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Supply chain coordination and decision-making under revenue sharing and cost-revenue sharing contracts with returns

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Abstract

The increasing prevalence of product returns poses challenges for businesses, the environment, and society. Efficient returns systems need to be developed. This article addresses the issue by presenting a game-theoretical modeling approach to optimize pricing and ordering decisions in supply chain contracts between manufacturers and retailers. Revenue-sharing and cost-revenue-sharing contracts are investigated in conjunction with two returns-handling strategies: one performed by the manufacturer and the other by the retailer. As a result, four distinct contract scenarios are derived. In each scenario, the manufacturer-leader and the retailer-follower engage in a Stackelberg game. Optimal solutions are obtained for the models. In addition, it has been shown that the supply chain can be coordinated if the manufacturer shares the revenue and returns handling costs with the retailer. Numerical analyses are conducted to illustrate the theoretical results.

Keywords: *returns, optimization, pricing, revenue sharing, supply chain coordination*

1. Introduction

Contract management is a crucial process in supply chain management (SC). It helps businesses negotiate and manage the terms of an agreement between two or more parties. It requires establishing different requirements for what each party delivers and ensures that both sides carry out their responsibilities in the deal [30]. The idea of a contract implies a coordination mechanism within a decentralized supply chain that incentivizes members of the supply chain to act in a manner consistent with behaviors typically seen in a centralized supply chain. Decentralization decreases system efficiency [45] and reduces SC's profit [33].

The operations management literature devotes significant attention to revenue sharing (RS) contracts, which have gained increasing favor among businesses [8]. By determining how profits and losses are

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distributed among members of the supply chain, the RS contract plays a crucial role in effectively coordinating SC [10]. RS contracts are typically modeled in two ways: one involves a wholesale-type price contract with RS, while the other involves a consignment contract with RS. The first approach constitutes a generalization of the wholesale price contract with the RS mechanism added. Occasionally, SC members engage in cost-revenue sharing (CRS) contracts, where they share revenue and various types of costs. In comprehensive investigations, the distribution of revenue is ultimately shared between the parties based on a negotiated fixed percentage.

The main goal of this theoretical study is to construct and analyze contracts characterized by wholesale-type agreements with RS and CRS policies and, additionally, False Failure Returns (FFR). FFR also known as non-defective returns refers to product returns that do not exhibit any cosmetic or functional defects [17]. The volume of consumer returns, including FFR, is experiencing a significant increase and has exceeded more than USD 200 billion in the US in 2021, accounting for approximately 20% of total online sales [19]. Businesses pay costs that are up to 66% of the original price of the product as a result of consumer returns. Regardless of whether the item is returned in perfect condition, the overall return process remains costly due to the labor, transportation, repackaging, and inspection requirements involved [16]. FFR belongs to the return fraud category. Return fraud can be committed intentionally or unintentionally by customers and is very costly for businesses. It includes the following activities in the SC, e.g. wardrobing if customers buy an item planning to use it once and then return it for a refund; bracketing if customers buy multiple items to return the ones they do not want or price arbitrage when customers buy two similar-looking but differently priced items and return the cheaper item as if it is the more expensive one. Therefore, there exists a strong argument to introduce new effective returns policies to manage FFR [18].

In this article, the SC is treated as a transactional process, namely the flow of goods in one direction and financial resources in the reverse one. The SC consists of two risk-neutral firms: the manufacturer and the retailer. They are involved in these transactions, and the retailer is connected to the manufacturer through the share of the sales profit. The manufacturer's decision-making concerns determining the wholesale price and choosing the method of sharing returns costs and the retailer's decisions are related to establishing the order quantity and retail price. The set of decision variables that produce an optimal result is found through negotiations, the result of which is recorded in contracts that specify the terms of the agreements. Indicating optimal quantities in contracts for both parties is based on their profit maximization. Optimality is understood as the identification of decisions for which corrections lead to a worsening of the outcome for either party in the transactions. This requirement can be represented as an equilibrium point in the Stackelberg game with the manufacturer as the leader and the retailer as the follower [10].

In the SC studied, the product delivery and sales handling in a forward decentralized channel transaction run from the manufacturer through the retailer to the consumer. The retailer faces the newsvendor problem, purchases the product at a wholesale price, and then sells it to the consumer at a retail price. There is one selling season with stochastic demand and a single opportunity for the retailer to order inventory from the manufacturer before the selling season begins. The non-defective products sold may be returned and exchanged for new ones by the consumer in the reverse channel. Any leftovers can be salvaged with no limit in capacity. The reverse channel can be considered according to strategy M or strategy R. In strategy R, the consumer returns the product to the retailer, who receives and handles the returns. In strategy M, the consumer returns the product directly to the manufacturer, who is responsible for handling the returns. In both M and R strategies, the forward channels are identical. After determining the returns handling strategy, the manufacturer selects an RS or CRS contract to be concluded between the channel members. More specifically, in the RS scenario, the retailer shares revenues with the manufacturer according to a revenue-sharing ratio negotiated among channel members. In the CSR scenario, depending on whether strategy R or strategy M is adopted, the retailer shares its revenues and additionally returns handling costs. Consequently, in the decentralized channel, four distinct contract structures regulate (Table 1), 1) revenue sharing (RS) under the M manufacturer handling strategy denoted by the RS-M contract scenario; 2) cost and revenue sharing (CRS) under the M manufacturer handling strategy denoted by the CRS-M contract scenario; 3) revenue sharing (RS) under the R retailer handling strategy denoted by the RS-R contract scenario; 4) cost revenue sharing (CRS) under the retailer handling strategy R denoted by the CRS-R contract scenario.

Table 1. Setup of contract scenarios

Sharing policy	Strategy M	Strategy R
Revenue sharing	RS-M	RS-R
Cost-revenue sharing	CRS-M	CRS-R

In the SC, the following sequence of events occurs:

- The manufacturer designs and proposes the contract structure to the retailer.
- If the retailer does not agree with the manufacturer's offer, the contract is not concluded.
- If the retailer agrees with the manufacturer's offer, the following Stackelberg game with the manufacturer as the leader begins, and the contract emerges from the outcome of this game.
  - Firstly, the Stackelberg leader, the manufacturer, sets the wholesale price, maximizing their own expected profit.
  - Then, the retailer decides on the retail price and order quantity, aiming to maximize their own expected profit.
- The manufacturer produces and delivers the product to the retailer before the selling season.
- Season demand occurs, the products are returned and exchanged according to the mismatch rate, and transfer payments are made between the firms based upon the agreed contract.

The manufacturer is assigned to make the contract offer, rather than the retailer, but it has no impact on the subsequent mathematical analysis. It is done mainly for expositional convenience [8].

The specific objectives of the article are to construct, mathematically formulate and optimize contract scenarios considering revenue or cost-revenue sharing contracts with returns handling made by the manufacturer or retailer, and to find whether these contracts coordinate the SC. To achieve these goals, we formulate the following research questions:

- **RQ1.** What are the mathematical models of the developed contract scenarios?
- **RQ2.** What are the optimal solutions to maximize the respective expected profits in the contract scenarios obtained?
- **RQ3.** Which contract proposed in the article is able to coordinate the SC?

