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An Innovative mathematical model and optimized solution approach for machine patterns in berth allocation problems

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Abstract

In this paper, we integrate two decision problems related to the management of port terminals, the berth allocation problem and the machine assignment problem. The berth allocation problem consists of assigning and scheduling incoming vessels to berthing positions, and the machine assignment problem consists of assigning a machine pattern/profile. The machines can be quay cranes, trucks, or any other machine. We present two MILP formulations, one with machine patterns for the quay and another for berths. The objective function aims to minimize the waiting time and the handling time of the vessels. To solve the problem, we developed a heuristic algorithm capable of solving a problem instance in seconds. To compare the results, we generate several instance problems based on real data and solve them with our MILP formulation, our heuristic, and a FIFO algorithm. We tested our heuristic with instances with more than 100 berths, 500 vessels, and 250 machines. The solver was unable at finding solutions for instances with more than 4 berths after three hours of processing. The heuristic was able to solve all the instances in less than 3 seconds. On average, the heuristic solution is 8% worse than the optimal solution.

Keywords: *Berth allocation problem, quay crane assignment problem, port machines assignment, mixed integer linear programming, heuristic method*

1. Introduction

Maritime transportation has always played a crucial role in the international exchange of goods, and reducing the time and cost of such transportation continues to be an important goal. In order to reduce transportation costs, terminal managers seek to increase cargo handling efficiency, with larger container ships for long-haul routes and terminals with better infrastructure and technologies able to efficiently handle them [5]. Containers, dry bulk and oil bulk are the majority of cargo handled by maritime shipping. The container is the driver of intermodal transportation, which permits easy handling between modal systems. Since intermodal transportation with containers is more efficient and cheap, containerization is increasing [22].

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The need for efficient management of logistic activities at modern terminals is a well-known problem and several papers addressed this problem in the literature, one of the most relevant is the Berth Allocation Problem (BAP) which consists of assigning and scheduling vessels to berthing positions along the quay, with the aim of minimizing the waiting time of the vessels [13]. Since quay cranes are one of the main terminal equipment used for container movement, an inefficient quay crane employment could impact negatively in handling operations, many papers address the integration between the BAP and the quay crane (QC) allocation or scheduling problem, this problem is known as Quay Crane Assignment Problem (QCAP) which consists of deciding how many QCs to assign and for how long for each vessel [7] and [10].

The majority of integrated BAP models address the QCAP, although some address others types of port structures or machines, such as yard allocation [21] or bulk unloaders [19]. In this paper, we present two new mathematical models that address the BAP integrated with the allocation of several types of transportation machines, such as quay cranes, mobile cranes, straddle carriers, forklifts, trucks, and others. In an attempt to address the intermodal transportation that could take place in a terminal, the models work with a pattern or profiles of machines for terminals or berths, with the aim of minimizing the waiting time and the handling time of the vessels. Besides addressing any type of machine, our model uses a continuous horizon time, which differs from most of the integrated models for BAP.

Most of the operational decisions in major ports are made by ship and port operators using solutions generated by the first-come, first-served model. The solutions generated with the proposed model have up to 78% less berth and machine usage time and 20% less on average, which leads to fewer fees and charges, reducing the transportation costs. The paper is organized as follows. The literature review related to the berth allocation problem is in Section 2. The problem formulation is presented and analyzed in Section 3. A heuristic to solve the problem is presented and discussed in Section 4. The computational analyses of the models, the heuristic, and numerical experiments are presented in Section 5. The final section concludes the paper.

2. Literature Review

One of the first mathematical models for the BAP can be found in [11], where the port berths have a fixed length and are considered fixed points along the quay. In this model, the authors assume a situation named static, where all vessels are in the port. This model is referred as static discrete BAP or static DBAP by the authors. The model aims to minimize the allocation time of the vessels on the berths to provide a better solution than the first in, first out technique (or FIFO) also called *first in, first served technique* (or FIFS). The FIFO technique consists of handling the vessels by arrival order, and it is widely used in commercial ports.

In [12] the BAP of [11] is extended to the dynamic version, where all vessels that have an arrival time on the planning horizon time are considered by the mathematical model. Those models are called dynamic DBAP by the authors. The dynamic DBAP models assume that the handling of any vessel can be done at any berth.

Cordeau et al. [4] formulates the BAP as a vehicle routing problem with time windows (MDVRPTW) and develops a heuristic based on the tabu search to solve the model instances. The heuristic also has a version for the continuous BAP (or CBAP), where the berths can have a variable length.

The Tactical Berth Allocation Problem (or TBAP) was first proposed in [17] where their TBAP model aims to represent the trade-off between the waiting time of the vessels and the costs of moving containers between the berths and yards. The model is based on the rectangular packing problem on a cylinder and uses a simulated annealing-based heuristic to solve the instances of the model. The objective of their model is to maximize the level of service (defined as the number of vessels serviced in two hours) and minimizes the costs related to the container movements between the berths and yards.

Giallombardo et al. [7] proposed another TBAP model to integrate the BAP with the quay crane schedule (CS). The model discretizes the time horizon into several time partitions of the same length and defines every variable and parameter with an index related to each time partition. The variables and parameters associated with the vessels and berths are very similar to the variables and parameters of the DBAP. The quay crane schedule (CS) presupposes that a reallocation of the quay cranes can occur at the end of each work shift change, and, therefore, the length of each time partition used in this model is given by the work shift duration. The yard cost depends on the berthing position of each vessel. This model is one of the first to consider the integration between the BAP and the quay crane scheduling, but the partition of the horizon time causes a large increase in the number of variables, and consequently, that increases the memory and processing needed to solve the instances of this model.

Zhen et al. [29] proposed an integrated template-planning model for berthing locations in continuous indexes and yard container stack arrangements. They provide a heuristic to solve instances of the model. Hendriks et al. [8] extend the BAP model for a multi-terminal port, which can allocate two connected vessels in different terminals by means of inter-terminal container transport. The objectives of the model are to balance the working load of the quay crane between the terminals over time and minimize the number of containers transported between terminals. Hendriks et al. [9] approach a TBAP for terminals with discrete berths integrated with the allocation of yard spaces. Those studies do not consider machine allocation.

Shang et al. [25] investigates the integrated berth allocation and quay crane assignment problem in container terminals under data uncertainties. A deterministic model was formulated by considering the setup time of quay cranes in discretized horizon time. Our proposed model considers a continuous horizon time, which significantly reduces the number of variables and does not generate time gaps in the solution. Zheng et al. [30] studies an integrated berth allocation and quay crane assignment model where quay crane maintenance is involved and establishes a MILP with the objective of minimizing the total turnaround time. The authors also provide a GA-based metaheuristic to solve large instances.

Xiang and Liu [28] integrated berth allocation and quay crane assignment problem considering uncertainties in the late arrival of ships. They formulated the problem as an almost robust model by introducing the weighted max penalty function with the objective of minimizing the total cost. They separate the problem into a deterministic master problem and a stochastic subproblem, they claim their results demonstrate the robustness of the model and the effectiveness of their proposed model and solution method. Our proposed model and solution method solve a single problem, capable of finding the allocation of vessels and any type of port machine.

Martin-Iradi et al. [15] reformulates a multiport berth allocation problem, which is a mixed-integer problem, into a generalized set partitioning problem in which each variable refers to a sequence of feasible berths in the ports that the vessel visits. The authors also propose a column generation and cut separation in a branch-and-cut-and-price procedure method to solve their problem, which they claim it was able to outperform commercial solvers. Our proposed model and solution method are capable of allocating any type of port machine.

Tang et al. [26] proposes a large neighborhood search algorithm to solve the continuous berth allocation and quay crane assignment problem. The continuous berth is separated into discrete segments via a proposed discretization strategy. They claim to obtain the optimal solution for small-scale instances and a more efficient solution than the genetic algorithms for large-scale instances. Our model considers a discrete set of berths, and we also present a discussion regarding berthing positions and machine allocation considering the quay geography. Besides our proposed heuristic, we also propose new mathematical models for the berth and machine allocation problems.

