac probability measures on S_7 centred at permutations σ_1 and σ_2 respectively, where σ_1 and σ_2 are randomly chosen in such a way that they satisfy the condition

$$\mathcal{LF}_2(\sigma_1\sigma_2^{-1}) \simeq d \cdot \max\{\mathcal{LF}_2(\sigma) : \sigma \in S_7\}.$$

$\mathbb{P}$ defined in this way is a convex combination of $\mathbb{P}_{\sigma_1}$, $\mathbb{P}_{\sigma_2}$ and $\mathbb{P}_{\text{uniform}}$. The condition bonding σ_1 and σ_2 is a metric analogue of the condition for the correlation coefficient of σ_1 and σ_2. In this paper, we consider $\alpha, \beta = 0, 0.05, 0.1, \dots, 0.95, 1$ and $d = 0, 0.1, \dots, 0.9, 1$. Set $N_{\max} = \max\{\mathcal{LF}_2(\sigma) : \sigma \in S_7\}$. The notation $\simeq$ means that σ_1 and σ_2 are randomly chosen from all possible pairs of permutations satisfying condition $\mathcal{LF}_2(\sigma_1\sigma_2^{-1}) \in [d \cdot N_{\max} - 0.05 \cdot N_{\max}, d \cdot N_{\max} + 0.05 \cdot N_{\max}]$.

The procedure for generating the random network of votings is as follows:

(i) choose a repetition number $j = 1, 2, \dots, 100$,

(ii) choose $d = 0, 0.1, \dots, 0.9, 1$,

(iii) randomly choose such σ_1 and σ_2 that $\mathcal{LF}_2(\sigma_1\sigma_2^{-1}) \simeq d \cdot N_{\max}$,

(iv) choose such $\alpha, \beta = 0, 0.05, 0.1, \dots, 0.95, 1$ that $\alpha + \beta \leq 1$,

(v) for every $s = 1, 2, \dots, 11$ randomly, according to $\mathbb{P} = \alpha\mathbb{P}_{\sigma_1} + \beta\mathbb{P}_{\sigma_2} + (1 - \alpha - \beta)\mathbb{P}_{\text{uniform}}$, choose $\alpha_n^s \in S_7$ – this is the vote of juror J_s,

(vi) create weighted network $N(d, \alpha, \beta, j)$ according to the procedure described in Section 3,

(vii) for $p = 1, 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1, 0.0$, create network $N(d, \alpha, \beta, j)_p$.

5. Fitting the parameters

In this section, we determine such parameters d, α and β that the statistical properties of a family of networks $\{N(d, \alpha, \beta, j) : j = 1, 2, \dots, 100\}$ fit best the properties of the network $N(\mathbf{W})$.

According to the constructions of networks $N(\mathbf{W})_p$, their connected components, as p decreases, correspond to groups of jurors voting in a more and more consistent way. Table 2 contains the number of connected components of networks $N(\mathbf{W})_p$.

Table 2. Number of connected components of networks $N(\mathbf{W})_p$

treshold parameter p	components number
1	1
0.9	1
0.8	1
0.7	1
0.6	2
0.5	2
0.4	3
0.3	7
0.2	8
0.1	10
0	11

Let $\mathrm{C}(N)$ denote the number of connected components of network N. We determine the numbers $d_{\mathrm{e}}^{\min}$, $d_{\mathrm{m}}^{\min}$, $d_{\mathrm{e}}^{\max}$ and $d_{\mathrm{m}}^{\max}$ given by

$$d_{\mathrm{e}}^{\min} = \min_{d,\alpha,\beta} \left\{ \sqrt{\sum_p \left(\mathrm{C}\left(N(\mathbf{W})_p\right) - \frac{\sum_{j=1}^{100} \mathrm{C}(N(d,\alpha,\beta,j)_p)}{100} \right)^2} \right\},$$

$$d_{\mathrm{m}}^{\min} = \min_{d,\alpha,\beta} \left\{ \sum_p \left| \mathrm{C}\left(N(\mathbf{W})_p\right) - \frac{\sum_{j=1}^{100} \mathrm{C}(N(d,\alpha,\beta,j)_p)}{100} \right| \right\},$$

$$d_{\mathrm{e}}^{\max} = \max_{d,\alpha,\beta} \left\{ \sqrt{\sum_p \left(\mathrm{C}\left(N(\mathbf{W})_p\right) - \frac{\sum_{j=1}^{100} \mathrm{C}(N(d,\alpha,\beta,j)_p)}{100} \right)^2} \right\} \text{ and}$$

$$d_{\mathrm{m}}^{\max} = \max_{d,\alpha,\beta} \left\{ \sum_p \left| \mathrm{C}\left(N(\mathbf{W})_p\right) - \frac{\sum_{j=1}^{100} \mathrm{C}(N(d,\alpha,\beta,j)_p)}{100} \right| \right\}$$

respectively.