The above questions concern decisions regarding price levels, order quantities, and SC coordination under demand uncertainty. In the contracts considered in this article, equations for optimal retail price and order quantities as well as a procedure for determining the wholesale price are obtained. SC coordination research is of significance as it addresses the inherent conflict of interests within a SC, where channel members may make decisions without taking into account the repercussions for other channel members. The research demonstrates that the examined RS contracts with returns are unable to fully coordinate the SC unless the returns handling costs are shared between the parties. That is, the CRS-M and CRS-R contracts have the potential for SC coordination and enhance SC performance, whereas the RS-M and RS-R contracts cannot do that. A numerical example is provided to illustrate the solutions obtained.

The model is based on a scenario that describes relationships between the SC members introduced in [36] and developed further in [5]. Liu et al. [36] consider wholesale or buyback transactions with price-free demand and retail price being the exogenous variable. Bieniek [5] supplements the model of [36] with endogenous price and linear additive demand, but still considers the wholesale price contract and limits the study to the manufacturer's handling strategy. The contribution of this article to the existing literature is as follows:

- considering RS and CRS contracts with returns in the SC instead of the wholesale price contract with returns;
- creating four different SC contract scenarios dependent on returns handling, namely manufacturer or retailer handling, and upon the distribution of revenue and costs between parties; mathematically formulating and optimizing these contracts;
- treating the retail price in the studied RS agreements as a decision variable and making demand dependent linearly on price with additive uncertainty;
- establishing among the introduced contracts those that have the ability to coordinate the SC.

These theoretical findings may serve as a basis for managers to develop an effective returns policy, which is currently of significance due to the widespread use of RS contracts and the growing issue of product returns, especially in e-commerce transactions, e.g. returns of books at Amazon, or apparel returns at AliExpress or Zara.

2. Related literature

The article refers to the RS contract and returns handling in supply chains. We describe the findings related to these issues. It should be noted that the growing number of works on this subject implies that we concentrate on the most recent research.

2.1. Revenue sharing contracts

The study of Cachon [8] provides a comprehensive overview of SC coordination using contracts, including the RS contract. The work establishes that coordination becomes more complex when price and ordering decisions are incorporated, as incentives designed to align one action may lead to distortions in the other. Subsequently, Cachon and Lariviere [10] examine the RS contracts within the SC model, where revenues are determined by each retailer's order quantity and price. The model considers scenarios where the supplier sells to a fixed price or price-setting newsvendor. The authors demonstrate that the RS contract can be coordinated within a SC involving a single retailer, albeit with an arbitrary allocation of the SC profit.

Bart et al. [3] present a review of the literature on RS contracts, offering an extensive overview of the research field and identifying potential avenues for further investigation. They present mathematical formulations for a wholesale price contract with an RS mechanism added, as well as a consignment contract with RS.

Wang et al. [55] introduce three novel types of wholesale price contract with RS and compare them with the classical RS contract introduced by Cachon and Lariviere [10]. They assume the price-dependent deterministic demand function and consider that the manufacturer is the leader in the Stackelberg game with uncertain knowledge of the system parameters. The authors state that if these parameters are known deterministically, the classical model provides perfect coordination. Palsule-Desai [38] study the Cachon and Lariviere [10] scenario with the share ratio dependent on general revenue or assume a fixed percentage revenue-sharing ratio.

Katok and Wu [31] introduce a new variant of the classical contracting of RS in which each party receives a certain sum of money for a unit sold instead of splitting the revenue according to a ratio. Instead of splitting the revenue in terms of the fixed ratio, one can do it with a non-fixed one. Gerchak et al. [20] use a sharing ratio dependent on the reported sales of the retailer. Cheng et al. [12] investigate a ratio dependent on the retail price.

Wang et al. [54] demonstrate that in a consignment contract with RS, the overall performance of SC and of individual firms is influenced by demand price elasticity and the retailer's share of channel costs. Taking into account a manufacturer as the leader and a retailer as the follower in the RS contract with consignment, Li et al. [35] develop a cooperative game model using the Nash bargaining approach to achieve profit sharing and cooperation between parties. Under the assumption of risk neutrality, the decentralized SC can be perfectly coordinated. Adida and Ratisoontorn [2] examine the consignment contract with RS with one manufacturer and two retailers. However, the model proved to be very complex, and numerical solutions were obtained exclusively. Gong et al. [22] analyze a few consignment contracts with RS to outsource logistics to a manufacturer and a third-party logistics provider providing logistics services between the manufacturer and the retailer. Choi and He [14] compared two types of consignment contracts in a sharing economy in which consumers trade with each other via a platform.

In a closed-loop SC context, Xie et al. [56] employ a Stackelberg game and combine RS contracts in the forward channel with channel investment cost-sharing (CS) contracts. They propose a contract with appropriately set RS and CS ratios, which enhances the profits of SC members in both channels. Ran et al. [41] focus on the Bullwhip effect and SC coordination in the context of CS and CRS contracts involving a supplier and retailer. The show that coordination enhances the performance of the SC, especially in the case of the CRS contract.

Considering whether a risk-averse SC is coordinated, Biswas et al. [6] employ a mean-variance approach to centralized and decentralized cases under buyback and RS contracts. They illustrate how the risk-averse behavior of an individual SC agent affects the contract selection mechanism.

Xue and Wang [58] explore a dual-channel SC with a risk-averse retailer and a risk-neutral and fairness-neutral manufacturer. They present a joint contract comprising RS and buyback contracts with the aim of coordination with the SC. They assume yield- and demand-uncertainty conditions that lead to a Pareto improvement.

In a two-tier SC with capital constraints and asymmetric information, Yan et al. [59] establish an equilibrium analysis using the Stackelberg game and the principal-agent theory. They find that a joint contract that combines RS and transfer payment can effectively coordinate SC. Finally, Tan et al. [49] propose a model incorporating value-added services and order cancellation behavior. They design a CRS contract that improves value-added services and profits within the decentralized channel.

2.2. Returns

FFR is implemented in the model of Ferguson et al. [17] who address the challenge of reducing FFR through SC coordination. They propose a target rebate contract and demonstrate that this contract leads to a Pareto improvement, substantial profit enhancement for both parties involved, and overall improvement for the SC.

Later, Su [46] develops a model of consumer returns policies in which consumers face valuation uncertainty realized after purchase. Consumers decide whether to purchase and returns the product and the seller sets the price, quantity, and refund amount. They study the impact of full- and partial-returns policies on SC performance and propose strategies for SC coordination.

Chiu et al. [13] show that a policy that includes wholesale price, channel rebate, and returns can coordinate a channel under additive and multiplicative price-dependent demands. Moreover, they prove that there are multiple equilibrium policies for channel coordination and explore how the equilibrium policy can be adjusted to achieve the Pareto improvement.

Chen [11] investigates a supply chain with a manufacturer and a retailer in which the manufacturer is the Stackelberg leader and the retailer is the follower. The work considers a returns policy with a wholesale-type return-discount contract which can coordinate the supply chain.

Huang et al. [29] suggest using a coordination contract to resolve a profit conflict between the manufacturer and the retailer that results from exerting to reduce FFR. They introduce a quantity discount contract, which specifies a payment to the retailer with an amount exponentially decreasing in the number of returns. They show that the contract is Pareto improving and show that if the contract is applied in a closed-loop supply chain, it can discourage the retailer from accepting returns.

Govindan and Popiuc [23] design an analytical model used to explore the implications of recycling in the reverse supply chain from an efficiency perspective. They consider the two- and three-echelon reverse supply chains that are coordinated by the RS contract. They specify the willingness of the customer to return antiquated units as a function of the discount offered by the retailer. These units can be exchanged for recycling devices with a remanufacturing value. The findings show that performance measures improve through coordination with RS contracts on both reverse supply chains.