The papers [1], [2], [23] and [14] contain a review of publications related to the BAP and QCAP.

The mathematic models of [18], [10] and [16] makes an hour-by-hour relocation of machines and [7] and [27] makes a turn-by-turn relocation in order to find the machine schedule. Our proposed model uses novel variables and constraints that can detect simultaneous allocations in different berths, and, with this information, the model can allocate the machine pattern to the handling of the vessels, without discretizing the horizon time. The proposed mathematical model of [3] uses variables and contraints to archive the machine allocation, while the proposed model uses machine patterns for berths, which generates a model with less variables and less complex constraints.

3. Mathematical Model

Before the introduction of the mathematical formulation, we address the issue of which berths will compete for the usage of machines. As an example, consider the situation of Figure 1, where the container port has berths distributed by sets of three, and each set has disconnected rails.

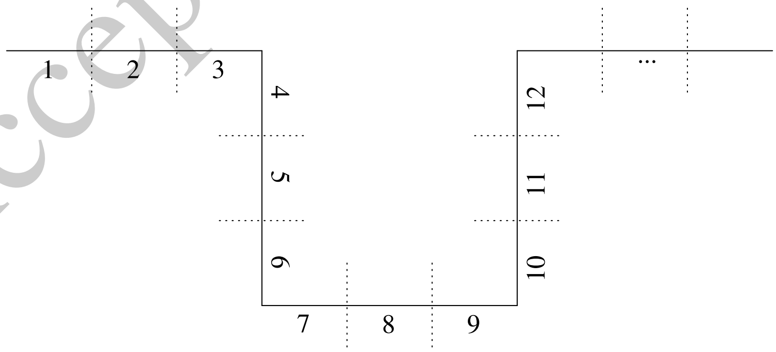

Figure 1. Exemple of berths division.

Berths only compete for the usage of quay cranes with the ones in the same rail. To address this issue, we introduce in this work the concept of regions of the quay. We define sets of berths that compete for the usage of a given type of machine. Each set has a number of available machines. In our example, for the quay crane, we have the sets given by Figure 2.

The sets generated by different types of machines are not necessarily equal. In our example, the sets

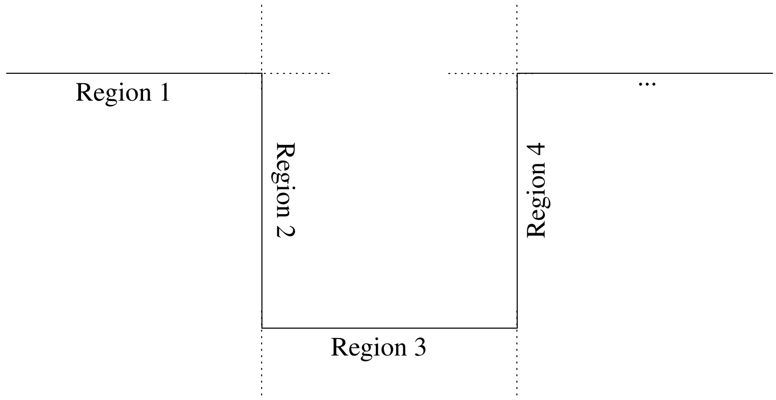

Figure 2. Exemple of terminal/quay regions.

of Figure 2 are not the same for trucks. Since trucks have free mobility between berths, we have a single set, which contains all berths and has the total number of trucks available.

With the concept of berth sets, we present the mathematical model created to optimize the handling of vessels on a terminal with multiple types of machines. To avoid confusion, we address the sets of berths generated by a type of machine as regions generated by that type of machine. Each type of machine generates its own rate of handling in the model, based in the number of machines assigned, and the handling rate of a vessel is given by the slowest rate generated by the machines. The Model 1 is defined with machine patterns for the terminal therefore, the concept of regions should be addressed in the creation of patterns. The Model 2 uses machine patterns for berths , therefore, the regions are addressed directly in the model.

Sets

- B: set of berths.
- N: set of vessels.
- O: set of service order.
- P: set of machine patterns.

Variables

- $x_{i,o,k} = \begin{cases} 1, \text{ if the vessel } i \text{ is the } o\text{-th in berth } k \\ 0, \text{ otherwise.} \end{cases}$
- $y_{i,k,p} = \begin{cases} 1, \text{ if the handling of vessel } i \text{ occurs in berth } k \text{ with pattern } p \\ 0, \text{ otherwise.} \end{cases}$
- T_i: allocation time of vessel i.
- t_i: handling time of vessel i.
- $s_{i,j} = \begin{cases} 1, \text{ if the end of vessel } i \text{ handling occurs before the beginning of} \\ \text{vessel } j \\ 0, \text{ otherwise.} \end{cases}$

- $w_{i,j} = \begin{cases} 1, \text{ if the beginning of vessel } i \text{ handling occurs before the end of} \\ \text{the vessel } j \\ 0, \text{ otherwise.} \end{cases}$

If the handling of any two vessels i and j have an intersection, which is, exists a moment where those vessels are simultaneously allocated (in different berths), then $s_{i,j} = w_{i,j} = 1$. **Parameters**

- β_i and $\beta_i^{'}$: weights related to the vessel i waiting handling time, respectively.
- a_i and b_i: arrival and departure time of vessel i, respectively.
- $t_{i,p}^{'}$: handling time of vessel i in berth k with pattern p.
- H: large number.

Mathematical Model

$$\min \sum_{i \in N} \beta_i (T_i - a_i) + \beta_i^{'} t_i \tag{1a}$$

$$s.t \;\; t_i \geq y_{i,k,p} \, t_{i,p}^{'} \qquad \forall i \in N, \forall k \in B, \forall p \in P \tag{1b}$$

$$\sum_{k \in B} y_{i,k,p} \geq \sum_{k \in B} y_{j,k,p} + (w_{i,j} + s_{i,j} - 1) - 1 \qquad \forall i \in N, \forall j \in N, \forall p \in P \tag{1c}$$

$$\sum_{k \in B} \sum_{p \in P} y_{i,k,p} = 1 \qquad \forall i \in N \tag{1d}$$

$$\sum_{p \in P} y_{i,k,p} \geq \sum_{o \in O} x_{i,o,k} \qquad \forall i \in N, k \in B \tag{1e}$$

$$T_i \geq a_i \qquad \forall i \in N \tag{1f}$$

$$T_i + t_i \leq b_i \qquad \forall i \in N \tag{1g}$$

$$\sum_{i \in N} x_{i,o,k} \leq 1 \qquad \forall o \in O, \forall k \in B \tag{1h}$$

$$\sum_{o \in O} \sum_{k \in B} x_{i,o,k} = 1 \qquad \forall i \in N \tag{1i}$$

$$\sum_{i \in N} x_{i,o,k} \leq \sum_{i \in N} x_{i,o-1,k} \qquad \forall o \in O \backslash \{1\}, k \in B \tag{1j}$$

$$T_i - H(2 - x_{i,o,k} - x_{j,o-1,k}) \geq T_j + t_j \quad \forall i \in N, \forall j \in N, \forall o \in O \backslash \{1\}, k \in B \tag{1k}$$

$$T_i + t_i - H s_{i,j} \leq T_j \qquad \forall i \in N, \forall j \in N \tag{1l}$$

$$T_i + H w_{i,j} \geq T_j + t_j \qquad \forall i \in N, \forall j \in N \tag{1m}$$

$$x_{i,o,k} \in \{0,1\} \qquad \forall i \in N, o \in O, \forall k \in B \tag{1n}$$

$$y_{i,k,p} \in \{0,1\} \qquad \forall i \in N, \forall k \in B, \forall p \in P \tag{1o}$$

$$T_i \geq 0 \qquad \forall i \in N \tag{1p}$$

$$s_{i,j}, w_{i,j} \in \{0,1\} \qquad \forall i \in N, \forall j \in N. \tag{1q}$$

The Function 1a is the sum of the time window and handling time of the vessels. Some vessels could have priority, therefore, a constant was added to the objective function. The Constraints 1b is the handling time calculation, given by the machine pattern used. As Constraints 1c ensure the same machine pattern for two vessels with simultaneous handling. The Constraints 1d allows only one pattern for each vessel. The Constraints 1e associates berths and machine patterns. The Constraints 1f ensure the allocation time before the arrival time. The Constraints 1g ensure the handling before the departure time. The Constraints 1h and 1i ensure the handling of each vessel occurs one time in one berth. The Constraints 1j ensure the position of each vessel. The Constraints 1k calculate the allocation time of the vessels. The Constraints 1l and 1m compute the case where the vessels have simultaneous allocation. If the vessels i and j are simultaneously allocated in different berths, then $s_{i,j} + w_{i,j} = 2$. The Constraints 1q ensure binary allocation variables and non-negative continuous time variables.