Note that $d_{\mathrm{e}}^{\min}$ and $d_{\mathrm{m}}^{\min}$ minimalise the Euclidean distance and the Manhattan distance between the number of connected components of $N(\mathbf{W})_p$ and the average number of connected components of $N(d,\alpha,\beta,j)_p$, respectively. Similarily, $d_{\mathrm{e}}^{\max}$ and $d_{\mathrm{m}}^{\max}$ maximalise these distances. Table 3 contains parameters d, α and β of models for which these distances are the smallest. The sum

$$\sqrt{\sum_p \left(\mathrm{C}\left(N(\mathbf{W})_p\right) - \frac{\sum_{j=1}^{100} \mathrm{C}(N(d,\alpha,\beta,j)_p)}{100} \right)^2}$$

varies between $d_{\mathrm{e}}^{\min} = 14.29699$ and $d_{\mathrm{e}}^{\max} = 24.75912$, whereas the sum

$$\sum_p \left| \mathrm{C}\left(N(\mathbf{W})_p\right) - \frac{\sum_{j=1}^{100} \mathrm{C}(N(d,\alpha,\beta,j)_p)}{100} \right|$$

varies between $d_{\mathrm{m}}^{\min} = 34.28$ and $d_{\mathrm{m}}^{\max} = 75.02$. In both cases best fitted models are those with parameter $d = 1$. Note that $d = 1$ means that σ_1 and σ_2, chosen during the procedure described in Section 4, satisfy $\mathcal{LF}_2(\sigma_1\sigma_2^{-1}) \simeq N_{max}$, therefore being almost revers permutations. This fact confirms the hypothesis that jurors of the 15^{th} International Henryk Wieniawski Violin Competition were far away from being consistent.

6. Conclusions and recommendations for further research

This section contains conclusions and open questions related to the subject of these studies.

The results of voting include a lot of information about preferences of voters and their structure. The application of network theory can highlight properties of networks constructed on the basis of jurors' votings. The obtained networks may be used to describe homogeneity or heterogeneity of jurors' votings.

During this research some questions arose.

Table 3. Best fitting parameters

Euclidean Distance	maximal value - 24.75912
$d = 1, \alpha = 0.4, \beta = 0.6$	14.29699
$d = 1, \alpha = 0.45, \beta = 0.55$	14.29773
$d = 1, \alpha = 0.5, \beta = 0.5$	14.30108
$d = 1, \alpha = 0.55, \beta = 0.45$	14.30696
$d = 1, \alpha = 0.6, \beta = 0.4$	14.30696
Manhattan Distance	maximal value - 75.02
$d = 1, \alpha = 0.1, \beta = 0.9$	34.28
$d = 1, \alpha = 0.15, \beta = 0.85$	34.32
$d = 1, \alpha = 0.85, \beta = 0.15$	34.32
$d = 1, \alpha = 0.9, \beta = 0.1$	34.4
$d = 0.9, \alpha = 0.1, \beta = 0.9$	34.78

1. How do statistical coefficients of networks $N(d, \alpha, \beta, j)_p$ depend on d, α and β?
2. How does behaviour of generic networks depend on the number of jurors and the number of contestants?
3. What are the asymptotic (with number of jurors and/or contestants tending to ∞) properties of generic networks?
4. How do best fitting parameters d, α and β change when increasing the number of repeats?
5. How will properties of generic networks change when we randomly choose permutation σ_1 and σ_2 with $\mathcal{LF}_2(\sigma_1\sigma_2^{-1})$ precisely set from all possible values of $\mathcal{LF}_2$?

These questions are a good starting point for further research.

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References

[1] Abdi, H. The Kendall rank correlation coefficient. In *Encyclopedia of Measurement and Statistics* (2007), N. Salkind, Ed., SAGE Publication Inc.

[2] Bartosiewicz, S. Prosta metoda wyboru zmiennych objaśniających w modelu ekonometrycznym. (Polish) [A simple method for selecting explanatory variables in an econometric model]. *Prace naukowe WSE 43 (65)* (1974), 93–101.

[3] Boddy, R., and Smith, G. *Statistical Methods in Practice: For Scientists and Technologists*. John Wiley & Sons, 2009.

[4] Cherifi, H., Gaito, S., Quattrociocchi, W., and Sala, A. *Complex Networks & Their Applications V - Proceedings of the 5th International Workshop on Complex Networks and their Applications (COMPLEX NETWORKS 2016)*. Springer Publishing Company, 2018.