In [57], the authors investigate a dual-channel supply chain contract with risk-averse agents under a mean-variance model. They obtain optimal price decisions in a centralized and decentralized SC and analyze the impact of risk tolerance on these decisions. They show that the price set by a risk-averse SC is lower than the one set by a risk-neutral one. Furthermore, vertical and horizontal competition results in channel inefficiency. They propose a two-way RS contract that can coordinate the dual-channel SC with risk aversion.

Guo et al. [25] review the recent state-of-the-art literature on SC contracts with reverse logistics systems. They investigated different types of such contracts and classified and examined the literature on SC structure and channel leadership. They identify research gaps and suggest the main categories of future research directions.

The authors of [27] explore SC, which offers a money-back guarantee policy and faces a stochastic demand. With a given SC contract offered by the supplier, the retailer makes decisions on the amount of orders and market coverage of the money back guarantee service. They studied three models: the wholesale price contract, the model with a buyback contract for unsold items, and a dual-buyback contract model for both unsold and consumer-returned items. They find that using the buyback contract for unsold items cannot achieve the Pareto improvement, while the double buyback contract can.

Heydari and Ghasemi [28] study a two-echelon reverse SC consisting of one remanufacturer and one collector. The collector offers a reward to convince consumers to return their used products. Uncertainty arises from the quality of the returned products and the remanufacturing capacity. To optimize the reverse SC, the collector determines the reward amount considering the idle and overload capacity of the remanufacturing process. A customized RS mechanism is developed to fairly share the risk of uncertainties between two members. They indicate that, when remanufacturing capacity is limited, the RS contract is able to share risks between participants and creates a win-win situation.

Gu et al. [24] consider a fresh product SC consisting of one supplier and one e-tailer. The supplier sells fresh products through e-Commerce in an online market, and the e-Commerce company offers a full refund returns policy to loss-averse consumers and exerts a fresh-keeping effort to keep the product at the optimal freshness level. The authors derive the unique optimal price, quantity, and fresh-keeping effort jointly in the centralized setting. They demonstrate that the e-tailer has an incentive to engage in a fresh-keeping effort and show that the return rate is independent of this effort and consumers' loss aversion. In the decentralized setting, they obtain the optimal wholesale price numerically and find that the buyback contract can coordinate the SC, but the RS contract cannot do it. They develop a RS and CS contract with a new contractual mechanism that can coordinate the SC.

Ghoreishi et al. [21] focus on the inventory model of the quantity of economic production under inflationary conditions with returns. They provide recommendations to managers on how to utilize price as a control mechanism to align the quantity sold with inventory levels while maximizing revenues.

Examining the principal-agent problem in handling FFR within a reverse SC featuring one manufacturer and two competing dealers, Sun et al. [47] design optimal incentive contracts under both symmetric and asymmetric information. They find that under the symmetric information, the dealers' effort levels align with those that maximize the expected overall profit of the SC.

In [7], various mechanisms are proposed to incentivize retailers to reduce returns such as offering a reduced wholesale price for returns that fall below a specified target. The authors find that when faced with competition through online stores, retailers do not make more effort to reduce returns. Furthermore, the profitability of the manufacturer is consistently higher with an online store.

Considering the possibility of reselling FFR, Pingping et al. [16] investigate transshipment and inventory decisions in a dual-channel SC. They examine three scenarios with FFR, taking into account transshipment and consumer switch, and demonstrate that scenario choices significantly affect optimal quantities.

In [36] and [5], a wholesale price contract with a return handling strategy is considered. In [36], the retailer's order quantity, the manufacturer's return handling strategy, and channel coordination are explored. The study shows that a buyback contract can coordinate the SC. In [5], the endogenous retail price and price-dependent demand are also considered, with a discussion of the possibility of negative demand realizations, although only in the case of the manufacturer's handling strategy.

The authors of [34] examine returns management strategies using a theoretical game model for an SC where the manufacturer acts as the Stackelberg leader and the retailer faces customer returns. The manufacturer can choose either a buyback or a wholesale price contract. The optimal returns management strategy is determined. Furthermore, each strategy is shown to lead to a Pareto improvement.

Constructing a dual-channel SC game model with an offline return service and online reviews, Guo et al. [26] analyze the impact of the return rate, the level of service, and the perceived quality of online reviews on SC decisions. The research reveals that the perceived quality of the reviews has a greater impact on the overall profit of the SC compared to the level of returns service.

In [50], a remanufacturing SC is established where the retailer offers a money back guarantee with a full refund. The study investigates pricing decisions in four cases considering the quality of the returns. It is demonstrated that controlling remanufacturing costs can lead to a win-win situation for both members of the SC in the majority of cases.

A research gap is identified in the course of the investigation of the RS and CRS contracts with the reverse channel structure and FFR based of the model of Liu et al. [36] and [5]. Mathematical modeling is performed, optimal quantities are derived, and the assessment whether these contracts can coordinate the SC is carried out.

3. Problem formulation

Consider an SC consisting of a single manufacturer and a single retailer which are independent. In the distribution channel, the manufacturer sells a new product, through the retailer who pays the wholesale price w, to the consumer who pays the retail price p to the retailer. The manufacturer incurs a unit cost of c for the product. The model parameters, assumptions, and decision variables are provided in Table 2, and the structure of the model is depicted in Figure 1.

In this SC, the manufacturer and the retailer accept consumer FFR and allow consumers to exchange returned items for new product variants. The manufacturer offers a contract to the retailer, who has the option to accept or reject the offer. At the beginning of the sales season, the manufacturer selects one of the following two returns handling strategies: handling the returns themselves (M) or delegating the task to the retailer (R). Additionally, the manufacturer determines the type of contract to be used: the RS contract, where the revenue is shared between the parties, or the CRS contract, where both the revenue and returns handling costs are shared. The share ratio, denoted as r, is fixed in both types of contracts and is determined during negotiations between channel members [40, 53]. Based on this ratio, the retailer receives r share of the revenue or cost and revenue generated for each unit, while the manufacturer's share is equal to $1 - r$ per unit. Consequently, the retailer encounters four distinct contract scenarios, belonging to the set $\{$RS-M, CRS-M, RS-R, CRS-R$\}$ with (Figure 2):

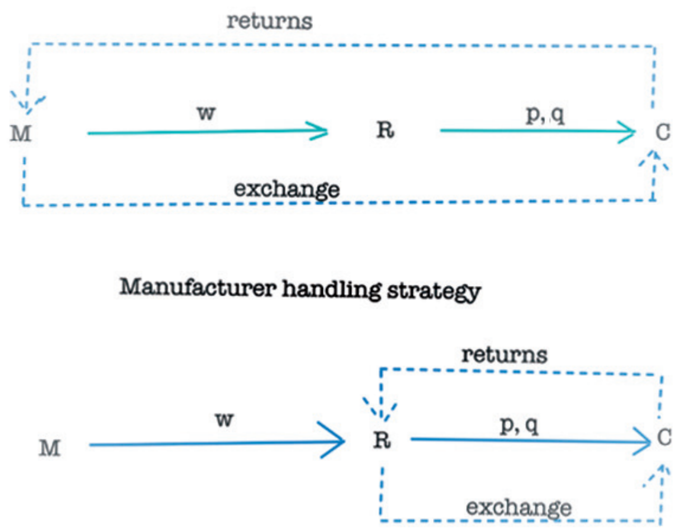

Figure 1. SC diagram under strategy M and R; C denotes a consumer in decentralized channel

1. RS-M contract scenario in which the manufacturer handles the returns, and the channel members agree to the RS contract with sharing ratio r.
2. CRS-M contract scenario in which the manufacturer handles the returns, and the channel members agree to the CRS contract with sharing ratio r.
3. RS-R contract scenario in which the retailer handles the returns, and the channel members agree to the RS contract with sharing ratio r.
4. CRS-R contract scenario in which the retailer handles the returns, and the channel members agree to the CRS contract with sharing ratio r.