Sets

- B: set of berths.
- B^r: berths subset of region r.
- B^r_k: subset of all k-combinations of B^r elements.
- N: set of vessels.
- N_k: set of all k-combinations of N elements without repetition.
- C_{N_k}: set of combination two-by-two of N_k elements.
- O: set of service order.
- P: set of machine patterns.
- R_α: set of regions of machine type α.

Variables

- $x_{i,o,k} = \begin{cases} 1, \text{ if the vessel } i \text{ is the } o\text{-th in berth } k \\ 0, \text{ otherwise.} \end{cases}$
- $y_{i,p} = \begin{cases} 1, \text{ if the handling of vessel } i \text{ occurs with pattern } p \\ 0, \text{ otherwise.} \end{cases}$
- T_i: allocation time of vessel i.
- t_i: handling time of vessel i.
- $s_{i,j} = \begin{cases} 1, \text{ if the end of vessel } i \text{ handling occurs before the beginning of} \\ \quad \text{vessel } j \\ 0, \text{ otherwise.} \end{cases}$
- $w_{i,j} = \begin{cases} 1, \text{ if the beginning of vessel } i \text{ occurs before the end of} \\ \text{the vessel } j \\ 0, \text{ otherwise.} \end{cases}$
- $m_{i,\alpha}$: number of α type machines allocated for the service of vessel i.

Parameters

- β_i and $\beta_i^{'}$: weights related to the vessel i waiting time and handling time, respectively.
- a_i and b_i: arrival time and maximum departure time of vessel i, respectively.
- $t_{i,p}^{'}$: handling time of vessel i with pattern p.
- M_α^r: number of α machines in region r.
- H: large number.

Mathematical Model

$$\min \sum_{i \in N} (\beta_i (T_i - a_i) + \beta_i^{'} t_i) \tag{2a}$$

$$s.t.\ t_i \geq y_{i,p} t_{i,p}^{'} \qquad \forall i \in N, \forall p \in P \tag{2b}$$

$$\sum_{p \in P} y_{i,p} = 1 \qquad \forall i \in N \tag{2c}$$

$$\sum_{p \in P} t_{i,k,p}^{'} \geq \sum_{o \in O} x_{i,o,k} \qquad \forall i \in N, k \in B \tag{2d}$$

$$T_i \geq a_i \qquad \forall i \in N \tag{2e}$$

$$T_i + t_i \leq b_i \qquad \forall i \in N \tag{2f}$$

$$\sum_{i \in N} x_{i,o,k} \leq 1 \qquad \forall o \in O, \forall k \in B \tag{2g}$$

$$\sum_{o \in O} \sum_{k \in B} x_{i,o,k} = 1 \qquad \forall i \in N \tag{2h}$$

$$\sum_{i \in N} x_{i,o,k} \leq \sum_{i \in N} x_{i,o-1,k} \qquad \forall o \in O \backslash \{1\}, k \in B \tag{2i}$$

$$T_i - H(2 - x_{i,o,k} - x_{j,o-1,k}) \geq T_j + t_j \qquad \forall i \in N, \forall j \in N, \forall o \in O \backslash \{1\}, k \in B \tag{2j}$$

$$T_i + t_i - H s_{i,j} \leq T_j \qquad \forall i \in N, \forall j \in N \tag{2k}$$

$$T_i + H w_{i,j} \geq T_j + t_j \qquad \forall i \in N, \forall j \in N \tag{2l}$$

$$\sum_{i \in \bar{N}} m_{i,\alpha} \leq \left[(k^2 + 1) - \sum_{v \in C_{\bar{N}}} (w_v + s_v) - \sum_{b \in \bar{B}} \sum_{i \in \bar{N}} \sum_{o \in O} x_{i,o,b} \right] M_\alpha^r \tag{2m}$$
$$\forall \alpha \in P, \forall \bar{N} \in N_k, \forall \bar{B} \in B_k^r, \forall r \in R_\alpha, \forall k \in \{1, 2, ..., |\bar{B}|\}$$

$$x_{i,o,k} \in \{0, 1\} \qquad \forall i \in N, \forall o \in O, \forall k \in B \tag{2n}$$

$$t_{i,k,p}^{'} \in \{0, 1\} \qquad \forall i \in N, \forall k \in B, p \in P \tag{2o}$$

$$T_i \geq 0 \qquad \forall i \in N \tag{2p}$$

$$s_{i,j}, w_{i,j} \in \{0, 1\} \qquad \forall i \in N, \forall j \in N. \tag{2q}$$

The Constraints 2b is the handling time calculation, given by the machine pattern used. The Constraints 2c allows only one pattern for each vessel. The Constraints 2d associates berths and machine patterns. The Constraints 2e ensure the allocation time before the arrival time. The Constraints 2f ensure the handling before the departure time. The Constraints 2g ensure the handling of each vessel occurs one time in one berth. The Constraints 2i ensure the position of each vessel. The Constraints 2j calculate the allocation time of the vessels. The Constraints 2k and 2l compute the case where the vessels have simultaneous allocation. If the vessels i and j are simultaneously allocated in different berths, then $s_{i,j} + w_{i,j} = 2$. Constraints 2m compute the maximum number of machines. For example, to a given region $\hat{r}$ with two berths we have the sets $B^{\hat{r}} = \{k_1, k_2\}$, defining C_2, N_2 and $B_2^{\hat{r}}$ as:

- $B_2^{\hat{r}} = \{k_1, k_2\}$.
- $N_2 = \{\{i, j\}\} \qquad \forall i \in N, j \in N, i \neq j$.
- $C_2 = \{(i, j)\} \qquad \forall i \in N, j \in N, i \neq j$.

Constraints 2m can be written as Constraints 3.

$$m_{i,\alpha} + m_{j,\alpha} \leq (5 - w_{i,j} - s_{i,j} - \sum_{o \in O} x_{i,o,k_1} - \sum_{o \in O} x_{j,o,k_2}) M_\alpha^{\hat{r}} \quad \forall i \in N, \forall j \in N, \forall \alpha \in P. \tag{3}$$

If $w_{I,j} = 1$ and $s_{I,j} = 1$ the handling of vessels i and j are simultaneous. If $\sum_{o \in O} x_{I,oh,k_1} = 1$ and $\sum_{o \in O} x_{j,o,k_2} = 1$, the handling of vessels i and j occurs in berths of the same region. Therefore, if $w_{I,j} = 1$, $s_{I,j} = 1$, $\sum_{o \in O} x_{I,o,k_1} = 1$ and $\sum_{o \in O} x_{j,o,k_2} = 1$, the machines of this region will be divided between the vessels i and j. If $\hat{r}$ has three berths, which are $B^{\hat{r}} = \{k_1, k_2, k_3\}$, defining C_2, C_3, N_2, N_3, $B_2^{\hat{r}}$ and $B_3^{\hat{r}}$ as:

- $B_2^{\hat{r}} = \{\{k_1, k_2\}, \{k_1, k_3\}, \{k_2, k_3\}\}$.
- $B_3^{\hat{r}} = \{\{k_1, k_2\}\}$.
- $N_2 = \{\{i, j\}\} \qquad \forall i \in N, j \in N, i \neq j$.
- $N_3 = \{\{i, j, p\}\} \qquad \forall i \in N, j \in N, p \in N, i \neq j \neq p$.
- $C_2 = \{(i, j)\} \qquad \forall i \in N, j \in N, i \neq j$.
- $C_3 = \{(i, j), (i, p), (j, p)\} \qquad \forall i \in N, j \in N, p \in N, i \neq j \neq p$.

