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An approach to spatial localization of dynamic objects in a large swarms based on a combination of GPS and Bluetooth technologies

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Abstract

In large swarms of dynamic objects (for example, tens of thousands of people in a stadium during mass events), it is difficult to equip all objects with full-fledged GPS sensors to determine coordinates and transmit them to the security system. Therefore, we propose using a combination of two technologies: GPS (active objects) and Bluetooth (passive objects). Passive objects periodically emit a radio signal with a unique identification number. Active objects detect passive objects and communicate with the security system. This interaction scheme gives rise to the problem of determining the coordinates of passive objects. We present a set of novel algorithms to estimate and refine the coordinates of passive objects through information redundancy, which occurs when a passive object is detected by several active objects. Under favourable conditions, the use of this set of algorithms is equivalent to equipping passive objects with GPS sensors.

Keywords: *coordinate measurement, large swarms, Bluetooth-technology, fuzzy filtering*

1. Introduction

Among the many security problems, public events occupy the most important place. Thousands of people gather in stadiums to take part in a political or sporting event or attend concerts of famous performers. In these cases, the number of participants can be very large. For example, the Melbourne Cricket Ground in Melbourne (Australia) seats 100 thousand people, the Camp Nou stadium in Barcelona (Spain) seats 99 thousand people, and the Soccer City stadium in Johannesburg (South Africa) seats 94 thousand people. In Eastern European countries, the capacity of national-level stadiums is between $55-75$ thousand people. In large crowds of people, even minor disorganising impacts, such as riots, smoke or loud bangs, can cause panic that quickly spreads to everyone involved. In such cases, people try to quickly leave the stadium, creating a crush in which they receive injuries that are often incompatible with life. One recent

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example is the October 1, 2022 incident at a stadium in Malang, Indonesia, in which 182 people died and 190 were injured. Such cases show that serious security problems need to be addressed. For example, it is necessary to determine which exits to open so that allow people to leave the stadium safely, or where to deploy security personnel, or where to turn on or off lights to control crowd movement. To solve these problems, it is necessary to know the coordinates of all participants in order to determine the threat of a stampede and prevent it. There are other security problems that require determining the coordinates of all dynamic objects in large swarm.

The problems that are closest in nature are the problems of controlling a swarm of objects based on coordinate information. Today, these problems are relevant and are widely discussed in the scientific literature. As a rule, each object in a swarm is equipped with a GPS sensor, which determines coordinates and transmits them to the leader object. However, these problems consider swarm sizes of up to hundred of objects. When the number of dynamic objects reaches 100 thousand, equipping them with GPS sensors requires significant financial costs and becomes a problem. In addition, GPS signal traffic increases greatly, which can lead to serious delays in determining the coordinates of objects. If an indoor swarm is considered, then, as a rule, the room is equipped with stationary Bluetooth receivers with known GPS coordinates, and dynamic objects are equipped with Bluetooth beacons. To determine the coordinates of dynamic objects, the control center solves the problem of triangulation or trilateration. However, even in this case, equipping the stadium with stationary Bluetooth receivers requires large expenses. In addition, if necessary, the coordinate determination system will be difficult to transfer to another stadium, since all equipment will need to be dismantled and reconfigured.

Therefore, to determine the coordinates of dynamic objects in large swarm, we proposed to equip only part of the objects (active objects) with a GPS sensor, and equip the remaining objects with simple cheap Bluetooth transmitters (passive objects). Each active object is additionally equipped with a Bluetooth receiver for receiving radio signals from passive objects, as well as a transmitter for communication with the security system. Each passive object emits a radio signal that contains only a unique identifier for that object. In accordance with the characteristics of Bluetooth transmitters [1], the signal propagates over a distance of up to $25 - 125$ meters, depending on the type of sensor. The active object receives signals from passive objects located in the detection zone and, together with its coordinates, transmits the identifiers of these objects to the Security Control Center (SCC), which calculates the GPS coordinates of all objects for further use in security problems. We add that security system employees can be considered as active objects that not only provide determination of the coordinates of passive objects, but can also receive control signals from the SCC to control the movement of passive objects.

There are several variants for the relative position of passive and active objects. If the signal from a passive object is not received by any active object, then the coordinates of this passive object cannot be determined and this object is lost. If the signal from a passive object is received by one active object, the coordinates of this passive object will be determined with a maximum error, which is equal to the sum of the error in determining GPS coordinates and the range of the Bluetooth transmitter. If several active objects receive a signal from a passive object, redundancy of information arises, which makes it possible to clarify the coordinates of this passive object.

For the latter variant, we proposed three algorithms.

The algorithm for determining the coordinates of passive objects is based on the use of information redundancy, which occurs when one passive object is detected by several active objects. The intersection of several active object's detection zones forms the area in which the passive object is located. The coordinates of the passive object correspond to the coordinates of one of the points in this area. The more active objects, the more intersections of detection zones, the smaller the size of the area and, therefore, the more accurately the coordinates of the passive object will be determined.

An algorithm for clarifying the coordinates of active objects, which uses the inverse dependence of the standard error on the number of information sources when averaging data. The algorithm ensures a reduction in coordinate determination errors (CDEs) of active objects, which in turn improves the accuracy of determining the coordinates of passive objects.

An algorithm for clarifying the coordinates of passive objects based on fuzzy filtering. The algorithm reduces the CDEs of active objects by averaging and smoothing coordinate estimates, which are presented as a membership function. This, in turn, provides an additional increase in the accuracy of determining the coordinates of passive objects.

In general, if sufficient information redundancy is provided, the set of proposed algorithms makes it possible to reduce the CDEs of passive objects to the level of CDEs of active objects, that is, to the error level of GPS sensors. In other words, the use of the proposed algorithms is almost equivalent to equipping passive objects with GPS sensors.

2. Literature Review

We did not find any known approaches that would consider similar problems. However, since our problem has common features with problems of swarm control and problems of indoor localization, we will consider approaches to solving them.

Approaches to solving problems of swarm control.

Yasin et al. [30] considers the problem of reducing computing power in a swarm of drones by adaptively switching the level of intelligence of drones in the case when the swarm is built in accordance with the "leader-follower" scheme. The follower has equipment to receive coordinates from the leader. If there is no connection with the leader, the follower can declare himself the leader. Otherwise, the follower adjusts its movement to follow the leader, preventing collisions with obstacles (other drones). In this problem, all swarm objects must be equipped with the same equipment, including GPS sensors.

Meng et al. [24] proposes a solution to the problem of matching the speeds of swarm objects, maintaining swarm connectivity and preventing collisions. In the work, the swarm is presented in the form of two groups: leaders and followers. Leaders independently evaluates their coordinates and manage their followers. Fedele et al. (2019) [11] uses a swarm model with coordinate relationship matrices, which assumes the same equipment of swarm objects. Fedele et al. (2020) [12] is a development of previous work. The study assumes that each object has a limited detection range. However, as in previous work, the swarm objects are not functionally different from each other.

Chen et al. [6] is devoted to increasing the accuracy for measuring coordinates in a swarm of objects due to redundant information about the relative coordinates of objects. In this work, swarm objects are also not separated based on their different equipment with instruments for measuring coordinates.

Fabra et al. [10] proposed the MUSCOP protocol, which ensures the coordination of UAVs to maintain the desired flight formation when performing planned missions. According to this protocol, the master object regularly synchronizes the subordinate objects from one waypoint to another. The swarm structure presupposes the presence of a main object and subordinate objects.

Izhboldina et al. [18] considers the problem of swarm control. The conceptual model of the problem assumes an operator, a base station and two communication networks: between objects and between the base station and objects. The paper proposes two swarm reconfiguration algorithms with different collision avoidance methods.

For an in-depth study, we can recommend the review of Majid et al. [22] of studies related to the problems of swarm robotics: aggregation, dispersion, self-assembly and self-reconfiguration, collective search for sources, collective mapping, collective search for food, collective transport, collective manipulation and others. 252 sources make up the bibliography of this review.

Approaches to solving the problem of indoor localization.

Pau et al. [25] considers the problem of building high-precision indoor positioning systems, in particular, the use of radio frequency radiogoniometry techniques, as well as the new version of the Bluetooth specification, which allows you to determine the angle of radiation sources.

Sollie et al. [26] is devoted to the problem of direction finding (determining the direction) of a moving Bluetooth device using an antenna array.

Adjei et al. [3] discusses a Bluetooth device that is designed to provide security for valuable items. Goh et al. [14] is devoted to solving a similar problem.

Bai et al. [4] considers a positioning system designed to determine the location of a user in a building. This system includes a sensor system of many sensors that detects the signal from the beacon.

Mendakulov et al. [23] is aimed at increasing the accuracy of measuring the coordinates of objects indoors by receiving a signal simultaneously at four frequencies.

Grinyak et al. [15] is devoted to the problem of 3D positioning of objects indoors using Bluetooth beacons and other devices. The effect is to use inexpensive, widespread devices that do not require special professional skills of personnel to determine the location.

Daudov et al. [8] examines the use of Bluetooth beacons for locating objects indoors. A Kalman filter was used to improve accuracy.

Zochowski [31] solves the problem of providing objects with information about their location inside buildings when the errors of satellite navigation systems are unacceptably large. The work assumes that the building is equipped with transmitters, the signals from which are received by the object's Bluetooth receiver. A trilateration algorithm is used to determine the location of objects.

Ho et al. [16] considers the use of smartphones to determine distance or location. Neural networks are used to improve the accuracy of measurements.

Fei et al. [13], to reduce human localization errors indoors, the authors proposed several algorithms, also based on the use of neural networks with taking into account the level of the received signal.

Subedi et al. [27] suggests using weighted centroid localization together with a received signal strength indicator from neighboring beacons to increase the accuracy of internal positioning.

Hou et al. [17] considers the use of wavelet and an smoothing filter to increase the positioning accuracy of a Bluetooth device in real line-of-sight and non-line-of-sight conditions.

Thaljaoui et al. [28] considers an indoor inter Ring Localization Algorithm (in the context of a smart home) based on a hybrid system that combines radio, light and sound information.

Davidson and Piche [9] provide an overview of the most common indoor positioning methods that can be implemented in a smartphone using Wi-Fi and Bluetooth. In particular, the study considers methods: reading fingerprints using a magnetic field, navigation using maps of building floors and additional sensors (counting steps, estimating the length and direction of steps, motion classifications, and others). We also recommend taking a look at the retrospective review of Li et al. [20] of machine learning-based positioning methods.

As a conclusion, the following can be noted. Swarm control problems focus on managing dynamic objects. Determining coordinates is not a difficult problem, since all objects, as a rule, are equipped the same and use GPS sensors. Some of the main problems are: collision avoidance, swarm formation, swarm reconfiguration. Known approaches either do not indicate the maximum size of the swarm, or consider a maximum of 70 objects. To solve the problem of indoor localization, as a rule, the buildings are equipped with stationary Bluetooth devices, the GPS coordinates of which are determined during installation. In addition, maps of Bluetooth signal strength in a room can also be used to more accurately determine the coordinates of an object, including the coordinates of a dynamic object. In general, a review of known approaches indicates the relevance and practical usefulness of our research.

3. Research problem

The flowchart of the proposed approach is shown in Figure 1.