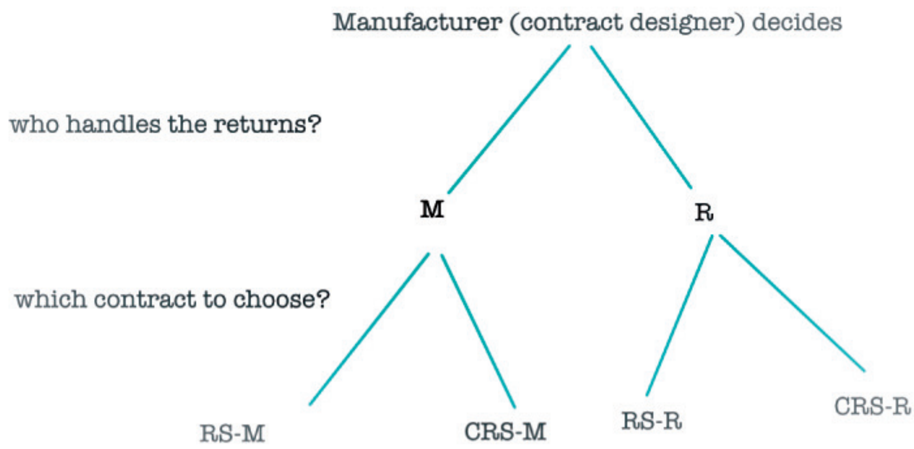

Figure 2. Design of contract scenarios in decentralized channel

Once the retailer accepts the manufacturer's offer, a Stackelberg game is initiated; in the firms engage in a strategic game characterized by sequential moves. In economics, the Stackelberg game refers to a scenario in which the leader firm takes the initial action, followed by the follower firms. The game can be rooted in quantity competition, but in the modified version, price competition is also possible. Importantly, the leader must be aware that the follower observes its actions, while the follower cannot commit to deviating from being a follower in the future, and the leader must be aware of this [52].

Table 2. Model parameters, notation and assumptions

Indices	
k	returns handling strategy, $k \in \{M, R\}$
j	contract, $j \in \{RS, CRS\}$
$j-k$	contract scenario, $k \in \{M, R\}$, $j \in \{RS, CRS\}$
cen	centralized channel
dec	decentralized channel
Decision variables	
p	retail price per unit set by the retailer
z	inventory factor set by the retailer
q	order quantity set by the retailer
w	manufacturer's wholesale price per unit set by the manufacturer
Optimal quantities	
p^k_{cen}	optimal retail price per unit under strategy k in the centralized channel
z^k_{cen}	optimal inventory factor under strategy k in the centralized channel
p^{j-k}_{dec}	optimal retail price per unit under strategy k, scenario j in the decentralized channel
z^{j-k}_{dec}	optimal inventory factor under strategy k, scenario j in the decentralized channel
w^{j-k}_{dec}	optimal manufacturer's wholesale price per unit under strategy k, contract j in the decentralized channel
Parameters and notation	
Π_{cen}	expected profit in the centralized channel
Π_m	manufacturer's expected profit in the decentralized channel
Π_r	retailer's expected profit in the decentralized channel
$r \in [0,1]$	sharing ratio
$c > 0$	unit production cost
v	unit salvage value $v < c$,
$\alpha \in [0,1]$	mismatch rate
$h_m \geq 0$	returns handling under strategy M
$h_r \geq 0$	returns handling under strategy R
$h^k_{\text{cr}} \geq 0$	consumer's average returns handling cost under strategy k
$a > 0, b > 0$	deterministic demand parameters
ε	random variable with $\mathrm{E}\varepsilon = 0$ (WLOG)
$F(x)$, $f(x)$	cumulative df and probability df of ε with support $[A, B]$, $A < 0, B > 0$
$g(x) = \dfrac{f(x)}{\bar{F}(x)}$	failure rate with $\bar{F}(x) = 1 - F(x)$
IFR	increasing failure rate
$\mu(z) = \mathrm{E}\min\{\varepsilon, z\} = \int_z^B (z-\varepsilon) f(\varepsilon) d\varepsilon$	$\dfrac{\mathrm{d}\mu(z)}{\mathrm{d}z} = \bar{F}(z)$
Assumptions	
1a. $2A + a - bc - b\alpha(h_m + h^{\text{M}}_{\text{cr}}) > 0$	assures demand non-negativity under strategy M
1b. $2A + a - bc - b\alpha(h_r + h^R_{\text{cr}}) > 0$	assures demand non-negativity under strategy R
2a. $c + \alpha h_m < w < p$	constraint on prices under M
2b. $c + \alpha h_r < w < p$	constraint on prices under R
3. $w \leq r\dfrac{A + a - b\alpha h^k_{\text{cr}}}{b}$	assures the existence of an optimal solution under strategy k, $k \in \{M, R\}$, in the decetralized channel
4. F is IFR	assures the unimodality of the profit functions

In this study, we adopt the manufacturer-dominated Stackelberg model, in which both the manufacturer and the retailer determine prices and order quantity to maximize their respective profits (or utility functions). This game structure enables the Stackelberg leader, the dominant manufacturer, to enjoy high profits stemming from their first-mover advantage over the follower. It is important to note that funds are not exchanged unless an item is sold. Initially, the manufacturer selects the wholesale price w, and subsequently, the retailer, acting as a follower, determines the order quantity q and the retail price p for the product.

The customer's demand for the product is assumed to depend on the price p, and it is additive denoted by $D(p, \varepsilon) = d(p) + \varepsilon$. Here, $d(p) = a - b(p + \alpha h^k_{\text{cr}})$ is a deterministic demand and $\varepsilon \in [A, B]$ is a random variable with cdf F independent of p and continuously differentiable pdf f. Thus, the demand is expressed by

$$D(p, \varepsilon) = a - b(p + \alpha h^k_{\text{cr}}) + \varepsilon \tag{1}$$

where a is the market size and b is the consumer's sensitivity to retail price p and $k \in \{M, R\}$ [32, 39, 60].

The consumer's preferences are captured by the mismatch rate α. We assume that the returns are sent back either to the manufacturer or the retailer. Subsequently, the returned product undergoes inspection or repackaging before it can be resold as a new item. The consumer's returns handling cost includes expenses such as reverse shipping fees, travel costs, and time costs, and is influenced by factors such as responsiveness and convenience in the returns process.