Constraints 2m can be written as Constraints 4, 5, 6 and 7.

$$m_{i,\alpha} + m_{j,\alpha} \leq (5 - w_{i,j} - s_{i,j} - \sum_{o \in O} x_{i,o,k_1} - \sum_{o \in O} x_{j,o,k_2}) M_\alpha^{\hat{r}} \quad \forall i \in N, \forall j \in N, \forall \alpha \in P, \tag{4}$$

$$m_{i,\alpha} + m_{j,\alpha} \leq (5 - w_{i,j} - s_{i,j} - \sum_{o \in O} x_{i,o,k_2} - \sum_{o \in O} x_{j,o,k_3}) M_\alpha^{\hat{r}} \quad \forall i \in N, \forall j \in N, \forall \alpha \in P, \tag{5}$$

$$m_{i,\alpha} + m_{j,\alpha} \leq (5 - w_{i,j} - s_{i,j} - \sum_{o \in O} x_{i,o,k_1} - \sum_{o \in O} x_{j,o,k_3}) M_\alpha^{\hat{r}} \quad \forall i \in N, \forall j \in N, \forall \alpha \in P, \tag{6}$$

$$m_{i,\alpha} + m_{j,\alpha} + m_{p,\alpha} \leq \left(10 - w_{i,j} - s_{i,j} - w_{i,p} - s_{i,p} - w_{j,p} - s_{j,p} - \sum_{o \in O} x_{i,o,k_1} - \sum_{o \in O} x_{j,o,k_2} - \sum_{o \in O} x_{q,o,k_3}\right) M_{\alpha}^{\hat{r}} \quad \forall i \in N, \forall j \in N, \forall p \in N, \forall \alpha \in P. \tag{7}$$

The Constraints 2n, 2o, 2p e 2q ensure binary allocation variables and non-negative continuous time variables. In the next section, we define the form of a machine pattern and several theorems to minimize the number of patterns before the optimization of a model instance.

3.1. Machine Patterns

A berth pattern has information regarding the number of machines allocated in one berth, and a terminal pattern has information regarding the number of machines allocated in all berths. The terminal pattern can be interpreted as a combination of berth patterns.

The berth pattern can be defined as a vector and a terminal pattern as a matrix or a set of vectors. In a terminal with m types of machine, and the set B with n berths, letting $q_{p,i,k}$ be the number of machines type i allocated in berth k of the pattern p where $k \in B$ and $\alpha \in \{1, ..., m\}$ and $\sum_{k \in B^r} q_{p,\alpha,k} \leq M_{\alpha}^{r}$, $\forall \alpha \in 1, ..., m$, $p \in P$, $r \in R_{\alpha}$. The Set 8 is an example of a terminal pattern.

$$\left\{ \begin{pmatrix} q_{p,1,1} \\ q_{p,2,1} \\ \dots \\ q_{p,m,1} \end{pmatrix}; \begin{pmatrix} q_{p,1,2} \\ q_{p,2,2} \\ \dots \\ q_{p,m,2} \end{pmatrix}; \dots; \begin{pmatrix} q_{p,1,n} \\ q_{p,2,n} \\ \dots \\ q_{p,m,n} \end{pmatrix} \right\}. \tag{8}$$

For example, a terminal with two berths, five trucks, and three quay cranes, where a truck has a flow rate of 50 containers per day and a quay crane has a flow rate of 100 containers per day.

$$\left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}; \begin{pmatrix} 4 \\ 1 \end{pmatrix} \right\}. \tag{9}$$

The Pattern 9 generates a flow rate of 50 containers per day in berth 1 and 100 containers per day in berth 2, where the rate of a berth is given by the slowest rate between the trucks and the quay cranes allocated in the respective berth. Consider another pattern given by

$$\left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}; \begin{pmatrix} 4 \\ 2 \end{pmatrix} \right\}. \tag{10}$$

The Pattern 10 generates a flow rate of 50 containers per day in berth 1 and 200 containers per day in berth 2. By replacingPattern 9 with Pattern 10 will improve a solution without violation. Therefore,Pattern 9 can be omitted from the Model 1.

We propose several theorems to help filter the machine patterns used to solve the models. The theorems and their proofs are based on the multi-objective optimization theory. As a reference for multi-objective optimization, we can cite [6], [24] and [20]. We begin by defining the efficiency of a machine pattern.

Definition 1 (Effective flow rate of a terminal pattern). The effective flow rate of the berth $k \in B$ with the terminal pattern $p \in P$ is given by: $\delta_{p,k} = min_{\alpha \in M}\,(q_{p,\alpha,k}\,\delta_\alpha)$, where $q_{p,\alpha,k}$ is the number of machine type $\alpha \in M$ of pattern p allocated on berth k and δ_α is the flow rate of machines type α.

Definition 2 (Efficient terminal pattern). Given $p \in P$ a terminal machine pattern. The pattern p is efficient if every pattern $\hat{p}$ defined as

$$q_{\hat{p},\alpha,k} = \begin{cases} q_{p,m,b} - 1, & \text{if } \alpha = m \text{ and } k = b \\ q_{p,\alpha,k}, & \text{otherwise.} \end{cases} \tag{11}$$

where $m \in M$ and $b \in B$, have $\delta^{\hat{p}}_m < \delta^p_m$.

In summary, an efficient pattern does not have idle machines. If the pattern p is not efficient, it is possible to remove at least one (idle) machine without changing the effective flow rate. Let $A_{p,\alpha,k}$ be the number of idle machines type $\alpha \in M$ of berth $k \in B$ and pattern $p \in P$.

Property 1. Inefficient pattern: The pattern p is an inefficient pattern if admitted $A_{p,\alpha,k} > 0$ for any $\alpha \in M$ and $k \in B$.

In summary, an inefficient pattern has idle machines.

Given a solution with at least one inefficient terminal pattern, the theorem 1 ensure the existence of an alternative solution, which contains only efficient patterns.

Theorem 1. Let U be the non-empty set of feasible solutions of a model instance. For every solution $u \in U$, where u has at least one inefficient terminal pattern, at least one solution $\hat{u} \in U$ has the same objective function value that contains only efficient terminal patterns.

Proof. Let $u \in U$ be a feasible solution of a model instance with at least one inefficient terminal pattern. Defining:

- p^u_i: The terminal pattern selected for the service of vessel i of solution u.
- T^u_i: The allocation time of vessel i of solution u.
- t^u_i: The handling time of vessel i of solution u.
- $z^u = \sum_{i \in N}(\beta T^u_i + \beta' t^u_i)$: The objective function value on solution u.

By hypothesis, there is at least one vessel $j \in N$ such that the pattern p^u_j is inefficient. Let:

$$q_{p^{\hat{u}}_j,\alpha,k} = q_{p^u_j,\alpha,k} - A_{\alpha,k}, \quad \forall k \in B, \alpha \in M. \tag{12}$$

Where $A_{m,k}$ is the largest positive value such that $\delta^{p^{\hat{u}}_j}_k = \delta^{p^u_j}_k, \forall k \in B$. In this way

$$t^{\hat{u}}_i = t^u_i \quad \forall i \in N. \tag{13}$$

Using Theorem 13 we have:

$$T_i^{\hat{u}} = T_i^u \quad \forall i \in N. \tag{14}$$

With Theorem 13 and 14 we have $z^{\hat{u}} = z^u$, which completes the proof. □

Given a solution with at least one inefficient terminal pattern, the theorem 2 ensures the existence of a better or alternative solution, which contains only efficient patterns.