Let us consider the general formulation of the problem in three-dimensional space. Let at the current moment of time $t \in T$ a swarm of moving objects be described by a set of objects $O = \{Ob_i \mid i = \overline{1,N}\}$ with coordinates $(x_i, y_i, z_i) \in X \times Y \times Z$. Objects from O are divided into two groups, depending on their equipment. The group of active objects $AO = \{Ob_j^a \mid j = \overline{1,M}\} \subseteq O$ is equipped with BlueTooth receivers of radio signals from passive objects, GPS sensors for determining their coordinates and transmitters of coordinate information in the SCC. The group of passive objects $PO = \{Ob_k^p \mid k = \overline{1,K}\} \subseteq O$ is equipped with short-range BlueTooth transmitters $(r \approx 10m)$. These transmitters, with a certain periodicity $\triangle t$, emit a radio signal with a unique identifier $Ob_k^p \in PO$, for example, number $k = \overline{1,K}$. Groups of objects are organized so that $AO \cap PO = \emptyset, AO \cup PO = O$. Objects $Ob_j^a \in AO$ receive radio signals only from objects $Ob_k^p \in PO$ that are in the detection zone. Then the object $Ob_j^a \in AO$ periodically transmits to the SCC the numbers of all detected objects $Ob_k^p \in PO$ and its unique number j and its coordinates, which are determined using GPS. Let us assume that the accuracy of determining GPS coordinates is $\xi \approx 5m$.

Thus, to determine the coordinates of passive objects, SCC receives data in the form of a tuple $I(Ob_j^a) = \langle (x_j, y_j, z_j); B_j \rangle$, where $B_j \subseteq PO$ is the set of passive objects, detected by the object $Ob_j^a \in AO$. The interaction scheme of active, passive objects and SCC in three-dimensional space is shown in Figure 2.

In this scheme, three options are possible, on which the CDEs will depend.

1. If a passive object is not identified by any of the active objects, it is considered missing. However, it may appear at the next moment in time as soon as it is detected by any active object.

2. If a passive object is detected by one active object, the maximum CDE is equal to the sum of the

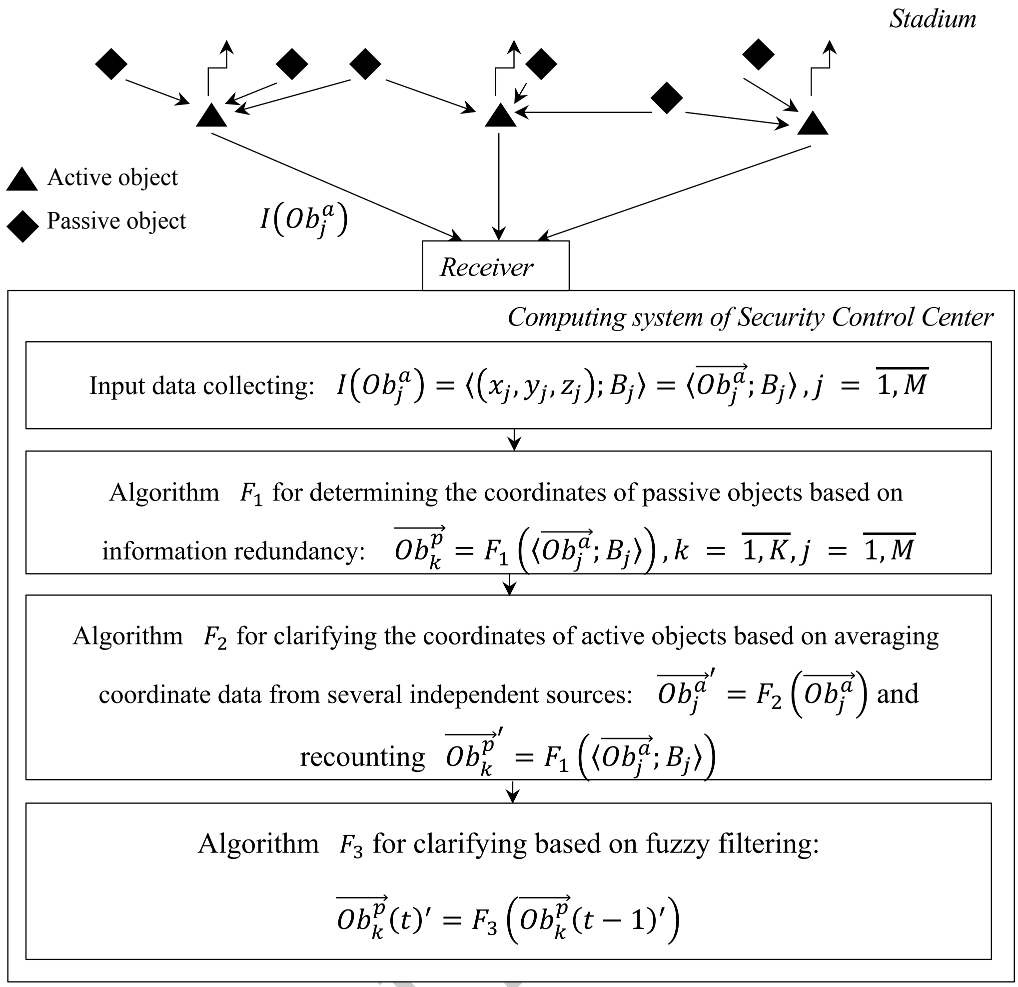

Figure 1. The flowchart of the proposed approach.

range of the BlueTooth transmitter and the maximum error of the GPS sensors. In this case, there is no information redundancy.

3. If a passive object is detected by several active objects, then the CDE of the passive object can be reduced due to information redundancy. We consider the last two cases as the main conditions for the proposed approach. The general problem of our research is to develop a set of algorithms for determining the coordinates of a passive object and refining them to ensure an error that would not be greater than CDE using GPS ($\xi \leq 5m$). In other words, under the most favorable conditions, a set of algorithms should provide a CDE of passive objects that can be compared with the error of a GPS sensor.

The development of three algorithms corresponds to three partial research problems. Composition of the algorithm complex:

1. An algorithm for determining the coordinates of passive objects based on information redundancy, which appears due to the detection of one passive object by several active objects, the coordinates of which are known with a GPS error ($\xi \leq 5m$). Since the result of the intersection of detection zones is some area, the coordinates of a passive object can be determined with an accuracy equal to the size of this area. The more active objects, the more intersections of detection zones, the smaller the size of the area and, therefore, the more accurately the coordinates of the passive object will be determined.

2. An algorithm for clarifying the coordinates of active objects based on averaging coordinate data

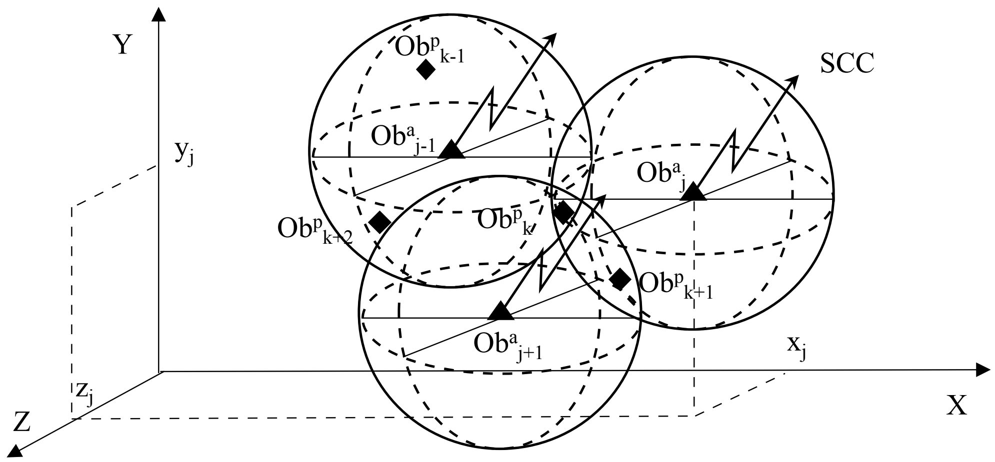

Figure 2. Scheme of interaction of active, passive objects and SCC in three-dimensional space.

from several independent sources. If you use several independent data sources, the errors of which are unbiased, then when averaging the data, the errors in determining the coordinates are inversely proportional to the square root of the number of sources. The algorithm increases the accuracy of determining the coordinates of active objects, which in turn increases the accuracy of determining the coordinates of passive objects.

3. Algorithm for clarifying the coordinates of passive objects based on fuzzy filtering. To clarify the current coordinates, the algorithm uses a time series of measurements.

Further, to simplify the description of the algorithms, we will use one-dimensional linear space, that is, we will consider the location of objects on the X coordinate axis. In the case of three-dimensional space, it is only necessary to replace the one-dimensional distance with the three-dimensional Euclidean metric. The essence of the algorithms will not change.

4. Algorithm for defining coordinates of passive objects

4.1. The main idea of the algorithm and simple example for its explaining

The main idea of the algorithm is to determine the coordinates of passive objects as areas of intersection of detection zones of active objects. The more active objects that have detected a passive object, the more intersections of detection zones, the smaller the size of the intersection area, and the smaller the CDE of the passive object.

We will assume that active objects $Ob_j^a \in AO$ detect of passive objects $Ob_k^p \in PO$ with different levels of confidence depending on the distance L between the objects. The further away a passive object is located, the less possibility detecting it. The scheme of objects placement and detection zones is shown in Figure 3.

The functions $\mu_j\colon X \to [0,1]$ describe the possibility of detecting passive objects $Ob_k^p \in PO$, which are located at points $x_k \in X$, by active objects $Ob_j^a \in AO$.

In the "transmitter-receiver" channel there is a discarding threshold α_j. We will assume that if $\mu_j(x) < \alpha_j \in [0,1]$, then the detection of the passive object Ob_k^p, which is located at point $x_k \in X$, fails. The threshold α_j depends on the conditions of emission, propagation and reception of the radio signal that is

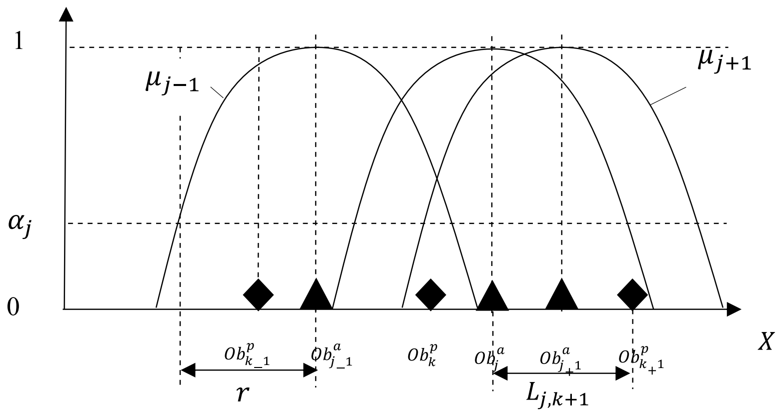

Figure 3. The scheme of objects placement and detection zones on linear space X.

received by the j-th active object. In practice, the threshold value fluctuates around the value $\alpha_j \approx 0.2$. Therefore, we will consider the threshold α_j to be a random variable that is distributed in accordance with the normal distribution law and has a mathematical expectation $M[\alpha_j] = 0.2$. The value of α_j mainly depends on the transmitter and receiver and can be determined experimentally.