Defining an inventory factor by $z = q - a + b(p + \alpha h^k_{\text{cr}})$ [39] we get $\mathrm{E}\min\{q, D(p, \varepsilon)\} = \mu(z) + a - b(p + \alpha h^k_{\text{cr}})$, $k \in \{M, R\}$, which lets us provide basic formulations of the expected profit functions of parties using the news-vendor frameworks. To begin, we will outline the profit equations for the centralized channel when employing strategies M and R, both of which involve decision-making by a central authority. Subsequently, we will present the expression for the expected profit in the centralized channel under strategy M:

$$\begin{aligned}\Pi^{\text{M}}_{\text{cen}}(p, q) &= (p - v - \alpha h_m)\mathrm{E}\min\{q, D(p, \varepsilon)\} - (c - v)q \\ &= (p - v - \alpha h_m)(\mu(z) + a - b(p + \alpha h^M_{\text{cr}})) - (c - v)(z + a - b(p + \alpha h^{\text{M}}_{\text{cr}}))\end{aligned} \tag{2}$$

and under strategy R:

$$\begin{aligned}\Pi^{\text{R}}_{\text{cen}}(p, q) &= (p - v - \alpha h_r)\mathrm{E}\min\{q, D(p, \varepsilon)\} - (c - v)q \\ &= (p - v - \alpha h_r)(\mu(z) + a - b(p + \alpha h^M_{\text{cr}})) - (c - v)(z + a - b(p + \alpha h^R_{\text{cr}}))\end{aligned}$$

The anticipated profits of the decentralized channel under strategy M and R, coupled with RS or CRS contracts, can be represented by the following expressions. For brevity, we will focus on presenting the equations for the expected profits of the manufacturer. It is important to note that the expected profit of the retailer can be obtained by calculating the difference between the centralized expected profit and the manufacturer's expected profit. Then,

- Under the RS-M contract scenario the manufacturer's expected profit is given by

$$\begin{aligned}\Pi^{\text{RS-M}}_m(w) &= (w - c + (1 - r)v)q + ((1 - r)(p - v) - \alpha h_m)\mathrm{E}\min\{q, D(p, \varepsilon)\} \\ &= (w - c + (1 - r)v)(z + a - b(p + \alpha h^{\text{M}}_{\text{cr}})) \\ &\quad + ((1 - r)(p - v) - \alpha h_m)(\mu(z) + a - b(p + \alpha h^{\text{M}}_{\text{cr}}))\end{aligned} \tag{3}$$

- In the CRS-M contract scenario, the expected profit of the manufacturer is given by

$$\begin{aligned}\Pi_m^{\text{CRS-M}}(w) &= (w-c+(1-r)v)q+(1-r)(p-v-\alpha h_m)\mathrm{E}\min\{q,D(p,\varepsilon)\}\\ &= (w-c+(1-r)v)(z+a-b(p+\alpha h_{\text{cr}}^{\text{M}}))\\ &\quad+(1-r)(p-v-\alpha h_m)(\mu(z)+a-b(p+\alpha h_{\text{cr}}^{\text{M}}))\end{aligned} \tag{4}$$

- In the RS-R contract scenario, the expected profit of the manufacturer is equal to

$$\begin{aligned}\Pi_m^{\text{RS-R}}(w) &= (w-c+(1-r)v)q+(1-r)(p-v)\mathrm{E}\min\{q,D(p,\varepsilon)\}\\ &= (w-c+(1-r)v)(z+a-b(p+\alpha h_{\text{cr}}^{R}))\\ &\quad+(1-r)(p-v)(\mu(z)+a-b(p+\alpha h_{\text{cr}}^{R}))\end{aligned} \tag{5}$$

- In the CRS-R contract scenario, the expected profit of the manufacturer is equal to

$$\begin{aligned}\Pi_m^{\text{CRS-R}}(w) &= (w-c+(1-r)v)q+(1-r)(p-v-\alpha h_r)\mathrm{E}\min\{q,D(p,\varepsilon)\}\\ &= (w-c+(1-r)v)(z+a-b(p+\alpha h_{\text{cr}}^{R}))\\ &\quad+(1-r)(p-v-\alpha h_r)(\mu(z)+a-b(p+\alpha h_{\text{cr}}^{R}))\end{aligned} \tag{6}$$

respectively. These results address **RQ1**.

In the forthcoming analysis, we provide closed-form expressions for optimal quantities and address the question of whether the specific SCs are coordinated or not, based on the following definition of SC coordination.

Definition 1. A contract coordinates the SC if the decentralized channel reaches the same pricing and ordering decision as the centralized system [10].

The objective of SC coordination is to enhance the overall performance of SC by aligning the plans and objectives of individual enterprises. Typically, it places an emphasis on optimizing inventory management and ordering decisions within distributed intercompany settings [43].

4. Optimal ordering and pricing decisions

Now, using the above expressions, we solve optimization problems in four contract scenarios.

4.1. Solutions under RS-M and CRS-M

As a benchmark, we provide the solution to the centralized channel where the central decision maker, either the manufacturer or the retailer, decides the order quantity and retail price, maximizing the total expected profit of the SC system. Generally, the centralized channel problem under strategy M is written as

$$\max_{p,\, z\in[A,B]} \Pi_{\text{cen}}^{\text{M}}(p,\, z).$$

We get the following theorem.

Theorem 1. If assumptions 1–4 hold, then for any $z \in [A, B]$ under strategy **M**, the central decision-maker's unique optimal price $p^{\mathrm{M}}_{\mathrm{cen}}$ is given by

$$p^{\mathrm{M}}_{\mathrm{cen}}(z) = \frac{\mu(z) + a + bc + b\alpha h_m - b\alpha h^{\mathrm{M}}_{\mathrm{cr}}}{2b} \tag{7}$$

and the unique optimal inventory factor $z = z^{\mathrm{M}}_{\mathrm{cen}}$ is determined by

$$(p^{\mathrm{M}}_{\mathrm{cen}}(z) - v - \alpha h_m)\bar{F}(z) = c - v.$$

All proofs are relegated to the Appendix.

Remark 1. Note that the demand $D(p, \varepsilon)$ defined by (1) is always non-negative if the optimal price $p^{\mathrm{M}}_{\mathrm{cen}}(z) \leq \dfrac{A + a}{b} - \alpha h^{\mathrm{M}}_{\mathrm{cr}}$ is $\varepsilon \geq A$. Therefore, using the inequality $\mu(z) \leq 0$, we have

$$p^{\mathrm{M}}_{\mathrm{cen}}(z) \leq \frac{a + bc + b\alpha h_m - b\alpha h^{\mathrm{M}}_{\mathrm{cr}}}{2b} \leq \frac{A + a}{b} - \alpha h^{\mathrm{M}}_{\mathrm{cr}}$$

which gives Assumption 1a.

Under the RS-M contract scenario using the backward induction method, first, the retailer's maximization problem is solved

$$\max_{p,\, z \in [A,B]} \Pi^{\text{RS-M}}_r(p,\, z) = -(w - rv)(z + a - b(p + \alpha h^{\mathrm{M}}_{\mathrm{cr}})) + r(p - v)(\mu(z) + a - b(p + \alpha h^{\mathrm{M}}_{\mathrm{cr}})), \tag{8}$$

and then the manufacturer's optimization problem

$$\max_{w} \Pi^{\text{RS-M}}_m(w)$$

is considered. We obtain a similar optimization under the CRS-M contract scenario. The solutions to the decentralized problems are as follows.