Theorem 2. Given the patterns $p^{'} \in P$ and $\hat{p} \in P$ both feasible, such that $\delta_{\hat{p},k} \geq \delta_{p^{'},k}, \forall k \in B$ and there is at least one $\hat{k} \in B$ such that $\delta_{\hat{k}}^{\hat{p}} > \delta_{\hat{k}}^{p^{'}}$, that is, if the pattern $\hat{p}$ dominates the pattern p by the multi objective function $\max \delta_k^p$ where $p \in P$ e $k \in B$, so one solution of this problem instance that contains the pattern is not optimal, or it is an alternative optimal solution.

Proof. Let U be the set of all feasible solutions of a problem instance. For all solution $u \in U$:

- p_i^u: Is the terminal pattern selected to serve the vessel i of solution u.
- o_i^u: The order of vessel i of solution u.
- b_i^u: The allocation berth of vessel i of solution u.
- T_i^u: The allocation time of vessel i of solution u.
- t_i^u: The handling time of vessel i of solution u.
- $z^u = \sum_{i \in N}(\beta T_i^u + \beta^{'} t_i^u)$: The objective function values of solution u.

Let $u \in U$ be a solution that contains at least one pattern $p^{'} \in P$ which is dominated by the pattern $\hat{p} \in P$ according to the multi-objective function $\max \delta_k^p$ where $p \in P$ and $k \in B$. Therefore, at least one $\hat{k} \in B$ has $\delta_{\hat{k}}^{\hat{p}} > \delta_{\hat{k}}^{p^{'}}$, and exists at least one $j \in N$ such that $p_i^u = p^{'}$. Defining the solution $\hat{u} \in U$ where:

- $o_i^{\hat{u}} = o_i^u, \forall i \in N.$
- $b_i^{\hat{u}} = b_i^u, \forall i \in N.$
- $p_i^{\hat{u}} = \begin{cases} \hat{p}, \text{ if } i = j \\ p_i^u, \text{ otherwise.} \end{cases}$

In this way:

$$t_i^{\hat{u}} \leq t_i^u, \forall i \in N. \tag{15}$$

So, it is easy to verify that:

$$T_i^{\hat{u}} \leq T_i^u, \forall i \in N. \tag{16}$$

Therefore, of Theorem 15 and 16 we have $z^{\hat{u}} \leq z^u$, which completes the proof. □

If all terminal pattern that uses all machines are inefficient, the theorem 3 establishes a number of machines that can be removed from the problem without losing solution quality.

Theorem 3. It is possible to remove of the problem $n^r_m = \min_{p\in P^*} \sum_{k\in B^r} A_{p,m,k}$ machines type m of the region $r \in R_m$, where $P^* \subset P = \{p^* \in P^* | \sum_{k\in B^r} q_{p^*,m,k} = M^r_m, \forall m \in M, \forall r \in R_m\}$ is the subset of machine patterns that uses all available machines and $A_{p,m,k}$ it is the maximum numbers of machine type m that can be removed from berth k of pattern p such that the pattern $\hat{p}$ defined as:

$$q_{\hat{p},m,k} = q_{p,m,k} - A_{p,m,k}, \quad \forall k \in B, m \in M. \tag{17}$$

where $\delta_{\hat{p},k} = \delta_{p,k}, \forall k \in B$.

Proof. Defining:

$$q_{\hat{p},m,k} = q_{p,m,k} - A_{p,m,k} \qquad p \in P^*, m \in M, k \in B, \tag{18}$$

where $A_{p,m,k} \in \mathbb{Z}_+$ is such that $\delta^{\hat{p}}_k = \delta^p_k, k \in B, \forall p \in P$.

Defining the set $\hat{P} = \{\hat{p} : q_{\hat{p},m,k} = q_{p,m,k} - A_{p,m,k}, \ \forall p \in P^*, m \in M, k \in B\}$. The set $\hat{P}$ is equivalent to the set P in relation to the rates of terminal patterns, that is, for all $pinP$ there exist a terminal pattern $\hat{p} \in \hat{P}$ such that $\delta^{\hat{p}}_k = \delta^p_k, \forall k \in B$. Besides that:

$$\sum_{k\in B^r} q_{\hat{p},m,k} <= q_{\hat{p},m,k} - n^r_m \quad \forall r \in R_m, \forall \hat{p} \in \hat{P}, \tag{19}$$

which completes the poof. □

Given a solution with at least one terminal pattern which does not make use of all machines, the Theorem 4 ensures the existence of a better or alternative solution, which contains only patterns that make use of all the machines.

Theorem 4. Let the pattern $p \in P \backslash P^*$, where $P^* \subset P = \{p \in P | \sum_{k\in B^r} q_{p,m,k} = M^r_m, \forall m \in M, \forall r \in R_m\}$, that is, p do not make use of all existing machines. The instance solution of the Model 1 which contains p it is not an optimal solution or is an alternative optimal solution of that instance.

Proof. Given any $p \in P \backslash P^*$ and let $n^r_m = M^r_m - \sum_{k\in B^r} q_{p,m,k}$, where $m \in M, r \in R_m$. By hypothesis, there exists at least one $\hat{m} \in M$ e $\hat{r} \in R$ such that $n^{\hat{r}}_{\hat{m}} > 0$.

Given $\hat{m} \in M$ and defining $\hat{k}^r_{\hat{m}}$ such that:

$$\min_{m\in M} (q_{p,m,\hat{k}^r_{\hat{m}}} \delta_m) = q_{p,\hat{m},\hat{k}^r_{\hat{m}}} \delta_{\hat{m}}. \tag{20}$$

If there is no $\hat{k}^r_{\hat{m}}$ satisfying the Equation 20, take any $\hat{k}^r_{\hat{m}} = k \in B^r$, that is, for any region $r \in R_m$, take a berth $\hat{k}^r_m$ with a bottleneck in machine type m, if such berth does not exist, take any berth $\hat{k}^r_m$ of region r. The pattern defining as:

$$q_{\hat{p},m,k} = \begin{cases} q_{p,m,k} + n^{\hat{r}}_{\hat{m}}, & \text{if } k = \hat{k}^r_m, \forall m \in M \text{ and } \forall r \in R_m \\ q_{p,m,k}, & \text{otherwise.} \end{cases} \tag{21}$$

For all $\hat{k}^r_{\hat{m}}$ satisfying the Equation 20 we have $\delta^{\hat{p}}_{\hat{k}^r_{\hat{m}}} > \delta^{p}_{\hat{k}^r_{\hat{m}}}$ and by Theorem 2 a solution of an instance of the Model 1 that contains p is not optimal, or it is an alternative optimal solution.

If none $\hat{k}^r_{\hat{m}}$ satisfy Equation 20 then $\delta_{\hat{p},k} = \delta_{p,k}, \forall k \in B$ and replacing p for $\hat{p}$. That generates an alternative optimal solution, which completes the proof.

□

4. Solution Method

This section contains the main heuristics used to solve the instances of the models. The heuristic first finds an upper time limit (T), this time limit will be used to calculate the set of vessels which will be assigned to a berth in the iteration. Each iteration assigns the vessels of (S) to berths, the set S is given by all not assigned vessels which has an arrival time less than T. The set S can contain more vessels than the total number of available berths, in that case, we prioritize vessels with less load. After allocating the vessels, a machine pattern is selected. After assigning all vessels of S, the value of T is updated and a new iteration is initiated.

To show an example of the Algorithm 1, consider Figure 3.