To clearly describe the approach to determining the coordinates of passive objects, let's consider a simplified example.

Example. Determining the coordinates of a passive object using information from two active objects.

Let in space X at some moment of time $t \to T$ there are two active objects $Ob_j^a, j = 1, 2$ and one passive object Ob_1^p. Active objects have coordinates $x_1, x_2 \in X$, and passive objects have coordinates $y \in X$, as shown in Figure 4. It is necessary to calculate an estimate of the $\widehat{y}$ coordinate of the passive object Ob_1^p.

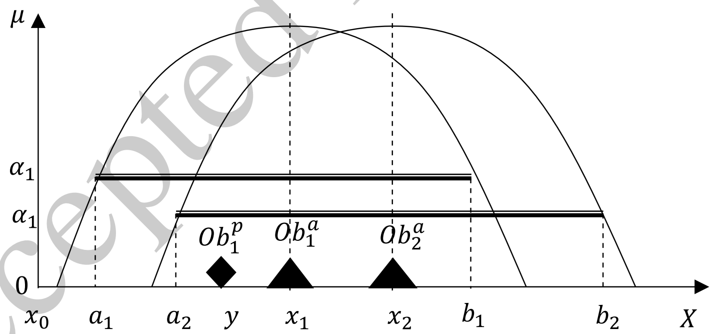

Figure 4. Interaction of two active objects with a passive object.

Let the j-th active object Ob_j^a have the ability to detect a passive object Ob_1^p in the interval $[a_j, b_j] \in P(X)$, where $P(X)$ is the set of all subsets of the set X. Therefore, between sets of active objects and the coordinates the relation $R_1 \subseteq AO \times X$ can be established. The relation R_1 induces an indexing map $v\colon AO \to P(X)$ [29], under which $v(Ob_j^a) = [a_j, b_j] \subseteq X$.

The detection interval $v(Ob_j^a)$ will correspond to α_j-level set $M_{\alpha_j}^j = \{x \in X \mid \mu_j(x) \geq \alpha_j\} \subseteq X$ of detection function $\mu_j(x)$. Then, if the passive object Ob_1^p has coordinate $y \in M_{\alpha_j}^j \subseteq X$, then $\chi_j(y) = 1$, where $\chi_j(\cdot)$ is the characteristic function of the α_j-level set $M_{\alpha_j}^j$, that is,

$$\chi_j(y) = \begin{cases} 0, y \notin M_{\alpha_j}^j, \\ 1, y \in M_{\alpha_j}^j. \end{cases}$$

For two active objects $Ob_j^a, j = 1, 2$, the entire space X will be divided into four intervals $X_m \subseteq X, m \in \overline{1, 4}$, as shown in Table 1.

Table 1. Intervals with binary code

№ interval	$\chi(X_m) = (\chi_1(x), \chi_2(x)), x \in X_m$ binary interval code	Interval on X	decimal interval code
1	$(0, 0)$	$X_1 = [x_0, a_1[\cup]b_2, \infty[$	0
2	$(1, 0)$	$X_2 = [a_1, a_2[$	2
3	$(1, 1)$	$X_3 = [a_2, b_1]$	3
4	$(0, 1)$	$X_4 = [b_1, b_2]$	1

For this example, any point $x \in X$ will belong to one of the intervals $X_m \subseteq X, m = \overline{1, 4}$. This point will correspond to a binary vector of values of characteristic functions $\chi(x) = (\chi_1(x), \chi_2(x)), x \in X_m$. Since the points $\forall x_i, x_j \in X_m$ are not distinguishable within the interval X_m, that is, $v^{-1}(x_i) = v^{-1}(x_j) \subseteq AO$, then the vector $\chi(X_m) = \chi(x_i) = \chi(x_j)$ can be considered a vector of the interval $X_m \subseteq X$. In this case, for the interval X_m, the vector $\chi(X_m)$ defines a subset of active objects $A(X_m) = \{Ob_j^a \in AO \mid \chi_m(x) = 1, x \in X_m\} \subseteq AO$, which will detect a passive object if it has a coordinate in a given interval. For example, for the interval $X_2 \subseteq X, A(X_2) = \{Ob_1^a\}$, and for $X_3 \subseteq X, A(X_3) = \{Ob_1^a, Ob_2^a\}$. For this set the relation $A(X_m) = v^{-1}(x), x \in X_m$ holds.

In accordance with the statement of the problem, the information that is transmitted to the SCC from the object Ob_j^a has the form: $I(Ob_j^a) = \langle x_j; B_j \rangle$, where $B_j \subseteq OP$ is a subset of passive objects detected j-th active object. This information contains numbers of passive objects with coordinates $y \in X$, for which $\chi_j(y) = 1$. The relation $R_2 \subseteq AO \times PO$ is established according to the information $I(Ob_j^a)$. The relation R_2 corresponds to the indexing map $\varphi : OA \to P(PO)$, where $P(PO)$ is the set of all subsets of the set of passive objects and $\varphi(Ob_j^a) \subseteq PO$. The inverse mapping can be written: $\varphi^{-1} : PO \to P(AO)$, where $P(AO)$ is the set of all subsets of the set of active objects. The mapping φ^{-1} for an arbitrary object Ob_1^p puts into correspondence the set of active objects $\varphi^{-1}(Ob_1^p) \subseteq AO$. Then the object Ob_1^p with coordinate $y \in X$ corresponds to the binary vector $\chi(Ob_1^p) = (\chi_1(y), \chi_2(y))$. For the example shown in Figure 4, the inverse mapping is defined as $\varphi^{-1}(Ob_1^p) = \{Ob_1^a, Ob_2^a\}$, and the vector $\chi(Ob_1^p) = (1, 1)$ is defined in according to Table 1.

It is natural to assume that if the object Ob_1^p has a coordinate $y \in X_m$, then the vectors $\chi(X_m)$ and $\chi(Ob_1^p)$ will coincide. This coincidence is valid for the precise determination of the detection zone $[a_j, b_j]$, that is, when the GPS coordinate x_j of the object Ob_j^a and the level α_j are clearly defined for a fixed detection function $\mu_j(x)$ of the j-th active object. In this case, the zero vector $\chi(Ob_1^p)$ will indicate that the passive object Ob_1^p is outside the detection zone.

Thus, using the information $I(Ob_j^a)$ we can state that the passive object Ob_1^p has a coordinate with a value in the interval X_m for which the condition $\chi(Ob_1^p) = \chi(X_m)$ holds. In other words, as an estimate $\widehat{y}$ of the coordinate $y \in X_m$, the coordinate of one of the points in the interval X_m can be chosen, for example, a point with the coordinate:

$$\widehat{y} = 0.5 \cdot (x_{\max}^m + x_{\min}^m), y \in X_m \subseteq X, \tag{1}$$

where: x_{max}^m, x_{min}^m – coordinates of the points at the ends of the m-th interval. For the example in Figure 4, the estimate of the coordinates of the passive object will be $\widehat{y} = 0.5 \cdot (b_1 + a_2)$.

As can be seen from expression (1), the estimate of the coordinate of a passive object $\widehat{y} \in X_m$ can be defined as one of the characteristics of the interval, for example, the middle or center of mass of the characteristic function of the interval $X_m \subseteq X$. Such a coordinate estimate will naturally have an error. The maximum error will be determined by the value of the norm metric $l_p = \|x^m_{max}, x^m_{min}\|_p$ [21].

4.2. Determination of the coordinates of a passive object using information from an arbitrary number of active objects

Let the set $AO = \{Ob^a_j \mid j = \overline{1, M}\} \subseteq O$ of active objects be located in space X. At the moment t, each of the active objects Ob^a_j detects a certain subset passive objects and transmits information $I(Ob^a_j) = \langle x_j; B_j \rangle$ to SCC. This information defines the mapping $\varphi^{-1} : PO \to P(AO)$. Then the set of active objects that detected the passive object Ob^p_k can be defined as a *coimage* of the relation R_2 in the form $\varphi^{-1}(Ob^p_k) = coim_{R_2}(O^p_k) \subseteq OA$. The set $coim_{R_2}(Ob^p_k)$ will correspond to the vector $\chi(Ob^p_k) = (\chi_1(x_k), \dots, \chi_M(x_k))$.

Since we assumed that all objects are motionless at time t, the space X can be divided into a set of intervals X_m in such a way that $X = \cup_{m=1}^{N_m} X_m, \forall m, s \in \overline{1, N_m}, X_m \cap X_s = \varnothing$. Since the mapping $v : AO \to P(X)$ is surjective, the set of intervals $\{X_m \mid m = \overline{1, N_m}\}$ forms a covering of the space X. Therefore, the current coordinate $x_k \in X$ of the passive object is Ob^p_k will definitely belong to one of the intervals $X_m, m = \overline{1, N_m}$. Each interval X_m will have its own vector $\chi(X_m) = (\chi_1(x), \dots, \chi_M(x)), x \in X_m$. The subset $v(Ob^a_j) = im_{R_1}(Ob^a_j) \subseteq X$ is the image of the active object Ob^a_j in the coordinate space X. If the passive object Ob^p_k is detected by several active objects from the subset $coim_{R_2}(Ob^p_k)$, then the mapping will be executed:

$$\Gamma(coim_{R_2}(Ob^p_k)) = \bigcap_{Ob^a_j \in coim_{R_2}(Ob^p_k)} im_{R_1}(Ob^a_j) = X_m \subseteq X. \tag{2}$$

Based on expression (2), if a passive object is detected by several active objects, the area $\Gamma(coim_{R_2}(Ob^p_k)) \subseteq X$ will satisfy the condition: $\forall j = \overline{1, M}, Card\{X_m\} \leq Card\{M^j_{\alpha_j}\}$, which allows you to clarify the coordinates Ob^p_k.

Note that the mapping:

$$\Gamma'(X_m) = \bigcap_{x \in X_m} v^{-1}(x) = A(X_m) \subseteq AO \tag{3}$$

together with mapping (2) form a Galois correspondence for the relation $R_1 \subseteq X \times AO$ with the corresponding properties [29].

Thus, the mappings between the coordinate space X, the space of active objects AO and the space of passive objects PO form a commutative diagram, as shown in Figure 5.

The procedure for determining the coordinates of an arbitrary passive object Ob^p_k at the current time will be determined by the mapping $g : PO \to P(X)$, which is specified by a composition of mappings of the form $g = \varphi^{-1} \circ v$. This mapping assigns to each passive object Ob^p_k a subset of coordinates from the interval X_m, that is, $g : PO \to P(X)$, and $g(Ob^p_k) = X_m \subseteq X$. In turn, the determination of the interval X_m is carried out from the condition $\chi(Ob^p_k) = \chi(X_m)$.

$$
\begin{tikzcd}
 & AO \arrow[dl, "\varphi"'] \arrow[d, "\nu"] \\
PO \arrow[r, "g"'] & X
\end{tikzcd}
$$

Figure 5. Diagram of spaces mappings.

Thus, the algorithm for estimating the coordinates of an arbitrary passive object Ob_k^p at the current time $t \in T$ consists of performing the following sequence of steps.

Step 1. Defining the coverage $\{X_m \mid m = \overline{1, N_m}\}$, which corresponds to the location of the set of active objects $AO = \{Ob_j^a \mid j = \overline{(1, M}\} \subseteq O$ and their detection capabilities Ob_k^p.