[5] Diaconis, P., and Graham, R. Spearman's Footrule as a Measure of Disarray. *Journal of the Royal Statistical Society B (Methodological) 39 (2)* (1977), 262–268.

[6] Dębowska, A. Konkurs Wieniawskiego 2016: Wewnętrzna wojna jurorów. Zwyciężczyni ledwo weszła do finału. (Polish) [2016 Wieniawski Competition. Internal jury war. Winner barely makes final]. http://wyborcza.pl/7,113768,20925157,konkurs-wieniawskiego-2016-wewnetrzna-wojna-jurorow.html?disableRedirects=true.

[7] Górski, A., Drożdż, S., Kwapień, J., and Oświęcimka, P. Complexity characteristics of currency networks. *Acta Physica Polonica B 37 (11)* (2006), 2987–2995.

[8] Grinberg, D. Notes on the combinatorial fundamentals of algebra. arXiv:2008.09862v1.

[9] Hołubiec, J., and Mercik, J. *Inside Voting Procedures*. Accedo Verlagsgesellschaft, 1994.

[10] Horvath, S. *Weighted Network Analysis: Applications in Genomics and Systems Biology*. Springer New York, 2011.

[11] Jackson, M. The Economics of Social Networks. In *Advances in Economics & Econometrics: Theory & Applications, Ninth World Congress of the Econometric Society, Volumne I* (2006), R. Blundell, W. Newey, and T. Persson, Eds., Cambridge University Press, pp. 1–56.

[12] Januszkiewicz, M., and Chorościak, E. Konkurs skrzypcowy im. Wieniawskiego w Poznaniu. (Polish) [Wieniawski Violin Competition in Poznań]. *Ruch Muzyczny* (2016), 50–55.

[13] Kendall, M. A new measure of rank correlation. *Biometrika 30 (1-2)* (1938), 81–93.

[14] Kendall, M. *Rank Correlation Methods*. C. Griffin, 1948.

[15] Kumar, R., and Vassilvitskii, S. Generalized distances between rankings. In *Proceedings of the 19th International Conference on World Wide Web, WWW 2010* (2010), M. Rappa, P. Jones, J. Freire, and S. Chakrabarti, Eds., ACM, pp. 571–580.

[16] Lehmer, D. Teaching combinatorial tricks to a computer. In *Combinatorial Analysis: Proceedings of Symposia in Applied Mathematics Volume X* (1960), R. Bellman and M. Hall Jr, Eds., Amer. Math. Soc., pp. 179–193.

[17] Lehmer, D. The machine tools of combinatorics. In *Applied Combinatorial Mathematics* (1964), E. Beckenbach, Ed., John Wiley & Sons, pp. 5–31.

[18] Newman, M. E. J. *Networks: an Introduction*. Oxford University Press, 2010.

[19] Newman, M. E. J., Barabási, A.-L., and Watts, D. *The Structure and Dynamics of Networks*. Princeton University Press, 2011.

[20] Nurmi, H. *Comparing Voting Systems*. Springer Netherlands, 1987.

[21] Ordeshook, P. C. *Game Theory and Political Theory*. Cambridge University Press, 1986.

[22] Park, H.-J., and Friston, K. Structural and Functional Brain Networks: From Connections to Cognition. *Science 342* (2013), 1–8.

[23] Piek, A., and Petrov, E. On a Weighted Generalization of Kendall's Tau Distance. *Annals of Combinatorics* (2021), 33–50.

[24] Qian, Z., and Yu, P. Weighted Distance-Based Models for Ranking Data Using the R Package rankdist. *Journal of Statistical Software 90 (5)* (2019), 1–31.

[25] R Core Team. *R: A Language and Environment for Statistical Computing*, 2020. https://www.R-project.org/.

[26] Sosnowska, H., and Zawiślak, P. Differences between jurors in classical music competitions: the MCDM and Network Theory approaches. *Multiple Criteria Decision Making 14* (2019), 93–107.

[27] Spearman, C. The proof and measurement of association between two things. *American Journal of Psychology 15 (1)* (1904), 72–101.

[28] West, D. B. *Introduction to Graph Theory*. Prentice Hall, 2000.

[29] Zawiślak, P. The Lehmer factorial norm on S_n. arXiv:2111.03951v1.

[30] Zawiślak-Sprysak, E., and Zawiślak, P. Generic networks of Votings. arXiv:2211.16204v1.

[31] Zawiślak-Sprysak, E., and Zawiślak, P. Generic networks of Votings. https://doi.org/10.22541/au.167247191.14187778/v1.