Theorem 2. If assumptions 1–4 hold, then for any $z \in [A, B]$

1. Under the RS-M contract scenario, the retailer's unique optimal price $p^{\text{RS-M}}_{\mathrm{dec}}$ is given by

$$p^{\text{RS-M}}_{\mathrm{dec}}(z) = \frac{r(\mu(z) + a - b\alpha h^{\mathrm{M}}_{\mathrm{cr}}) + bw}{2br}, \tag{9}$$

   and the retailer's optimal inventory factor $z = z^{\text{RS-M}}_{\mathrm{dec}}$ is uniquely determined by

$$(p^{\text{RS-M}}_{\mathrm{dec}}(z) - v)\bar{F}(z) = w - v. \tag{10}$$

2. In the CRS-M contract scenario, the retailer's unique optimal price $p^{\text{CRS-R}}_{\mathrm{dec}}$ is given by

$$p^{\text{CRS-M}}_{\mathrm{dec}}(z) = \frac{r(\mu(z) + a - b\alpha h^{\mathrm{M}}_{\mathrm{cr}}) + rb\alpha h_m + bw}{2br}, \tag{11}$$

   and the retailer's optimal inventory factor $z = z^{\text{CRS-M}}_{\mathrm{dec}}$ is uniquely determined by

$$r(p^{\text{CRS-M}}_{\mathrm{dec}}(z) - v - \alpha h_m)\bar{F}(z) = w - rv. \tag{12}$$

By anticipating the best response of the retailer described in Theorem 2, the manufacturer determines the wholesale price that maximizes its expected profit. The problem of optimizing the manufacturer's profit at wholesale price can be transformed into maximization of z after expressing the wholesale and retail prices in terms of z using (9) and (10) in the RS-M contract scenario and (11) and (12) in the CRS-M contract scenario, respectively. Therefore, finally, we solve optimization problems

1. Under the RS-M contract scenario
$$\max_{z\in[A,B]} \Pi_m^{\text{RS-M}}(z),$$
where $\Pi_m^{\text{RS-M}}$ is defined by (3) with
$$p = p_{\text{dec}}^{\text{RS-M}}(z) = \frac{\mu(z) + a - b\alpha h_{\text{cr}}^{\text{M}} + bvF(z)}{b(1 + F(z))},$$
and
$$w = w_{\text{dec}}^{\text{RS-M}}(z) = r\frac{(\mu(z) + a - b\alpha h_{\text{cr}}^{\text{M}})\bar{F}(z) + 2bvF(z)}{b(1 + F(z))},$$

2. Under the CRS-M contract scenario
$$\max_{z\in[A,B]} \Pi_m^{\text{CRS-M}}(z),$$
where $\Pi_m^{\text{CRS-M}}$ is defined by (4) with
$$p = p_{\text{dec}}^{\text{CRS-M}}(z) = \frac{\mu(z) + a - b\alpha h_{\text{cr}}^{\text{M}} + (b\alpha h_m + bv)F(z)}{b(1 + F(z))},$$
and
$$w = w_{\text{dec}}^{\text{CRS-M}}(z) = r\frac{(\mu(z) + a - b\alpha h_{\text{cr}}^{\text{M}})\bar{F}(z) - b\alpha h_m\bar{F}(z) + 2bvF(z)}{b(1 + F(z))}.$$

Due to the complexity of the above optimization problems, they can be solved numerically. The objective functions are continuous on the closed interval; therefore, they obtain their extreme values by the Weierstrass extreme value theorem [42]. However, these values do not have to be unique. Similar problems in the subsequent part of the article can be treated likewise. The above results address **RQ2**.

Now, we examine whether the decentralized channel makes the same pricing and ordering decision as the centralized system, namely, if the SC is coordinated.

Theorem 3. 1. Consider the set of CRS-M contracts with $w = rc$, $r \in (0, 1)$. Then, $\Pi_m^{\text{CRS-M}} = (1 - r)\Pi_{\text{cen}}^{\text{M}}$ and $\Pi_r^{\text{CRS-M}} = r\Pi_{\text{cen}}^{\text{M}}$. Furthermore, the pair $(p_{\text{cen}}^{\text{M}}, z_{\text{cen}}^{\text{M}})$ is the retailer's optimal price and service level, i.e., those contracts coordinate the SC.

2. RS-M contracts cannot coordinate the SC.

The result of the first statement of the above theorem is similar to those given in [10] but in the classical theorem, the returns were not considered. The retailer's share ratio r is shown to be the same as the share of the SC profit and the share of revenue and costs. Moreover, coordination requires a wholesale price below the manufacturer's cost c who profits by participating in the retailer's revenue and spending less by sharing the manufacturer's returns handling costs with the retailer. Selling below cost is necessary because revenue sharing systematically drops the retailer's marginal revenue curve below the centralized supply chain's. CRS contract can coordinate the supply chain; therefore, it encourages the retailer to order more, i.e., to behave like in the centralized arrangement. Then the retailer works not only for his benefit but also for the benefit of the entire supply chain. Therefore, the CRS contract can be more efficient for the overall supply chain than the RS contract. Theorem 3 addresses **RQ3**.

4.2. Solutions under RS-R and CRS-R

Initially, we present the solution for the centralized channel where the optimal order quantity and retail price are determined by a central decision maker. The findings are summarized in the following theorem.

Theorem 4. If assumptions 1–4 hold, then for any $z \in [A, B]$ under strategy R, the central decision maker's unique optimal price $p^{\mathrm{R}}_{\mathrm{cen}}$ is given by

$$p^{\mathrm{R}}_{\mathrm{cen}}(z) = \frac{\mu(z) + a + bc + b\alpha h_r - b\alpha h^R_{\mathrm{cr}}}{2b}$$

and the unique optimal inventory factor $z = z^{\mathrm{R}}_{\mathrm{cen}}$ is determined by

$$(p^{\mathrm{R}}_{\mathrm{cen}}(z) - v - \alpha h_r)\bar{F}(z) = c - v$$

Remark 2. Assumption 1b can be obtained in a similar way as assumption 1a presented in Remark 1.

The solutions to the decentralized problems are as follows.

Theorem 5. If assumptions 1–4 hold, then for any $z \in [A, B]$

1. Under the RS-R contract scenario, the retailer's unique optimal price $p^{\text{RS-R}}_{\mathrm{dec}}$ is given by

$$p^{\text{RS-R}}_{\mathrm{dec}}(z) = \frac{r(\mu(z) + a - b\alpha h^R_{\mathrm{cr}}) + b\alpha h_r + bw}{2br}$$

and the retailer's optimal inventory factor $z = z^{\text{RS-R}}_{\mathrm{dec}}$ is uniquely determined by

$$(p^{\text{RS-R}}_{\mathrm{dec}}(z) - v - \alpha h_r)\bar{F}(z) = w - v.$$

2. Under the CRS-R contract scenario, for any $z \in [A, B]$, the retailer's unique optimal price $p^{\text{CRS-R}}_{\mathrm{dec}}$ is given by

$$p^{\text{CRS-R}}_{\mathrm{dec}}(z) = \frac{r(\mu(z) + a - b\alpha h^R_{\mathrm{cr}}) + rb\alpha h_r + bw}{2br}$$

and the retailer's optimal inventory factor $z = z^{\text{CRS-R}}_{\mathrm{dec}}$ is uniquely determined by

$$r(p^{\text{CRS-R}}_{\mathrm{dec}}(z) - v - \alpha h_r)\bar{F}(z) = w - rv$$

Similarly, as in strategy M, the problem of optimizing the manufacturer's profit over the wholesale price can be transformed into maximization over the inventory factor. Finally, we solve the following optimization problems.