Step 1.1. For each $X_m \subseteq X$, determining the subset of active objects $A(X_m) \subseteq AO$ that detected a passive object in this interval, as well as determining the corresponding vector of characteristic functions $\chi(X_m) = (\chi_1(x), \ldots, \chi_M(x)), x \in X_m$.

Step. 1.2. Definition of the set of coordinates of the interval $X_m \subseteq X$ as $\Gamma(A(X_m)) = \bigcap_{(Ob_j^a \in A(X_m))} im_{(R_1)}(Ob_j^a) \subseteq X$.

Step 2. In accordance with the information $I(Ob_j^a), j = \overline{1, M}$, determining the subset of active objects that detected the passive object Ob_k^p as the inverse mapping $\varphi^{-1}(Ob_k^p) = coim_{R_2}(Ob_k^p) \subseteq AO$, as well as the definition of the vector of characteristic functions $\chi(Ob_k^p) = (\chi_1(x_k), \ldots, \chi_M(x_k))$.

Step. 3. Finding the interval $X_m^* \subseteq X$ into which the coordinate of the object Ob_k^p falls in accordance with the condition of matching sets $A(X_m^*)$ and $\varphi^{-1}(Ob_k^p)$, that is, when $\chi(X_m^*) = \chi(Ob_k^p)$.

Step. 4. Determination of the estimate of the coordinate $\widehat{y_k}$ of the object Ob_k^p as one of the characteristics of the interval X_m^*, for example, as the midpoint:

$$\widehat{y_k} = \frac{1}{2} \cdot (\min_{x \in X_m^*} \Gamma(A(X_m^*)) + \max_{x \in X_m^*} \Gamma(A(X_m^*))). \tag{4}$$

Thus, the estimate of the coordinate $\widehat{y_k}$ of any passive object Ob_k^p, which is located in the detection zone of at least one active object, can be defined as one of the characteristics of the interval X_m^* in which the coordinate of this object falls. Therefore, let us consider further how we can define the set of intervals $X_m \subseteq X$.

Defining set of intervals for space coverage.

Previously, we established that for each interval X_m we can uniquely determine a subset of active objects $A(X_m) = \{Ob_j^a \in AO \mid \chi_m(x) = 1, x \in X_m\} \subset AO$, which will detect a passive object located in this interval. In this case, $\forall x_i, x_j \in X_m$ we have $v^{-1}(x_i) = v^{-1}(x_j) = A(X_m)$, that is, the vector $\forall x_i, x_j \in X_m, \chi(X_m) = \chi(x_i) = \chi(x_j) = const$. Note that $\forall X_m \subseteq X, Card(A_m) \leq Card\{AO\} = M$. The function $Card(A_m) \in [0, M]$ determines the intensity of coverage of space X by active objects, for example, as shown in Figure 6 for the case of three active objects.

Since the set of intervals $\{X_m \mid m = \overline{(1, N_m}\}$ is ordered in increasing order m and $X = \bigcup_{m=1}^{N_m} X_m, \forall m \in \overline{1, N_m - 1}, X_m \cap X_{m+1} = \varnothing$, and each interval $X_m \subseteq X$ corresponds to its own subset of active objects $A_m \subseteq AO$, then we have the opportunity to determine the boundaries of the intervals $X_m \subseteq X$. To

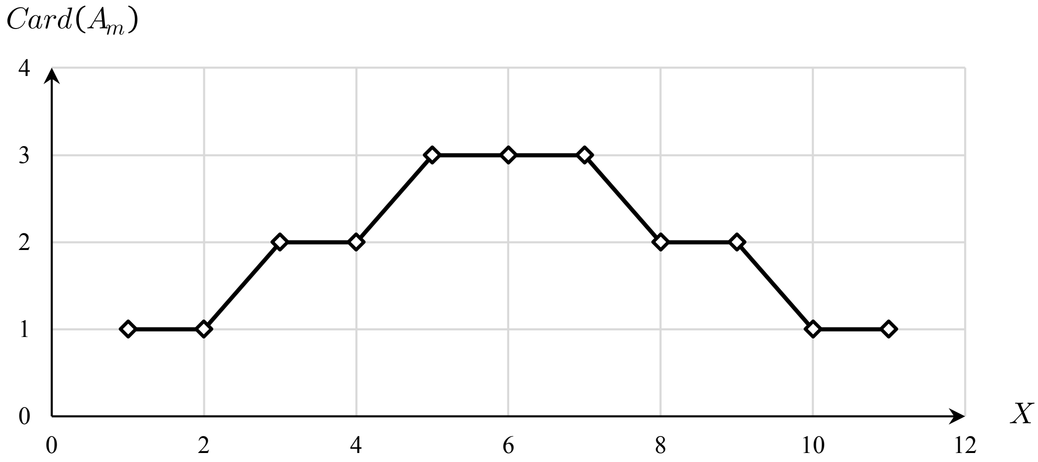

Figure 6. Function $Card(A_m)$ of coverage intensity of space X.

do this, on the set AO we define an adjacency (similarity) function of two sets of the form $v(A_i, A_j) : A_i \times A_j \to [0, 1]$, where $A_i, A_j \subseteq AO$ are subsets of active objects on the intervals $X_i, X_j \subseteq X$. This function is a positive definite, symmetric function, for which the expression $\forall A_i, A_j \subseteq AO, v(A_i, A_j) \leq v(A_i, A_i)$, turns into equality only if $A_i = A_j$ [21].

The function $v(A_i, A_j)$ is, by definition, bounded above by one. Therefore, this function can be obtained from the expression $v(A_i, A_j) = 1 - d(A_i, A_j)$, where $d(A_i, A_j)$ is the distance that is defined on the set of subsets of active objects. It is advisable to consider this distance as the Tanimoto distance (biotope distance) of the form [21]:

$$d(A_i, A_j) = \frac{Card(A_i \triangle A_j)}{Card(A_i \cup A_j)} = 1 - \frac{Card(A_i \cap A_j)}{Card(A_i \cup A_j)}, \tag{5}$$

where $A_i \triangle A_j = (A_i \cup A_j) \setminus (A_i \cap A_j)$ - symmetric set difference.

Then the adjacency function $v(A_i, A_j)$ will have the following form:

$$v(A_i, A_j) = \frac{Card(A_i \cap A_j)}{Card(A_i \cup A_j)} \in [0, 1], \tag{6}$$

This function will satisfy the properties of boundedness: $\forall A_i \cap A_j = \varnothing, v(i, j) = 0, \forall A_i = A_j, v(i, j) = 1$ and monotonicity: $\forall A_i \cap A_j \subseteq A_k \cap A_p, v(i, j) \leq v(k, p)$. The function $v(A_i, A_j)$ for the sets A_i, A_j reflects the degree of similarity (adjacency) of these sets, and the higher the value of the function, the more they coincide. Note that the function $v(A_i, A_j)$ can be considered for the set of subsets $\{A_i \mid i = \overline{1, n}\}$ in the following form:

$$v(A_i \mid i = \overline{1, n}) = \frac{Card(\bigcap_{i=1}^{n} A_i)}{Card(\bigcup_{i=1}^{n} A_i)}. \tag{7}$$

The adjacency function (7) shows the degree of similarity of n subsets. It is a non-increasing function of n and can be used to construct a hierarchical set of coverings $\{X_m \mid m = \overline{1, N_m}\}$ for subsequent management and coordination of groups of objects from O.

Using the adjacency function $v(A_i, A_j)$ allows us to determine the coordinates of the intervals $X_m \subseteq$

X covering the space X. To prove this, consider a discrete space X with a sampling step $\delta > 0$, where $x_0 \in X$ is a fixed origin on X. Then $x_{l+1} = x_l + \delta$ is the discrete coordinate $x_{l+1} \in X, l = \overline{1, L}$. For $\delta \to 0$, the space X can be considered as a continuous space X. For adjacent coordinates $\forall x_l, x_{l+1} \in X$, consider the function $v(A_l, A_{l+1})$, where $A_l = v^{-1}(x_l) \subseteq AO$ – a set of active objects that detect a passive object at point x_l. If points $x_l, x_{l+1} \in X$ are indistinguishable from the view point of coverage by active objects, then the function $v(A_l, A_{l+1}) = 1$. However, as soon as the sets do not coincide, the relation $v(A_l, A_{l+1}) < 1$ holds and, therefore, the coordinates x_l, x_{l+1} belong to different subsets $X_m \subseteq X$. Then, based on the properties of the function $v(A_l, A_{l+1})$ on the space X, we will have the graph shown in Figure 7.

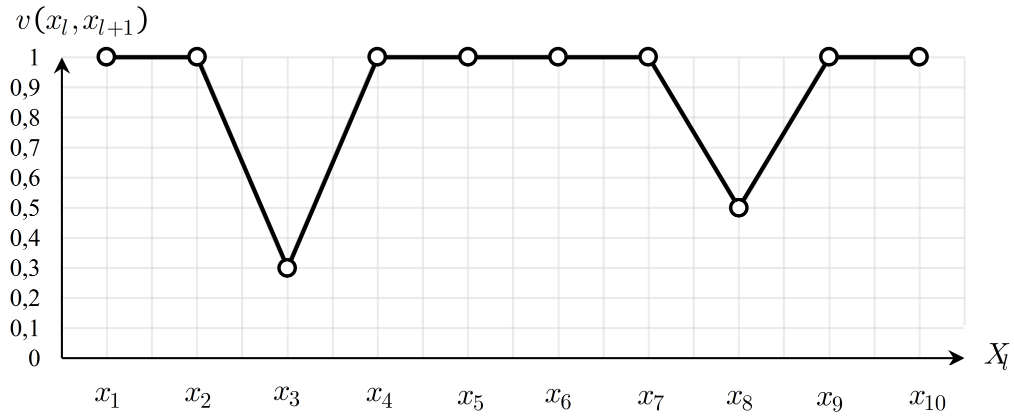

Figure 7. Form of function $v(A_l, A_{l+1})$)

Points x_{l^*}, for which $v(A_{l^*}, A_{l^*+1}) < 1$, are boundary points of the intervals $X_m \subseteq X$. We use the distance function (5) to determine the step function $\sigma(x_l) : X \to N$ of the form:

$$\sigma(x_l) = \sum_{x_l \in X} \delta(x_l), \delta(x_0) = 1, \tag{8}$$

where N – set of natural numbers, $\delta(\cdot)$ - Dirac function, $\delta(x_l) = \begin{cases} 0, & d(A_l, A_{l+1}) = 0, \\ 1, & d(A_l, A_{l+1}) > 0. \end{cases}$

The step function $\sigma(x_l)$ is a non-decreasing function of the coordinates $x_l \in X$ and for the function in Figure 7 it will have the form as shown in Figure 8.

The coordinates of points from the set $\{x_l^* \in X \mid \sigma(x_l) \neq \sigma(x_{l-1})\}$, where the function $\sigma(x_l)$ jump occurs, determine the coordinates of the beginning and end of the corresponding interval X_m covering the space X. The value of the function $\sigma(x_l)$ determines the number of the interval to which the coordinate $x_l \in X$ and, accordingly, the vector $\chi(X_m)$ belongs.

As can be seen from the presented algorithm, the accuracy of determining the coordinates of coverage intervals directly depends on the accuracy of determining the coordinates of active objects. As shown in the next section, the presence of several active objects in space X allows us to refine their coordinates.