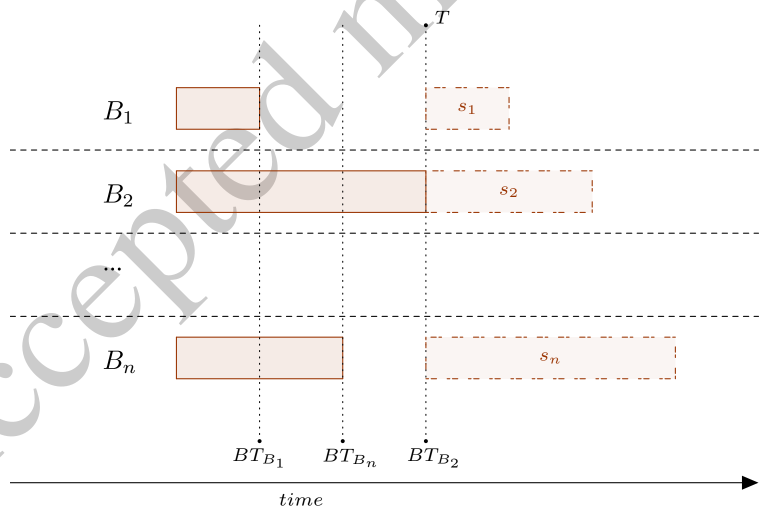

Figure 3. Example of Heuristic 1 - Iteration 1.

Figure 3 shows three berths, B_1, B_2 and B_n. Each rectangle represents a vessel and its length represents the handling time. The solid rectangles are vessels treated in previous iterations, and the dotted rectangles are vessels treated in the actual iteration. Each berth became available in the time BT_{B_i}, which is given by the end of the handling of previous vessels. The Step 19 of the Algorithm 1 results in $I = \{B_1, B_2, B_n\}$.

In the Step 21, the best machine pattern p^* is determined. With p^*, the values of $T^*_i; t^*_i$ and z^* are calculated. In this step all berths are available at the time $T = max_{b \in B} BT_b$, and every machine is available to work at any berth, therefore, we can choose any feasible machine pattern. The set of vessels

1: S_b is the vessel schedule of Berth $b \in B$;
2: N is the set of Vessels;
3: $q_{p,\alpha,k}$ is the number of machines of pattern p, type α in berth k;
4: $T_i; t_i; z$ are the allocation time, handling time, objective function value, respectively. Where $i \in N$;
5: $T_i^*; t_i^*; z^*; p^*$ are the best allocation time, handling time, objective function value and machine pattern, respectively. Where $i \in N$;
6: BT_b is the opening time of berth $b \in B$;
7: $o_b \leftarrow 1$ for $b \in B$, is the position of the schedule of berth b;
8: $T \leftarrow \min_{b \in B}(a_{p_b} + t_{p_b})$, is the initial time reference;
9: $mB_{b,\alpha}$ is the number of machines type α in the berth b, where $b \in B$ and $\alpha \in P$;
10: $n \leftarrow 0$;
11: Initialize the values of $T_i^*; z^*; mB_{b,\alpha}$ as zero;
12: $t_i \leftarrow$ estimate handling time of the vessel $i \in N$;
13: **while** $n < |N|$ **do**
14:   **for** $b \in B$ **do**
15:     **if** $o_b > |S_b|$ **then**
16:       Remove the berth b of set B;
17:     **end if**
18:   **end for**
19:   $I \leftarrow \{b : BT_b < T, b \in B\}$;
20:   $S \leftarrow \{S_b(o_b) : b \in I\}$;
21:   Find $T_i^*; t_i^*; z^*; p^*$ for $i \in S$ by testing the machine patterns for terminals;
22:   Ordered I and S according to the value of BT in descendant order;
23:   **for** $i \in I$ **do**
24:     $T \leftarrow BT_i + t_{S(i)}$;
25:     Find $T_j; t_j; z; p$ for $j \in S$ and $p \in P$ by selection patterns with $\sum_{i \in S} q_{p,\alpha,i} \leq \sum_{i \in S} mB_{i,\alpha}$;
26:     **if** $z < z^*$ **then**
27:       $z^* \leftarrow z$;
28:       $BT_i \leftarrow T_{S(i)} + t_{S(i)}; mB_{i,\alpha} \leftarrow q_{p,\alpha,S(i)}\ T_j^* \leftarrow T_j; t_j^* \leftarrow t_j; p^* \leftarrow p$ where $j \in S$.
29:     **end if**
30:     Remove $S(i)$ from S;
31:   **end for**
32:   $T \leftarrow \max_{b \in B}\{T^*_{S_b(o_b)} + t^*_{S_b(o_b)}\}$;
33:   $BT_b \leftarrow T^*_{S_b(o_b)} + t^*_{S_b(o_b)}$ for all $b \in B$;
34:   $mB_b \leftarrow q_{p^*,\alpha,S(i)}$ for all $b \in B$ and machine type α;
35:   $o_b \leftarrow o_b + 1$ for all $b \in I$;
36:   $n \leftarrow n + |I|$;
37: **end while**

Algorithm 1. Algorithm to Solve the Berth and Machine Allocation Problem

is $S = \{s_1, s_2, s_n\}$. In the Step 22 the set S is ordered by berth time BT_b in descendant order, which is $S = \{s_2, s_n, s_1\}$. The Step 23 begins with $T = max_{b \in B} BT_b$, in the example, $T = BT_{B_2}$. Then, the vessel in B_2 has its position fixed, and it is removed from S, and the values of allocation time, handling time and machine pattern are given by $T^*_{B_2}, t^*_{B_2}$ and p^*, respectively. In the next step, the value of T returns to the previous position, which is $T = BT_{B_n}$. Figure 4 shows the second iteration of the Loop 23.

In the second iteration of loop 23, $T = BT_{B_n}$ and $S = \{s_n, s_1\}$. Adding the constraint limiting

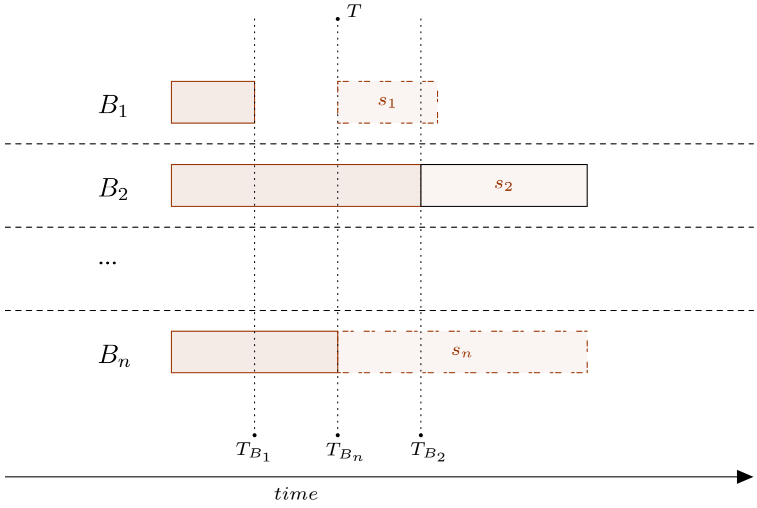

Figure 4. Example of Heuristic 1 - Iteration 2.

the machines of vessels s_1 and s_n to the sum of machines in B_1 plus the machines in B_n plus the idle machines, which is $q_{p,s_1,\alpha} + q_{p,s_n,\alpha} \leq mB_{B_1,\alpha} + mB_{B_n,\alpha} + (|M_\alpha| - \sum_{b \in B} mB_b)$ for all machine type α to the problem. In Step 21 the best machine pattern, which does not violate the new constraint, is selected. If the objective function value z, is better than z^*, which is $z < z^*$ than $z^* \leftarrow z$, $T^*_{s_1} \leftarrow T_{s_1}$, $t^*_{s_1} \leftarrow t_{s_1}$, $T^*_{s_n} \leftarrow T_{s_n}$, $t^*_{s_n} \leftarrow t_{s_n}$ and $p^* \leftarrow p$. If $z \geq z^*$ or the solution is infeasible, nothing changes for $T^*_{s_1}, t^*_{s_1}, T^*_{s_n}, t^*_{s_n}$ and q^*. The vessel s_n is fixed with the values of $T^*_{s_n}, t^*_{s_n}$. In the example, the value of z is greater than the value of z^*, therefore, the vessel s_n is fixed with $T_{s_n} = BT_{B_2}$, which can be seen in Figure 5.