1. In the RS-R contract scenario

$$\max_{z\in[A,B]} \Pi_m^{\text{RS-R}}(z)$$

where $\Pi_m^{\text{RS-R}}$ is given by (5) with

$$p = p_{\text{dec}}^{\text{RS-R}}(z) = \frac{r(\mu(z) + a - b\alpha h_{\text{cr}}^R + bvF(z)) + b\alpha h_r F(z)}{br(1 + F(z))}$$

and

$$w = w_{\text{dec}}^{\text{RS-R}}(z) = \frac{r(\mu(z) + a - b\alpha h_{\text{cr}}^R)\bar{F}(z) - b\alpha h_r \bar{F}(z) + 2brvF(z)}{b(1 + F(z))}$$

2. Under the CRS-R contract scenario

$$\max_{z\in[A,B]} \Pi_m^{\text{CRS-R}}(z)$$

where $\Pi_m^{\text{CRS-R}}$ is given by (6) with

$$p = p_{\text{dec}}^{\text{CRS-R}}(z) = \frac{\mu(z) + a - b\alpha h_{\text{cr}}^R + b\alpha h_r F(z) + bvF(z)}{b(1 + F(z))}$$

and

$$w = w_{\text{dec}}^{\text{CRS-R}}(z) = r\frac{(\mu(z) + a - b\alpha h_{\text{cr}}^R)\bar{F}(z) - b\alpha h_r \bar{F}(z) + 2bvF(z)}{b(1 + F(z))}$$

The above results refer to **RQ2**.

Now, we consider whether RS-R and CRS-R contracts coordinate the SC.

Theorem 6. 1. Consider the set of CRS-R contracts with $w = rc$, $r \in (0, 1)$. Then, $\Pi_m^{\text{CRS-R}} = (1 - r)\Pi_{\text{cen}}^{\text{R}}$ and $\Pi_r^{\text{RS-R}} = r\Pi_{\text{cen}}^{\text{R}}$. Furthermore, the pair $(p_{\text{cen}}^{\text{R}}, z_{\text{cen}}^{\text{R}})$ is the retailer's optimal price and service level, i.e., those contracts coordinate the SC.

2. RS-R contracts cannot coordinate the SC.

The interpretation of the above theorem under strategy R is in line with those under strategy M. Theorem 6 addresses **RQ3**.

5. Numerical example

As the primary purpose of this numerical example is to demonstrate the results of the proposed Stackelberg game and its solution algorithm, meaningful game parameters are reasonably set according to the constraints of the models. That is, we examine whether the derived equations work well. In part, the parameters are drawn from [37]. We present numerical results assuming $a = 10$, $b = 0.6$, $c = 1$, $\alpha = 0.1$, $r = 0.4$, and, moreover, ε follows a uniform distribution on $[-1, 1]$. We are interested in the effect of return handling costs on the model solutions.

Based on the numerical example, it can be observed that in centralized cases the optimal profits tend to decrease as the returns handling costs h_m or h_r increase. The decline in centralized channel profits can be attributed to the decrease in inventory factor and the increase in price (Table 3).

Table 3. Sensitivity analysis with respect to $h_{m,r}$ with $h_{\text{cr}}^{M,R} = 2$ and $v \in \{0.2, 0.3\}$: centralized channel

$h_{m.r}$	$z_{\text{cen}}^{M,R}$	$p_{\text{cen}}^{M,R}$	$\Pi_{\text{cen}}^{M,R}$	v
1.0	0.8112	8.7759	34,6956	0.2
1.5	0.8107	8.8009	34.4657	
2.0	0.8101	8.8258	34.2366	
2.5	0.8095	8.8508	34.0082	
3.0	0.8090	8.8757	33.7805	
3.5	0.8080	8.9007	33.5536	
4.0	0.8078	8.9256	33.3275	
4.5	0.8072	8.9506	33.1021	
5.0	0.8067	8.9755	32.8775	
1.0	0.8329	8.7775	34.7786	0.3
1.5	0.8324	8.8025	34.5487	
2.0	0.8319	8.8274	34.3195	
2.5	0.8314	8.8524	34.0910	
3.0	0.8309	8.8774	33.8633	
3.5	0.8304	8.9023	33.6364	
4.0	0.8298	8.9273	33.4102	
4.5	0.8293	8.9523	33.1848	
5.0	0.8288	8.9772	32.9601	

Considering the RS-M and CRS-M contract scenarios, the optimal profits exhibit a similar monotonic behavior concerning the returns handling cost as observed in the centralized case. Consider the monotonicity of optimal profits for various values of the handling costs of consumer returns and different salvage values, namely $h_{\text{cr}} \in \{3.0, 3.5, 4.0\}$ and $v \in \{0.2, 0.3\}$ with fixed other parameters. In both scenarios, increasing h_m results in a decrease in optimal profits for any of combinations of h_{cr} and v (Tables 4 and 5).

Table 4. Sensitivity analysis with respect to h_m with $h_{\text{cr}}^{\text{M}} \in \{3.0, 3.5, 4.0\}$, $v \in \{0.2, 0.3\}$ under RS-M

h_m	z_{dec}^{RS}	p_{dec}^{RS}	w_{dec}^{RS}	Π_m^{RS}	Π_r^{RS}	Π_{dec}^{RS}	h_{cr}^{M}	v
1.0	0.1292	10.331	1.8445	24.1349	7.3233	31.4581	3.0	0.2
1.5	0.1242	10.3448	1.8571	23.9636	7.2759	31.2395		
2.0	0.1192	10.3588	1.8697	23.7928	7.2298	31.0225		
1.0	0.1291	10.2991	1.8390	23.9733	7.2751	31.2484	3.5	
1.5	0.1241	10.313	1.8516	23.8025	7.2279	31.0305		
2.0	0.1191	10.3269	1.8642	23.6323	7.1819	30.8142		
1.0	0.1291	10.2673	1.8336	23.8123	7.2271	31.0394	4.0	
1.5	0.1241	10.2811	1.8461	23.6420	7.1804	30.8225		
2.0	0.1190	10.295	1.8587	23.4723	7.1348	30.6072		
1.0	0.1419	10.3320	1.8417	24.1868	7.3446	31.5315	3	0.3
1.5	0.1369	10.3459	1.8542	24.0153	7.2973	31.3125		
2.0	0.1318	10.3597	1.8667	23.8442	7.2511	31.0953		
1.0	0.1419	10.3002	1.8362	24.0252	7.2965	31.3217	3.5	
1.5	0.1369	10.314	1.8487	23.8542	7.2493	31.1035		
2.0	0.1318	10.3279	1.8613	23.6837	7.2032	30.8869		
1.0	0.1419	10.2683	1.8307	23.8642	7.2485	31.1126	4	
1.5	0.1368	10.2822	1.8432	23.6937	7.2018	30.8955		
2.0	0.1318	10.2960	1.8558	23.5238	7.1561	30.6798		