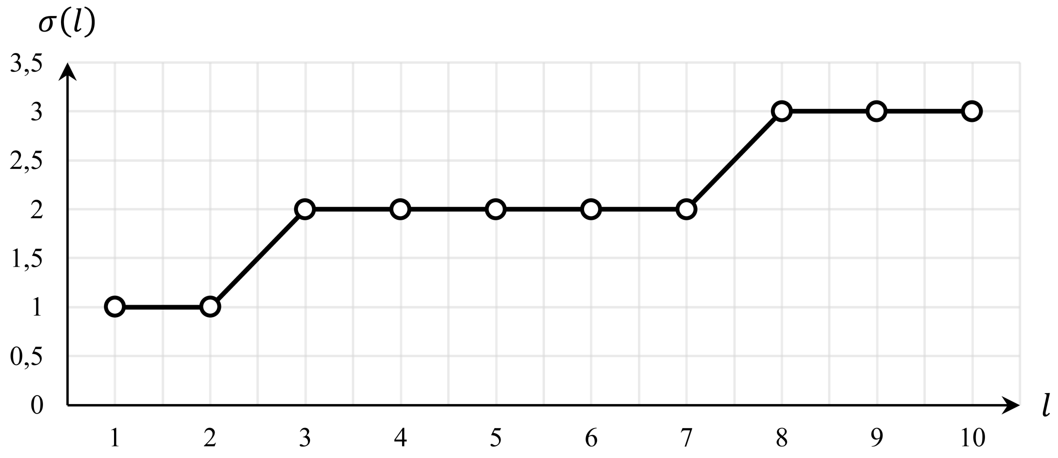

Figure 8. Step function $\sigma(l)$

5. Algorithm for clarifying the coordinates of active objects

The main idea of the algorithm is as follows. In our problem, the coordinates of active objects are determined using GPS sensors. In turn, these coordinates are used to determine the coordinates of passive objects. However, we can determine the coordinates of active objects using the adjacency function. In particular, if you use the adjacency function $v(B_1, B_2)$, defined on two subsets of passive objects B_1 and B_2, and also select some reference active object, then you can determine the relative coordinates of the entire group of active objects. If we assume that the errors of these sources are unbiased, then we can use the inverse dependence of the standard error value on the number of independent sources when averaging coordinate data. In [7], Douglas Curran-Everett showed that the standard deviation (the error in determining the coordinates for an active object) is inversely proportional to the increase in sample size (the number of passive objects that are detected by this active object).

In other words, here it becomes possible to clarify the coordinates of active objects, which will allow reducing the CDEs of passive objects, since the accuracy of determining the interval X_m^* largely depends on the accuracy of determining the coordinates of active objects that form the coverage $\{X_m \mid m = \overline{1, N_m}\}$.

Consider the problem of determining the relative distance between two active objects. To solve this problem, the adjacency function (6) can be used. Let us assume that passive objects are approximately uniformly distributed in space X. Then it is obvious that the greater the distance between two active objects $Ob_j^a, j = 1, 2$, the smaller the value of the adjacency function $v(B_1, B_2)$, where $B_1, B_2 \subseteq PO$. That is, in the case of a large distance between active objects, fewer passive objects will fall into the area of intersection of their detection zones. The dependence of the distance on the adjacency function $L(v(B_1, B_2))$ is shown in Figure 9.

The accuracy of the determination $L_{1,2} = L(v(B_1, B_2))$ depends on the number and location of passive objects that are detected by active objects $Ob_j^a, j = 1, 2$. Subject to two conditions:

random placement of passive objects in accordance with the normal distribution law and

random size of the detection zone $M_{\alpha_j}^j, j = 1, 2$ with a normal distribution law of the threshold α_j

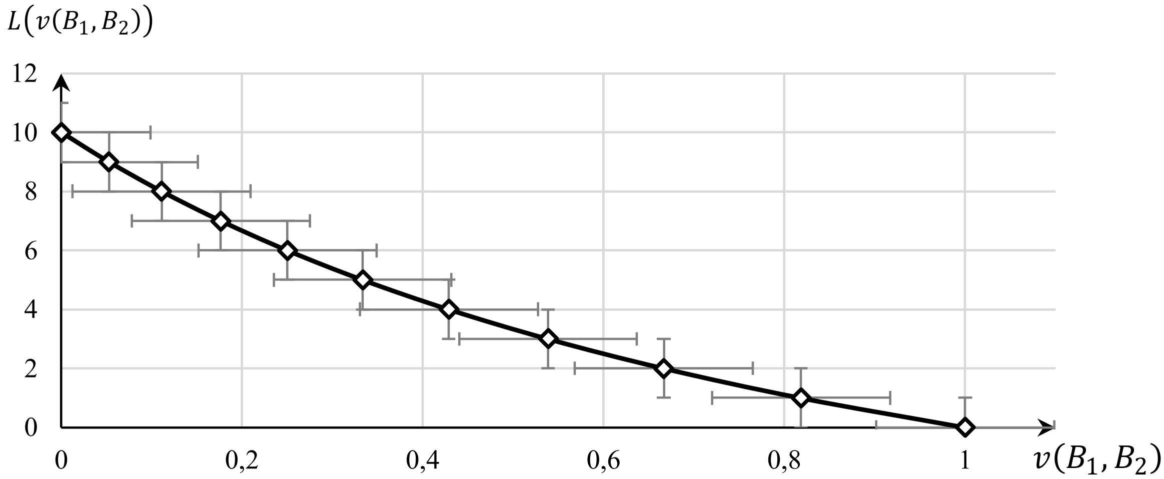

Figure 9. Dependence of the distance between two active objects on the adjacency function $v(B_1, B_2)$ indicating the standard error

and mathematical expectation $M[\alpha_j] = 0.2$,
the maximum error η of measuring the true distance $L_{1,2}$ will be $\eta_{max} \cong 2 \div 2.5m$. The error η will increase almost linearly with increasing distance $L_{1,2}$ between active objects. In addition, the error η will increase as the power of the set of passive objects decreases.

Note that the maximum error $\eta_{max} \approx 2.5m$ in determining the relative coordinates of active objects using the function $L(v(B_1, B_2))$ will be less than the maximum error in determining GPS coordinates: $\eta_{max}^{GPS} \approx 10m$. This allows you to clarify the coordinates of active objects.

Let's consider the principle of compensation for measurement errors.

Let $L_{i,j} = |x_i - x_j|$ - relative distance between objects $Ob_i^a, Ob_j^a \in AO$. GPS coordinates are measured relative to the origin $x_0 \in X$. Then the coordinate of the object Ob_i^a will be equal to $L_{0,i} = |x_i - x_0| = x_i$. Let us determine the coordinate $x_m \in X$ of the center of the group of active objects AO as follows:

$$x_m = L_{0,m} = \frac{1}{M} \cdot \sum_{i=1}^{M} L_{0,i}. \tag{9}$$

If we take into account that $L_{0,i} = x_i = \xi_i + x_i^u$ (where x_i is the GPS coordinate of the i-th active object measured with error, x_i^u is the true coordinate of the i-th active object, ξ_i is the determination error of GPS-coordinates), then the coordinate estimate of the center of a group of active objects can be presented as:

$$x_m = \frac{1}{M} \cdot \sum_{i=1}^{M} (x_i^u + \xi_i) = \frac{1}{M} \cdot \sum_{i=1}^{M} x_i^u + \frac{1}{M} \cdot \sum_{i=1}^{M} \xi_i = x_m^u + \xi_m, \tag{10}$$

where x_m^u - true coordinate of the center of a group of active objects.

From expression (10) it follows that the error $\xi_m = \frac{1}{M} \cdot \sum_{i=1}^{M} \xi_i$ in determining the coordinates of point $x_m \in X$ will decrease if the error in determining GPS coordinates $(M[\xi_i] = 0)$ will be unbiased and if the redundancy of covering the space X by active objects is sufficient. In other words, for large M the error $\xi_m = \lim_{n \to M} \sum_{i=1}^{M} \xi_i \to \varepsilon$, where $\varepsilon \ll \eta_{max}^{GPS}$.

Using this property allows you to minimize the CDEs of active objects and, accordingly, the CDEs of passive objects. Let's select some active object Ob_1^a as a reference and calculate the relative distances $L_{1,j}$ from it to any other object Ob_j^a using the formula:

$$\widehat{x_j} = \widehat{x_1} + L_{1,j} \cdot sign(x_j - x_1), \tag{11}$$

where $\widehat{x_1}$ is an estimate of the coordinate of the reference object Ob_1^a, which is obtained based on the coordinate of the point $x_m \in X$ of the "center of the group of active objects", in which the error in determining GPS coordinates is compensated.

As follows from expression (11), the accuracy of determining the coordinates of the active object Ob_j^a depends on the estimate of the coordinates of the reference object $\widehat{x_1}$, the relative distance $L_{1,j}$ and on the sign of the ratio between the GPS coordinates x_i,x_1, which are measured relative to the initial point $x_0 \in X$.

The estimate of the coordinate $\widehat{x_1}$ can be determined from expression (10) using expression (11) as follows:

$$x_m = \frac{1}{M} \cdot \sum_{j=1}^{M} L_{0,i} = \frac{1}{M} \cdot \sum_{j=1}^{M} (\widehat{x_1} + L_{1,j} \cdot sign(x_j - x_1)) = \widehat{x_1} + \frac{1}{M} \cdot \sum_{j=1}^{M} \cdot sign(x_j - x_1)).$$

Then the estimate of the coordinate of the reference object Ob_1^a can be defined as:

$$\widehat{x_1} = x_m - \frac{1}{M} \cdot \sum_{j=1}^{M} L_{1,j} \cdot sign(x_j - x_1). \tag{12}$$

Thus, the estimate of the coordinate of an arbitrary active object $Ob_i^a, i = \overline{1, M}$ will be determined by the following expression:

$$\widehat{x_i} = x_m - \frac{1}{M} \cdot \sum_{j=1}^{M} L_{1,j} \cdot sign(x_j - x_1) + L_{1,i} \cdot sign(x_i - x_1). \tag{13}$$

Using the x_m coordinate and relative distances from active objects to the reference object in this expression allows us to reduce errors in measuring the coordinates of a group of active objects and refine the coverage $\{X_m \mid m = \overline{1, N_m}\}$.

In general, the considered algorithm consists of the following steps.

Step 1. Selecting the reference object Ob_1^a.

It is necessary to select a reference object from the set of available active objects. This object can be anything, but the choice is discussed in the results discussion section.

Step 2. Determining the $x_m \in X$ coordinate of the center of the group of active objects using formula (9).

Step 3. Determining the estimate of the coordinate of the reference object $\widehat{x_1}$ using formula (12).

Step 4. Determination of estimates of the coordinates of all other active objects $\widehat{x}_i, i = \overline{1, M}$ using formula (13).

Example: Reducing CDEs of active objects

Let us assume that at the current time $t \in T$ we have coverage $\{X_m \mid m = \overline{1, N_m}\}$ for three active objects $Ob_j^a, j = \overline{1, 3}\}$, as shown in Figure 10. To visually distinguish the graphs in this figure, their maximum levels are shown with a slight shift up and down. For this coating, the initial data are presented in Table 2.