In the iteration 3, $T = BT_{B_1}$ and $S = \{s_1\}$. Figure 5 shows the iteration 3.

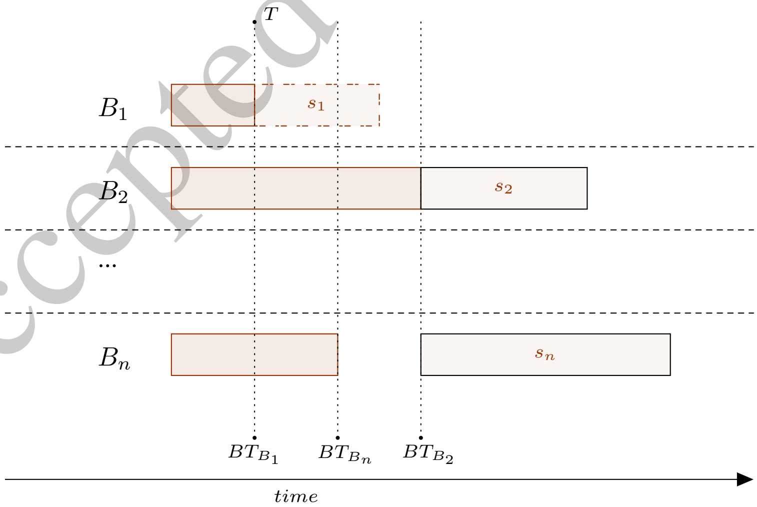

Figure 5. Example of Heuristic 1 - Iteration 3.

The iteration 3 begins adding the constraint limiting the machines of vessel s_1 to the sum of machines in B_1 plus the idle machines, which is $q_{p,s_1,\alpha} \leq mB_{B_1,\alpha} + (|M_\alpha| - \sum_{b \in B} mB_b)$ for all machine type α. With that constraint and the initial berth time set to $BT = T_{B_1}$ the best machine pattern is selected. In the example, $z < z^*$, therefore, $z^* \leftarrow z$, $T^*_{s_n} \leftarrow T_{s_n}$, $t^*_{s_n} \leftarrow t_{s_n}$, $p^* \leftarrow p$. The value T_{s_1} is fixed as $T_{s_1} = BT_{B_1}$, s_1 is removed from S and the Loop 23 ends. Figure 6 shows the final allocation of s_1, s_2 and s_n.

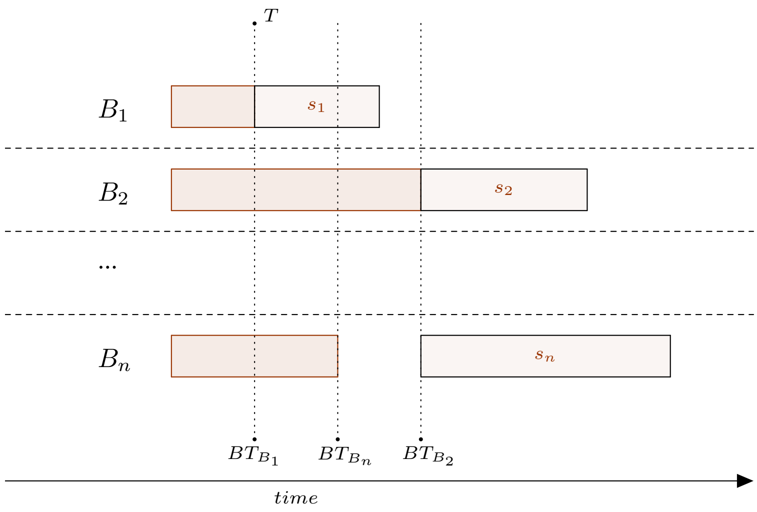

Figure 6. Example of Heuristic 1 - Final.

After the selection of the machine pattern to S, the values of BT, mB, p, and T are updated, and in the next iteration of the main loop, T will be used to select the new group of vessels.

5. Tests and Results

The model, heuristic and FIFO has been implemented in MatLab using the commercial solver CPLEX 12.3 when needed and tested on the same set of instances. Instances generated to validate our models are based on real data, from a small terminal with two berths, a small number of vessels, and machines to a large multi-terminal with more than a hundred berths, vessels, and machines. Table 1 brings the data interval of the instances of Table 2.

Table 1. Parameters interval

Parameter	interval
Number of berths	$[2; 125]$
Number of vessels	$[4; 600]$
Types of machines	$[1; 3]$
M_α	$[2; 300]$
a_i	$[1; 100]$
Q_i	$[1; 100000]$
$c_{i,\alpha}$	$[1; 2]$
$d_{i,\alpha}$	$[2; 10]$
δ_α	$[3000; 9000]$
β	4
β'	1

Experiments have been run with a time limit of 1 hour for instances with less than 4 berths and 3 hours otherwise. The solver was able to find solutions only in some small-scale instances, but not all. As the number of berths and vessels increases, the solver was only able to solve instances close to trivial, where the arrival of vessels is sparse. In cases with more than 4 berths, the solver was not able to find a solution in any instance.

Table 2 brings the results of some instances. In Table 2 the name of each case is composed by: *[number of berths]* + B + *[number of vessels]* + N +*[quantity of machine Type 1]* + *[quantity of* *machine Type 2*] + *[quantity of machine Type 3*].

In Table 2 the column *Case* bring the instance name, all columns of *Solver*, *Heuristic* and *FIFO* brings the respective results. The *O.F.* columns are the value of the objective function of models 1 and 2, respectively, *gap* is the difference between the primal and the dual function obtained by the respective strategy. The *time* columns are the processing time in seconds, the column *Solver vs Heu* is the comparison between solver and heuristic solutions and *Heu vs FIFO* is the comparison between heuristic and FIFO solutions.

A negative value in column *Solver vs Heu* indicates a better solution of the solver over the heuristic by the respective percentage and a negative value the otherwise, same for the column *Heu vs FIFO*. The FIFO implementation needed only a few seconds for finding solutions even in large-scale instances, therefore, the processing time was omitted in Table 2.

The instances from *2B4N54* to *3B8N735* have up to three berths, nine vessels, and 3 types of machines. In those instances, the heuristic obtained solutions close to the optimal obtained by the solver. In 16 of 51 instances, the solutions of the heuristic and solver are the same. All heuristic solutions are at most 40% worse than the solver solution. The instance *3B5N34* has the worst result, with 40% higher value than the solver solution. In all instances, the heuristic solution is better or equal to the FIFO solution.

The instances from *10B35N2834* to *4B50N1806* have from 4 to 10 berths, from 35 to 80 vessels and 2 types of machines. The solver found a high gap feasible solution with up to three hours of processing. The heuristic obtained solutions better or equal than FIFO in at most 3 seconds, in one instance the solution is up to 78% better. The FIFO and the heuristic solution have the same objective function value, due to the arrival time of the vessels in those instances being sparse.

The instances from *8B50N30* to *125B600N300250* have from 8 to 125 berths, from 50 to 600 vessels and 2 types of machines. The solver was unable at finding solutions after three hours of processing. The heuristic was able to obtain better or equal solutions than FIFO in at most three seconds, only two instances have the same solutions and the others have solutions up to 82% better.

Most of the optimal results of the small-scale instances prioritize the allocation of vessels with low handling time, which are related to the choice of $\beta \geq \beta'$. Choosing $\beta \geq \beta'$ prioritizes the allocation time over handling time. If two vessels are waiting in a berth queue, the sum of the allocation time of both vessels will be minimal by choosing to allocate the vessel with the smaller handling time first, which leads to a better objective function. Choosing $\beta \geq \beta'$ also leads to a better occupation of the berths, since we want to allocate the vessels early as possible, it is better to split the number of machines into different berths to be possible to allocate the vessels early. The usage of $\beta \geq \beta'$ seems more adequate, since choosing the $\beta' \geq \beta$ can lead to the concentration of the machines in a few berths to reduce the handling time of the vessels. The insight obtained from the small-scale instances was used in the heuristic. The heuristic prioritizes the allocation of vessels with less handling time in the step 22.