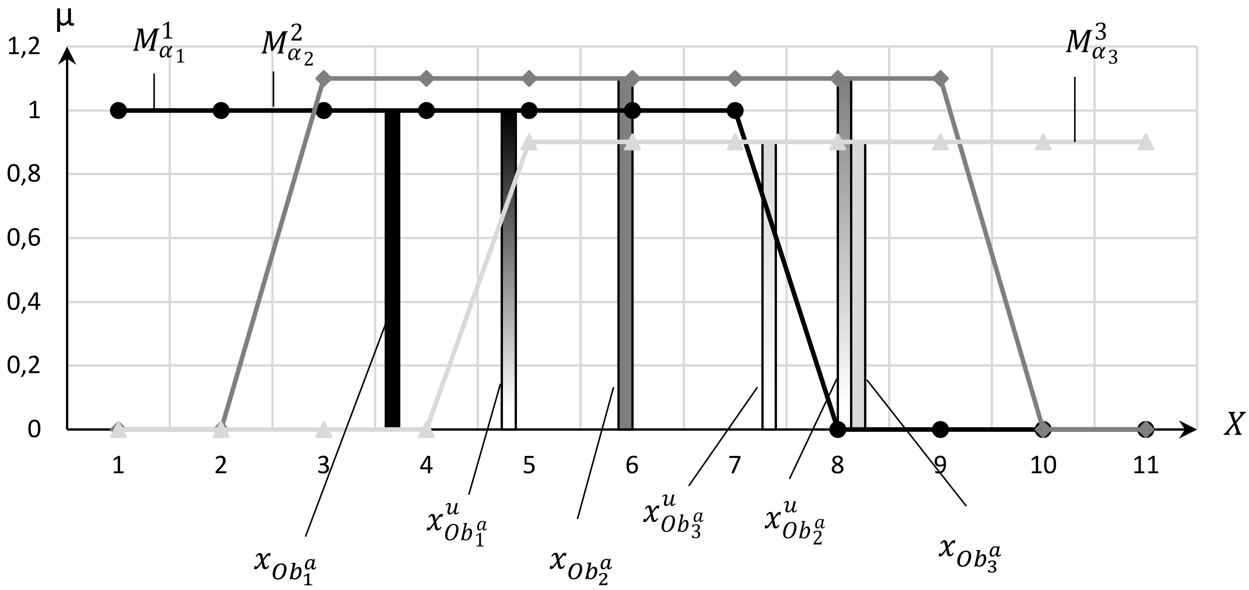

Figure 10. Coverage $\{X_m \mid m = \overline{1, N_m}\}$ for three active objects

Table 2. Initial data for the coating, which is presented in Figure 10

Ob_j^a	x_j	x_j^u	ξ_j
1	4	5	1
2	6	8	2
3	8	7	1
Average error 1.3333			

Under the condition of uniform distribution of passive objects, the adjacency function $v(B_i, B_j)$ and the corresponding distance estimates $L_{i,j}$ are presented in Table 3.

Table 3. The value of the adjacency function $v(B_i, B_j)$ and distance estimates $L_{i,j}$

$v(B_i, B_j)$	Ob_1^a	Ob_2^a	Ob_3^a	$L_{i,j}$	Ob_1^a	Ob_2^a	Ob_3^a
Ob_1^a	1.0000	0.4000	0.5556	Ob_1^a	0.0000	4.3186	2.7663
Ob_2^a	0.4000	1.0000	0.7500	Ob_2^a	4.3186	0.0000	1.2868
Ob_3^a	0.5556	0.7500	1.0000	Ob_3^a	2.7663	0.0000	1.2868

Let the error in measuring distances $L_{i,j}$ using the adjacency function be uniformly distributed (see Figure 9) and not exceed the value η_{max}, as shown in Table 4. In this case, the result of clarifying the coordinates of active objects is presented in Table 5.

Table 4. Error values for measuring distances $L_{i,j}$ using the adjacency function

ξ_{max}	Ob_1^a	Ob_2^a	Ob_3^a
Ob_1^a	0.0000	-0.5000	0.2000
Ob_2^a	-0.5000	0.0000	-1.2000
Ob_3^a	0.2000	-1.2000	0.0000

Table 5. Estimates of coordinates of active objects after clarification

	Ob_1^a	Ob_2^a	Ob_3^a
$\widehat{x}_j$	3.7384	7.5569	6.7047
ξ_j	1.2616	0.4431	0.2953
Average error 1.6667			

Comparing the values of the average errors given in Table 2 and Table 5, we see that for the case of only three active objects, the developed algorithm allows reducing CDEs by 2 times. If there are more active objects, the error reduction will be more significant. Note that the algorithm also reduces the CDEs of passive objects, since they are calculated using the coordinates of active objects.

6. Algorithm for clarifying the coordinates of passive objects based on fuzzy filtering

The main idea of the algorithm is to describe the coordinate of a passive object in the form of a membership function and to clarify the current coordinate (for $t \in T$) of the passive object based on the value of the previous coordinate (for $(t-1) \in T$) and of the value of the characteristic function, which describes the size of the interval (for $t \in T$), which contains the true value of the coordinate.

It was shown above that the coordinate of a passive object $Ob_k^p \in PO$ can be determined based on the interval $X_m^* \subseteq X$ from the coverage $\{X_m \mid m = \overline{1, N_m}\}$, and the center of mass of the characteristic function $\chi_k^*(x,t) : X \to \{0,1\}$ of this interval can be taken as an estimate of the coordinate Ob_k^p. It was also shown that the coordinates of passive objects can be refined by clarifying the coordinates of active objects. All these calculations take place at a fixed time $t \in T$, that is, they must be performed for each time sample in the overall SCC computation process.

If all objects from the set $O = AO \cup PO$ are moving, the interval X_m^* can be clarified using a fuzzy filtering algorithm [5]. In this case, the coordinate of the object Ob_k^p at time $t \in T$ will be represented as a membership function $\mu_k(x,t) : X \to [0,1]$, which will be described by the expression:

$$\mu_k(x,t) = \mu_k(x,t-1) + \beta \cdot (\chi_k^*(x,t) - \mu_k(x,t-1)), \tag{14}$$

where $\beta \in [0,1]$ – filter gain.

Then the estimate of the coordinate of the passive object Ob_k^p on the discrete set $X = \{x_l, l = \overline{1,N}\}$ at time $t \in T$ can be determined as follows:

$$\widehat{y}_k(t) = \frac{\sum_{l=0}^{N} x_l \cdot \mu_k(x_l,t)}{\sum_{l=0}^{N} \mu_k(x_l,t)}. \tag{15}$$

In this case, we have the opportunity to provide clarification and smoothing of the estimate of the coordinate of an arbitrary passive object $Ob_k^p \in PO$. To demonstrate this effect, consider an example.

Example. 3. Clarifying of the estimate of the coordinates of a passive object based on fuzzy filtering.

Let the movement of a group of objects $O = AO \cup PO$ (where $AO = \{Ob_j^a, j = \overline{1,3}\}$, $PO = \{Ob_1^p\}$) for time $T = \{t_i, i = \overline{1,5}\}$ has the form shown in Figure 11.

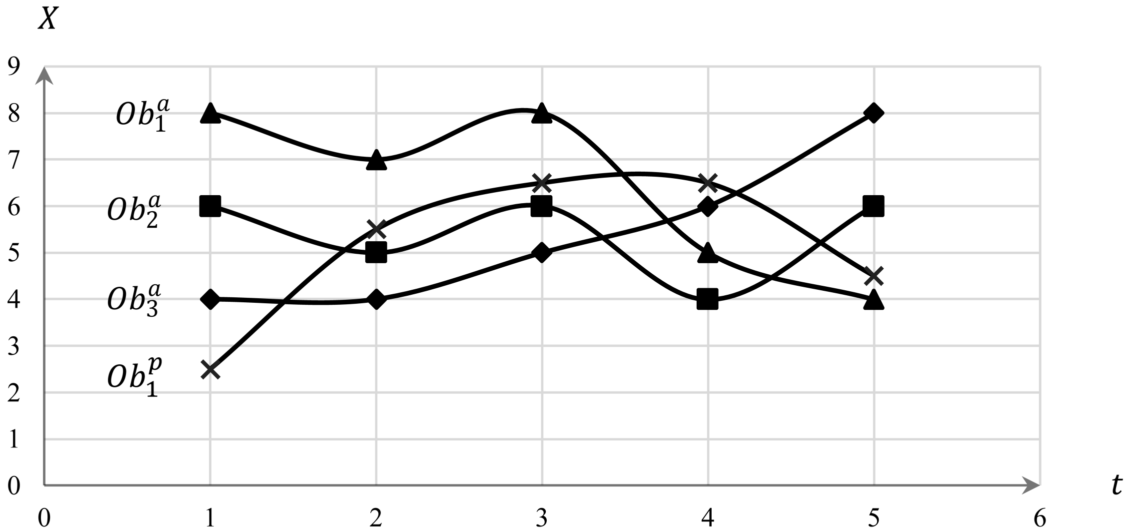

Figure 11. Movement of a group of active and passive objects

Functions $Card(A_i(X_m))$ (where $A_i(X_m) = \{Ob_j^a \in AO \mid \chi_m(x) = 1, x \in X_m, t_i \in T\} \subseteq AO$) for each $t_i \in T$ are shown in Figure 12.

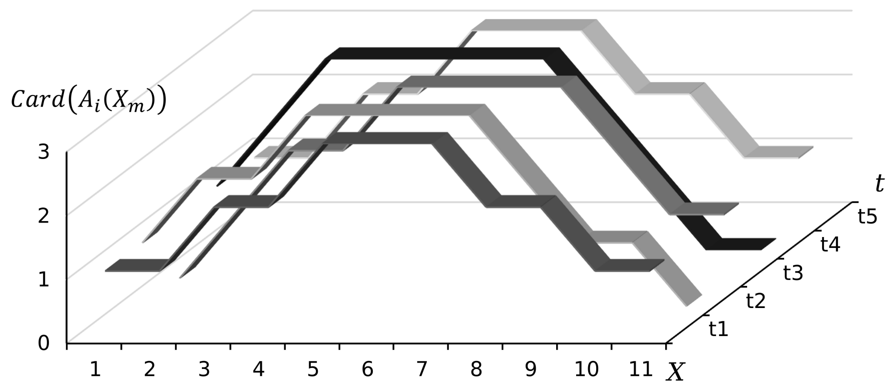

Figure 12. Functions $Card(A_i(X_m))$ for $T = \{t_i, i = \overline{1,5}\}$

Using the function $Card(A_i(X_m))$ for each moment of time $t_i \in T$, the coverage $\{X_m \mid m = 1, N_m\}$ was determined and using expression (4) the estimate of the coordinate $\widehat{y}(t_i)$ of the passive object Ob_1^p. To clarify the coordinate, a fuzzy filter (14) with a gain factor $\beta = 0.8$ was used, which made it possible to calculate a clarified estimate of the coordinate $\widehat{Y}(t_i)$ in accordance with expression (15). Comparative estimates of the coordinates of the passive object Ob_1^p, as well as errors, are presented in Table 6 and in

Figure 13.

Table 6. Comparative estimates of passive object coordinates

T	True coordinate $y(t_i)$	Estimation without filter $\widehat{y}(t_i)$	Estimation error without filter $\triangle\widehat{y}(t_i)$	Estimation with filter $\widehat{Y}(t_i)$	Estimation error with filter $\triangle\widehat{Y}(t_i)$
t_1	2.50	1.50	1.00	1.50	1.00
t_2	5.50	6.00	0.50	5.36	0.14
t_3	6.50	6.50	0.00	6.33	0.17
t_4	6.50	5.00	1.50	5.21	1.29
t_5	4.50	3.50	1.00	4.14	0.36

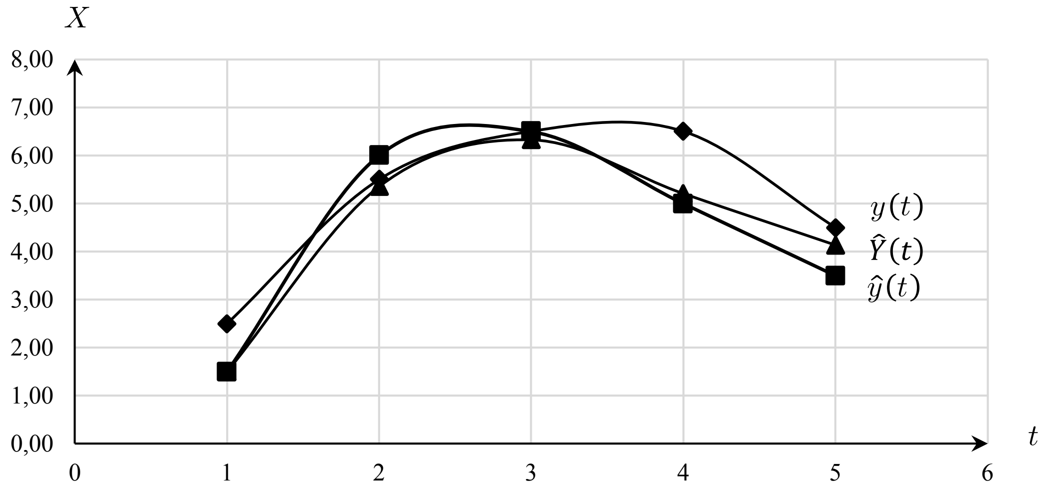

Figure 13. Comparative estimates of the coordinates of a passive object

Figure 14 shows two graphs of the error in estimating the coordinates of a passive object Ob_1^p: for the case without filter (14) $\triangle\widehat{y}(t_i)$ and with filter (14) $\triangle\widehat{Y}(t_i)$.

Analysis of the example data shows that in the case when a passive object is detected by three active objects, the use of a fuzzy filter (14) improves the accuracy of estimating the coordinates of this passive object by more than 26.

7. Discussion

We would like to discuss several questions regarding the minimization of object CDEs in the developed algorithms. We would also like to discuss the use of the developed algorithms in practice.

1. Conditions for minimizing CDEs of objects.

As follows from the description of the algorithm for determining the coordinates of passive objects, one measurement is enough to determine the coordinates. In this case, CDEs depend on two main factors.

1.1. *Configuration of the current coverage* $\{X_m \mid m = \overline{1, N_m}\}$. The maximum error is determined by the size of the maximum interval:

$$\triangle_{max} = \max_m \triangle^m_{max},$$

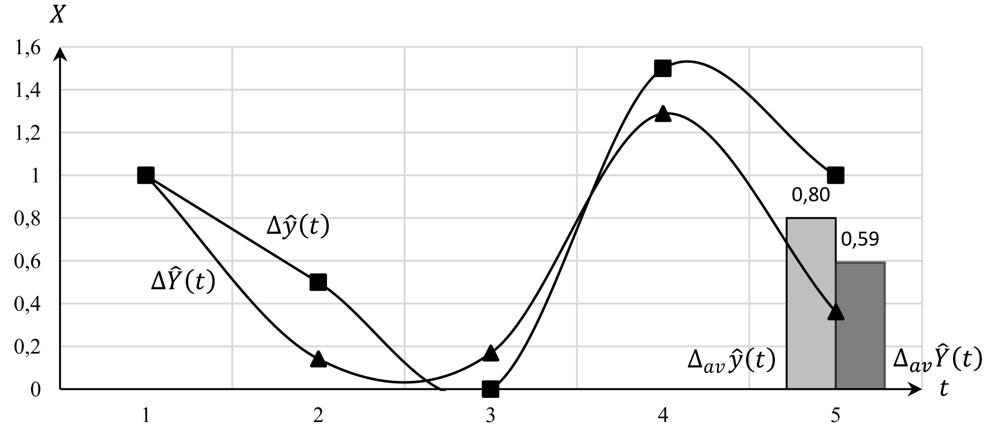

Figure 14. Graphs of CDEs of a passive object, where $\triangle_{av}\widehat{y}(t), \triangle_{av}\widehat{Y}(t)$ are the average error values $\triangle\widehat{y}(t_i)$ and $\triangle\widehat{Y}(t_i)$ respectively

where $\triangle_{max}^{m} = 0.5 \cdot (x_{max}^{m} - x_{min}^{m}), x_{max}^{m}, x_{min}^{m}$ - coordinates of the endpoints of the m-th coverage interval.

The larger the size of the intervals, the larger the possible CDE of the passive object $Ob_k^p \in PO$.

1.2. *Knowledge of the location of active objects* $Ob_j^a \in AO$ on X. The greater the error in determining the true coordinates of objects Ob_j^a, the greater the error in determining the boundaries of intervals and the greater the value of average errors in determining the coordinates of objects $Ob_k^p \in PO$. If a passive object is located on the boundary of an interval, it can be assigned to an adjacent interval, which increases the coordinate measurement error. In this case the maximum error will be:

$$\triangle_{max} = 2 \triangle_{max}^{m} + \triangle_{max}^{m+1},$$

where $\triangle_{max}^{m+1}$ - maximum error of the interval adjacent to the m-th.

To ensure the minimum CDE of a passive object, it is necessary to minimize the size of the X_m interval. This can be achieved through the following conditions:

the location of active objects must be rational, that is, active objects must provide such coverage of space X that will allow the formation of the maximum number of small intervals X_m in space X;

the number of active objects Ob_j^a can be increased (see also below);

errors in measuring the coordinates of active objects Ob_j^a should be minimized, which will ensure a reduction in the CDEs of the passive object.

The problem of managing the location of active objects to ensure minimal errors should be considered.

2. Conditions for minimizing errors when clarifying the coordinates of active objects.

As follows from the description of the algorithm for clarifying the coordinates of active objects, error reduction is ensured through two mechanisms:

integration of sensors for information about the coordinates of active objects with different natures of errors, namely GPS and Bluetooth errors.

use of redundancy of information sources when measuring coordinates.

In particular, the algorithm for clarifying the coordinates of active objects involves determining the coordinates of the reference active object Ob_1^a in accordance with expression 12. If in expression 12 we take into account the errors in measuring the x_m coordinate of the center of a group of active objects and the errors in determining the relative distance between active objects, then expression 12 can be written as follows:

$$x_1 = x_m^u - \frac{1}{M} \cdot \sum_{j=1}^{M} L_{1,j}^u \cdot sign(x_j - x_1) + \xi_j^{av} - \eta_j^{av},$$

where $\xi_j^{av} = \frac{1}{M} \cdot \sum_{j=1}^{M} \xi_j$ - average error in measuring the GPS coordinates of the active object Ob_j^a;

ξ_j – error in measuring the GPS coordinates of the active object Ob_j^a;

$\eta_j^{av} = \frac{1}{M} \cdot \sum_{j=1}^{M} \eta_j \cdot sign(x_j - x_1)$ - the average error in determining the relative distance between the reference object Ob_1^a and the object Ob_j^a using the adjacency function;

η_j - error in determining the relative distance between the reference object Ob_1^a and the object Ob_j^a using the adjacency function.

Errors ξ_j and η_j are independent and have zero mathematical expectation, and therefore, their averaging ensures a decrease in the average error. These errors for a group $(M = 11)$ of active objects are shown in Figure 15. As calculations show, average errors significantly reduce the CDE of the reference active object Ob_1^a.

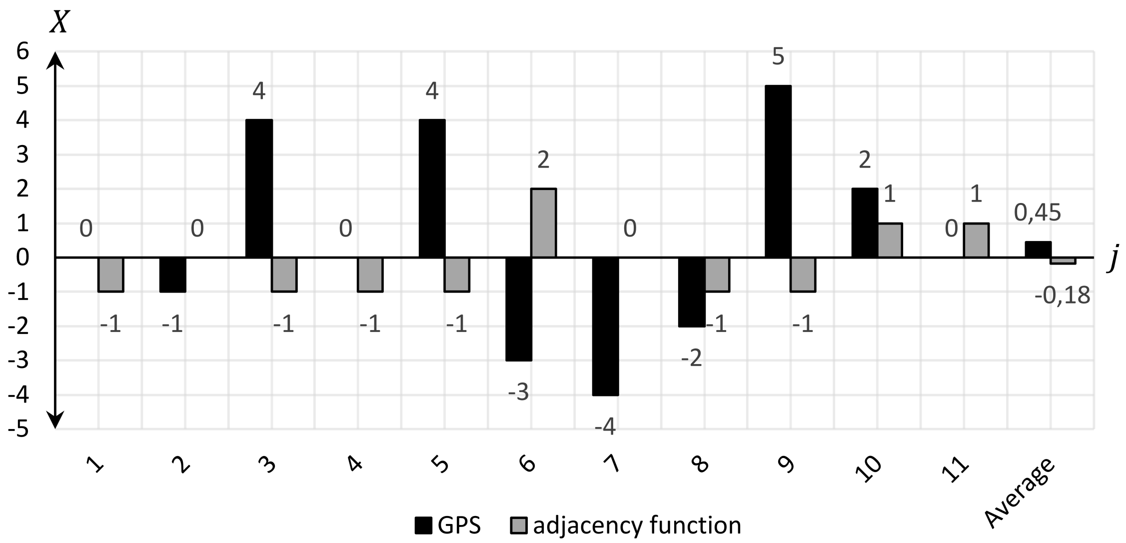

Figure 15. Coordinate measurement errors: using GPS (ξ_j) and using adjacency function (η_j)

With this in mind, the following positions can be used to minimize the CDEs of active objects:

2.1. To determine $L_{1,j}$, it is advisable to use the relative distance, which is determined using the adjacency function 6 (6).

2.2. To eliminate the systematic error that may arise when determining the relative distance $L_{1,j}$, it is advisable to form a group of active objects so that they interact with each other. In other words, it is desirable to provide a non-zero adjacency function for each pair of active objects. To select interacting groups of active objects, you can use one of the approaches to clustering a set of AO. Alternatively, you can form a conditional coverage on X by dividing this space on intervals X_m in increasing coordinates $x \in X$ and selecting a group of active objects in X_m from the condition $Ob_j^a \in A(X_m) \subseteq OA$.

Note that the coordinates of active objects in expressions 12 and 13 also depend on the value of the function $sign(x_j - x_1)$, which is determined from the GPS coordinates x_j, x_1. The error in measuring GPS coordinates is $\xi \approx 5m$. Therefore, an error in determining the $sign(x_j - x_1)$ may occur when the objects Ob_j^a and Ob_1^a are located at a distance of less than $10m$. Let's denote this event as A_1.