6. Conclusions

A mixed integer programming formulation has been presented to address the integration of the berth allocation problem with the machine assignment problem. The model has been validated on instances based on real data using a solver. These tests show that the problem is hardly solvable even in small instances. The proposed theorem can be used to reduce the computational complexity in some instances. As the number of berths and vessels increase, the solver was unable to find solutions, even with the application of the theorem, due to the high computational complexity. A heuristic algorithm to efficiently solve instances of the problem has also been presented, the heuristic is able to provide good feasible solutions in minutes.

The solver and heuristic solutions had similar results in the small instances. On average the solver obtained solutions 8% better than the heuristic, and both are more than 20% better than the FIFO strategy, on average. In larger instances, the solver was unable at finding solutions and the heuristic obtains solutions around 21% better than the FIFO strategy. The heuristic was able to solve instances with more than 100 berths, 500 vessels, and 250 machines in three seconds. These dimensions are similar to the biggest terminals in the world.

Data availability statement: The data that support the findings of this study are openly available in A Mathematical Model for the Berth Allocation Problem with Variable Service Time and Continuous Time Horizon at http://doi.org/10.17632/kdr7cn53k4.5, reference number 10.17632/kdr7cn53k4.5.

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Table 2. Tests and results

	Solver					*Heuristic*		*FIFO*	*Solver*	*Heu*
Case	**O.F.1**	**gap 1**	**O.F.2**	**gap 2**	**time**	**O.F.**	**time**	**O.F.**	*vs Heu*	*vs FIFO*
2B4N43	23,9	0%	21,6	0%	0	24,9	0	39,8	13%	-60%
2B4N44	16,5	0%	16,5	0%	0	16,5	0	16,5	0%	0%
2B4N52	119,4	0%	119,4	0%	0	125,2	0	125,2	5%	0%
2B4N54	10,5	0%	10,5	0%	0	10,5	0	11,1	0%	-6%
2B4N63	43,7	0%	43,7	0%	0	43,6	0	43,6	0%	0%
2B5N27	141,4	0%	141,4	0%	1	178,7	0	194,4	21%	-9%
2B5N32	181,1	0%	181,1	0%	1	181,1	0	227,0	0%	-25%
2B5N86	40,0	0%	40,0	0%	0	45,9	0	45,9	13%	0%
2B6N43	81,9	0%	82,0	0%	1	88,7	0	94,7	8%	-7%
2B6N53	72,1	0%	72,1	0%	1	76,8	0	86,3	6%	-12%
2B6N66	72,3	0%	72,3	0%	0	72,5	0	105,4	0%	-45%
2B6N75	42,8	0%	42,8	0%	1	46,3	0	46,3	7%	0%
2B8N33	534,8	0%	534,8	0%	254	565,0	0	643,8	5%	-14%
2B4N272	81,8	0%	81,8	0%	0	85,8	0	147,1	5%	-71%
2B4N375	86,9	0%	86,9	0%	0	86,9	0	92,7	0%	-7%
2B4N432	131,7	0%	131,7	0%	0	136,5	0	138,7	4%	-2%
2B4N678	100,6	0%	100,6	0%	0	101,0	0	101,0	0%	0%
2B4N714	242,4	0%	242,4	0%	0	242,4	0	296,1	0%	-22%
2B5N354	53,8	0%	53,8	0%	0	63,7	0	63,7	16%	0%
2B5N363	76,2	0%	76,2	0%	0	80,0	0	80,0	5%	0%
2B5N772	78,9	0%	78,9	0%	0	84,0	0	84,0	6%	0%
2B5N837	56,6	0%	56,6	0%	0	58,6	0	148,2	3%	-153%
2B6N235	143,5	0%	143,5	0%	1	160,6	0	251,9	11%	-57%
2B6N455	71,8	0%	71,8	0%	0	85,2	0	89,6	16%	-5%
2B6N628	221,0	0%	221,0	0%	1	231,0	0	282,9	4%	-22%
2B7N256	509,3	0%	509,3	0%	6	509,3	0	657,5	0%	-29%
2B7N346	130,1	0%	130,1	0%	6	137,4	0	165,7	5%	-21%
2B8N237	73,0	0%	73,0	0%	1	74,6	1	74,6	2%	0%
2B8N428	592,3	0%	592,3	0%	47	592,3	0	742,5	0%	-25%
2B9N476	215,5	0%	215,5	0%	368	215,5	0	319,3	0%	-48%
3B4N26	56,7	0%	56,7	0%	0	56,7	0	56,7	0%	0%
3B4N34	30,4	0%	30,4	0%	0	39,8	0	39,8	24%	0%
3B4N36	27,8	0%	27,8	0%	0	28,5	0	39,6	3%	-39%
3B4N55	8,8	0%	8,8	0%	0	8,8	0	20,4	0%	-133%
3B4N66	31,1	0%	31,1	0%	0	31.1	0	32,5	0%	-11%
3B4N72	89,1	0%	89,1	0%	0	109,3	0	160,3	18%	-47%
3B4N73	64,1	0%	64,1	0%	0	64,1	0	64,1	0%	0%
3B4N86	5,6	0%	5,6	0%	0	5,6	0	15,3	0%	-173%
3B5N34	20,8	0%	20,8	0%	1	34,9	0	35,5	40%	-2%
3B5N35	84,6	0%	84,5	0%	10	91,1	0	91,1	7%	0%
3B5N45	62,8	0%	62,8	0%	0	85,2	0	104,9	26%	-23%
3B5N57	75,4	0%	74,0	0%	18	90,8	0	95,3	19%	-5%
3B6N22	216,3	0%	216,3	0%	129	228,7	0	228,7	5%	0%
3B6N37	96,6	0%	94,6	0%	8	116,5	0	116,5	19%	0%
3B6N55	68,2	0%	67,6	0%	15	74,1	0	74,1	9%	0%
3B7N43	133,0	0%	133,0	0%	8	182,4	0	182,6	27%	0%
3B7N45	112,1	0%	110,6	0%	523	164,6	0	258,9	33%	-57%
3B4N433	26,0	0%	26,0	0%	1	26,0	0	27,7	0%	-7%
3B4N725	81,4	0%	81,4	0%	0	86,3	0	112,0	6%	-30%
3B4N832	44,1	0%	44,0	0%	0	46,8	0	59,2	6%	-26%
3B5N478	42,2	0%	42,2	0%	1	44,4	0	53,9	5%	-21%
3B6N457	151,5	0%	151,5	0%	3	162,3	0	205,7	7%	-27%
3B8N735	272,8	0%	272,0	0%	1442	284,1	0	288,6	4%	-2%
10B35N2834	360,0	97%	359,9	96%	35196	71,2	1	71,2	-405%	0%
9B55N2923	1024,8	97%	1024,8	97%	3619	449,2	1	449,2	-128%	0%
8B70N1726	892,0	99%	891,9	98%	3670	26,7	1	26,7	-3237%	0%
7B80N1520	3296,7	90%	3296,7	90%	3170	104,9	1	104,9	-3043%	0%
7B40N1221	144,2	96%	144,2	96%	3612	15,4	1	15,4	-839%	0%
6B65N1120	1477,4	99%	1477,4	99%	3688	39,5	1	39,5	-3639%	0%
5B60N1207	13180,7	95%	13180,7	95%	3626	7012,4	1	12477,5	-88%	-78%
4B50N1806	570,9	98%	570,9	98%	12991	39,4	1	39,4	-1349%	0%
8B50N30						4866,5	0	8864,1		-82%
16B50N47						628,2	0	628,2		0%
20B60N5445						776,8	0	801,3		-3%
30B100N6745						565,8	1	654,4		-16%
30B100N7550						4415,3	0	6045,8		-37%
30B200N6099						575,5	1	602,0		-5%
60B300N150120						777,8	2	1057,9		-36%
125B600N300250						1622,0	3	2080,8		-28%