If we accept the assumption of a uniform distribution of the relative distance, then, in accordance with the property of geometric probability, the probability A_1 will be $Pr(A_1) = Pr\{L_{1,j} \leq 10m\} = 0.5$. The dependence of this probability on the actual distance between active objects is shown in Figure 16.

Figure 16. Probability of error in determining the sign of the relative distance, where $|x_1^u - x_j^u|$ – true distance between objects Ob_1^a and Ob_j^a

Based on the distribution in Figure 16, the mathematical expectation of the error in determining the sign of the relative distance between active objects Ob_j^a and Ob_1^a can be $d = 2.14m$. The probability of incorrect determination of the larger coordinate from x_j, x_1 (Event A_2) is estimated as $Pr(A_2) = Pr\{x_j > x_1\} = 0.5$. Taking into account the independence of the above events, the error in determining the sign function $\triangle_L^{sign}$ can be estimated as $\triangle_L^{sign} \leq d \cdot Pr(A_1) \cdot Pr(A_2) \approx 0,53m$. Thus, the error in measuring the coordinates of active objects, due to the error in determining the sign function, can be considered insignificant.

<u>3. Conditions for minimizing errors in fuzzy filtering.</u>

As follows from the description of the algorithm for clarifying the coordinates of passive objects in dynamics, this algorithm provides smoothing of estimates. This is especially important when the passive object is on the boundary of subsets of coverages that correspond to neighboring times $t_i, t_{i+1} \in T$.

It should also be noted that to reduce errors, the fuzzy filtering algorithm requires adjusting the gain $\beta \in [0, 1]$ (see expression (14)). The choice of this gain factor must correspond to the dynamics of objects. The higher the dynamics of objects, the greater the gain factor β should be, which is equivalent to "greater confidence" in the new (t_{i+1}) measurement of the passive object's coordinates. In general, as mentioned above, the use of fuzzy filtering can increase the accuracy of estimating the current coordinate by $30 - 35\%$.

4. Features of using algorithms for two- and three-dimensional coordinates

Above, we considered at determining the coordinates of dynamic objects in linear space X. We did this to simplify the presentation of the proposed approach to measuring coordinates and minimizing errors. For the measurement, the simplest Euclidean norm metric $l_1(x_i, x_j) = |x_i - x_j|$ was used. However, this does not negate the possibility of using the proposed approach in more complex realistic conditions, for example, in two-dimensional $(X \times Y)$ and three-dimensional $(X \times Y \times Z)$ space. Modern stadiums, where public events take place, are high-rise buildings and have several levels of height. The same goes for a swarm of unmanned aerial vehicles or a battlefield.

In these cases, the proposed approach changes only in terms of the use of the coordinate space metric, which can be the l_1 norm metric. Computational procedures will become more complicated, but will not be fundamentally changed. In addition, the global satellite positioning system can also be replaced by any other coordinate determination system with the only condition: the range of this system must be greater than the size of the territory in which objects will move. Such a system can be implemented on the basis of a drone, which could provide good stabilization of its own position and sufficient accuracy in determining the coordinates of active objects.

5. Implementation of the proposed algorithms in practice. The algorithms considered above require centralized implementation in the SCC computing system. Here we will mention several engineering problems that need to be solved when implementing the proposed algorithms in practice.

5.1. Organization of processes for determining coordinates. During the operation of the entire coordinate determination system, data is transferred from transmitters to receivers. Data transmitters must be installed on both passive and active objects. Data receivers must be installed on active objects and in the SCC. Data transmission can be organized using two methods: asynchronous and synchronous.

The asynchronous method assumes that the transmitter transmits data constantly or at preset time intervals. The receiver receives data at times that are not consistent with the data transmission times. After receiving data, the receiver becomes unavailable: it spends some time processing them and only then resumes receiving data. When using the asynchronous method, the receiver may lose new data from the transmitter, since at some point in time it may be unavailable for receiving. There are several ways to minimize data loss. For example, we can increase the speed of data processing by the receiver, which will reduce the time the receiver is unavailable. We can also find the optimal intervals ratio of data transmission time, data reception time, and data processing time in the receiver. Other engineering problems can be formulated in which the probability of data loss and the cost of design solutions can serve as optimality criteria.

The synchronous method assumes that the transmitter transmits data upon preliminary request of the receiver. This method eliminates possible data losses, but its implementation seems complicated, since passive objects will receive the synchronization signal simultaneously and will also respond simultaneously. Therefore, the active object must first identify nearby passive objects in order to send a synchronization signal to the address. This can complicate the design of transmitters and receivers. In addition, for the synchronous method, the choice of synchronization frequency requires taking into account the maximum possible speed of movement of objects in accordance with Kotelnikov's theorem [19]. One might assume that the speed at which people move should allow the system to process all the data, but a large number of people question this assumption. It is also possible to parallelize calculations here to increase processing speed.

Based on this, we prefer the asynchronous method of organizing the interaction of passive and active objects, as well as active objects and SCC. In any case, engineering solutions should be selected from several options and evaluated taking into account complexity, reliability and costs.

5.2. Determining the rational ratio of the number of active and passive objects is an important task, the solution of which is possible using an optimization procedure with several conflicting criteria. Assuming that the number of passive objects does not change, then reducing the number of active objects reduces GPS and data traffic in the SCC. However, on the other hand, this reduces the accuracy of determining the coordinates of passive objects and increases the chance of losing passive objects in the SCC. In general, the solution to this task depends on specific operating conditions: the number of objects, threats potential, configuration of object movement paths, financial restrictions and other conditions.

5.3. How can you use data on passive objects that are lost in SCC? The list of passive objects that have been lost in the SCC is also important for safety purposes. These objects can be classified, for example, as potential saboteurs who headed to the location of sabotage. The last coordinate of such an object can be considered as the location where the mobile security team should be sent.

5.4. Is it possible to use Bluetooth with the received signal strength indicator (RSSI) to equip passive objects? Many Bluetooth developers declare that the RSSI technology allows measuring the distance more accurately. This is true for the case of indoor localization or for the case when the coordinates of the Bluetooth receivers are known precisely. According to our estimates, using Bluetooth with RSSI is not entirely justified in our task. Firstly, RSSI measures the distance, not the coordinates. In the case of a two-dimensional space, using RSSI does not allow solving the problem of determining the coordinates, since the passive object lies in a circle with a diameter of approximately 10 meters. RSSI provides an accuracy of determining the distance of up to 5 meters. It is possible that using RSSI will increase the accuracy, but this question requires additional research and assessment. Secondly, the level of the received signal strongly depends on the conditions of radio wave propagation. Indoors, these conditions are much more favorable than the conditions of our task, when mass events can be held in adverse weather conditions, and Bluetooth transmitters are located under clothing. Third, no matter which Bluetooth RSSI positioning method is chosen, the fundamental requirement is to initially receive the broadcast signal [2]. This received Bluetooth signal forms the basis for assessing the distance from the object being measured. Providing coverage of a large area with this signal can be a problem.

5.5. Some considerations regarding the required performance of the system for determining the coordinates of active and passive objects. Here we will proceed from two positions:

Kotelnikov's theorem [19], which determines the frequency with which it is necessary to quantize a set of changing processes;

requirements for the adequacy of data presentation from the view-point of the security service.

The required performance of the entire computing system in SCC directly depends on the frequency of updating data about the movement of objects in the stadium, which in turn depends on the quantization frequency. The time interval between two adjacent samples is used by the computing system to receive signals from all active objects and calculate the coordinates of all objects. The data display time can be ignored here, since calculations and display can be done in parallel. Kotelnikov's theorem [19] states that a continuous signal with a limited spectrum can be accurately restored from its discrete samples if the sampling frequency exceeds the maximum frequency of the signal at least twice. In turn, the frequency of the original signal is determined by the possible speed of movement of objects in linear space.

Let's assume that at the moment of panic some groups of people can move at an average speed of $10km/h$ or $2.8m/s$. This is the speed of easy running, but here we must take into account the limited space, as well as the fact that there may be women and children among the people. Therefore, this speed can be considered an upper estimate. The maximum linear size of the playing field of the Melbourne Cricket Ground is 174 meters. Given the need to monitor the paths indoor and the outer perimeter, we can use double the size: 350 meters. Then in one second, people will move 0.8% of the linear size of the control area. This number can be considered a percentage error associated with signal sampling.

We assume that a linear error of about 3 meters and a percentage error of about 1% per second will satisfy the requirements of the security service. Then, taking into account the Kotelnikov's theorem [19], the quantization frequency should be 2 hertz. In our opinion, this is a fairly strong requirement for the performance of the computing system. One can doubt the performance of the synchronous method of transmitting data from active objects, since this method will significantly slow down the measurements. In the case of the asynchronous method, 5 microseconds are required to determine the coordinates of each of 100 thousand objects. It is difficult to determine the requirements for the processor frequency. Much will depend on the optimality of the programs and the executable code. Based on empirical experience, modern processors can provide such a data processing speed. In addition, the presented algorithms allow parallel calculations. In any case, the solution to this engineering problem requires simulation modeling and taking into account the possibility of temporary data loss.

5.6. Other problems that may arise during the implementation of the proposed algorithms. Determining the coordinates of passive objects is a complex scientific problem that gives rise to many technical problems. When implementing the proposed algorithms, the main problems are associated with signal interference and stable work of transmitters. In the case of data transmission from passive objects to active objects, noise immunity depends on the distance between the objects and the Bluetooth signal level, which in turn depends on the battery charge. Firstly, in our research we proceeded from the fact that the range of the Bluetooth transmitter is 5 meters, i.e. significantly underestimated compared to the real range. This solution will ensure a sufficiently high signal-to-noise ratio at the input of the active object receiver. Secondly, determining the battery capacity to provide the voltage required for effective signal transmission will depend on the duration of the mass event. The required capacity can be determined based on practical experience. In addition, standard conditions for radio compatibility of all transmitting and receiving devices must be met. An important additional problem is intentional jamming. We see here an independent research problem regarding the determination of the maximum level of interference and the search for the best place for jamming in terms of effective suppression of the coordinate determination system.

8. Conclusions

Determining the coordinates of dynamic objects in the case of their large swarms (up to 100 thousand) requires significant costs to equip the objects with appropriate instruments. We proposed an approach that involves stratifying objects into active and passive. Active objects are equipped with GPS sensors, Bluetooth signal receivers and data transmitters in the SCC. Passive objects are equipped with simple low-power Bluetooth transmitters that periodically emit a radio signal with a unique identification number. Active objects detect passive objects that are in their detection zone. We have proposed three algorithms that determine the coordinates of passive objects and ensure their clarifying due to information redundancy, compensation of unbiased errors from independent sources, and the use of fuzzy filtering. As a result, as calculations show, provided that active objects are rationally located, the proposed algorithms provide an accuracy in determining the coordinates of passive objects comparable to the accuracy of a GPS sensor. This allows you to significantly reduce the cost of equipment when creating a security system, as well as reduce GPS traffic. Knowing the coordinates of a large number of objects opens up new areas of research necessary to ensure the safety of people, for example, determining locations for mobile security teams or managing the flow of objects in the event of critical situations. The scientific and engineering problems of finding the most rational or optimal parameters of algorithms also require solutions, for example, determining the rational number of active objects that could provide a given quality of coverage of the territory with minimal equipment costs.

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