Table 5. Sensitivity analysis with respect to h_m with $h_{\text{cr}}^{\text{M}} \in \{3.0, 3.5, 4.0\}$, $v \in \{0.2, 0.3\}$ under CRS-M

h_m	$z_{\text{dec}}^{CRS\text{-}M}$	$p_{\text{dec}}^{CRS\text{-}M}$	$w_{\text{dec}}^{CRS\text{-}M}$	$\Pi_m^{CRS\text{-}M}$	$\Pi_r^{CRS\text{-}M}$	$\Pi_{\text{dec}}^{CRS\text{-}M}$	h_{cr}^{M}	v
1.0	0.1392	10.34	1.8085	24.1543	7.2869	31.4412	3	0.2
1.5	0.13913	10.3578	1.8031	23.9927	7.2389	31.2316		
2.0	0.13912	10.376	1.798	23.8317	7.1910	31.0227		
1.0	0.13913	10.3078	1.8031	23.9927	7.2389	31.2316	3.5	
1.5	0.13912	10.326	1.7976	23.8317	7.191	31.0227		
2.0	0.13911	10.3441	1.7922	23.6712	7.1432	30.8144		
1.0	0.13912	10.27596	1.7976	23.8317	7.19098	31.0227	4	
1.5	0.13911	10.2941	1.79217	23.67115	7.14325	30.8144		
2.0	0.1391	10.3123	1.7867	23.5112	7.0957	30.6068		
1.0	0.15209	10.3408	1.8058	24,2067	7,3104	31,5172	3.0	0.3
1.5	0.1521	10,359	1,8003	24,0451	7,2624	31,3075		
2.0	0,1522	10,3771	1,7949	23,884	7,2145	31,0985		
1.0	0,1521	10,309	1,8003	24,0451	7,2624	31,3075	3.5	
1.5	0,15216	10,3271	1,7949	23,8840	7,2145	31,0985		
2.0	0,15219	10,3453	1,7894	23,7235	7,1667	30,8902		
1.0	0,15216	10,2771	1,7949	23,8841	7,2145	31,0985	4.0	
1.5	0,1522	10,2953	1,7894	23,7235	7,1667	30,8902		
2.0	0,1522	10,3135	1,784	23,5635	7,1191	30,6826		

Under the RS-R and CRS-R contract scenarios, both the manufacturer and retailer's profits are influenced by h_r. Notably, increasing returns handling costs negatively impact the retailer's and manufacturer's channel profit under both the RS-R and CRS-R with $h_{\text{cr}} \in \{0.5, 1.0, 1.5\}$ and $v \in \{0.2, 0.3\}$ (Tables 6 and 7).

Table 6. Sensitivity analysis with respect to h_r with $h_{\text{cr}}^{R} \in \{0.5, 1.0, 1.5\}$, $v \in \{0.2, 0.3\}$ under RS-R

h_r	$z_{\text{dec}}^{\text{RS-R}}$	$p_{\text{dec}}^{\text{RS-R}}$	$w_{\text{dec}}^{\text{RS-R}}$	$\Pi_m^{\text{RS-R}}$	$\Pi_r^{\text{RS-R}}$	$\Pi_{\text{dec}}^{\text{RS-R}}$	h_{cr}^{R}	v
3.0	0.1854	10.6130	1.6543	24.4022	7.319	31.7212	0.5	0.2
3.5	0.1934	10.6387	1.6227	24.2538	7.2671	31.5209		
4.0	0.2016	10.6646	1.5913	24.1059	7.215	31,3209		
3.0	0.1855	10.5813	1.649	24.2399	7.2708	31.5107	1.0	
3.5	0.1936	10.6070	1.6174	24.0921	7.219	31.3111		
4.0	0.2018	10.6330	1.5859	23.9447	7.167	31.1117		
3.0	0.1856	10.5496	1.6436	24.0782	7.2227	31.3009	1.5	
3.5	0.1937	10.5754	1.6120	23.931	7.1710	31.102		
4.0	0.2019	10.6014	1.5806	23.7840	7.1192	30.9032		
3.0	0.1992	10.6147	1.651	24.4566	7.3423	31.7988	0.5	0.3
3.5	0.2074	10.6405	1.6204	24.3085	7.2902	31.5988		
4.0	0.2158	10.6665	1.5891	24.1609	7.2380	31.3989		
3.0	0.1993	10.583	1.6466	24.2943	7.2940	31.5883	1.0	
3.5	0.2076	10.6089	1.61506	24.1468	7.2421	31.3889		
4.0	0.2160	10.6349	1.5837	23.9997	7.1900	31.18968		
3.0	0.1995	10.5513	1.6412	24.1326	7.2459	31.37848	1.5	
3.5	0.2078	10.5772	1.6097	23.9856	7.1941	31.1797		
4.0	0.2162	10.6033	1.57836	23.8390	7.14215	30.9812		

Based on these observations, it can be inferred that the choice of the contract scenario and the specific collection of model parameters affect the optimal solutions to the decentralized problems heavily, rendering strategies M and R incomparable in general. Finally, let us note that in the decentralized channel

Table 7. Sensitivity analysis with respect to h_r with $h_{\text{cr}}^R \in \{0.5, 1.0, 1.5\}$, $v \in \{0.2, 0.3\}$ under CRS-R

h_r	$z_{\text{dec}}^{\text{CRS-R}}$	$p_{\text{dec}}^{\text{CRS-R}}$	$w_{\text{dec}}^{\text{CRS-R}}$	$\Pi_m^{\text{CRS-R}}$	$\Pi_r^{\text{CRS-R}}$	$\Pi_{\text{dec}}^{\text{CRS-R}}$	h_{cr}^R	v
3.0	0.1392	10.5714	1.81399	24.3165	7.3351	31.6516	0.5	0.2
3.5	0.13914	10.5896	1.8085	24.1543	7.2869	31.4412		
4.0	0.13913	10.6078	1.8031	23.9927	7.2389	31.2316		
3.0	0.13914	10.5396	1.8085	24.1543	7.2869	31.4412	1.0	
3.5	0.13913	10.5578	1.8031	23.9927	7.2389	31.2316		
4.0	0.13912	10.576	1.79762	23.8317	7.191	31.0227		
3.0	0.13913	10.5078	1.8031	23.9927	7.2389	31.2316	1.5	
3.5	0.1391	10.526	1.7976	23.8317	7.19098	31.0227		
4.0	0.1391	10.5441	1.7922	23.6712	7.1432	30.8144		
3.0	0.15206	10.5726	1.8112	24.3689	7.3587	31.7275	0.5	0.3
3.5	0.15209	10.5908	1.8058	24.2067	7.3104	31.5172		
4.0	0.1521	10.609	1.8003	24.0451	7.2624	31.3075		
3.0	0.1521	10.5408	1.8058	24.2067	7.3104	31.5172	1.0	
3.5	0.1521	10.559	1.8003	24.0451	7.2624	31.3075		
4.0	0.1522	10.5771	1.7949	23.8840	7.2145	31.0985		
3.0	0.15212	10.509	1.8003	24.0451	7.2624	31.3075	1.5	
3.5	0.15215	10.5271	1.79486	23.8840	7.2145	31.09849		
4.0	0.15218	10.5453	1.7894	23.7235	7.1667	30.8902		

the position of the manufacturer as the leader is beneficial in the examined Stackelberg game, and, that is why higher manufacturer's profit in relation to the retailer's profit is achieved which can be seen in the studied experiments.

6. Discussion

The theoretical findings of this research offer insights into the issue of consumer returns within the SC. We proposed, explored, and optimized a few types of SC contracts, resulting in the development of the sets of optimal solutions. Our study is closely related to the research conducted by Liu et al. [36] and Bieniek [5]. In these works, the authors examine wholesale price and buyback contracts with a reverse channel structure. In contrast, our study considers two types of contracts, namely RS and CRS, in conjunction with two returns handling strategies, R and M, leading to the creation of four distinct contract scenarios. For each contract scenario, we determine an optimal order quantity, optimal wholesale price, and optimal retail price. Notably, our model differs from that of Liu et al. [36] in that we also identify optimal retail prices which are endogenous in our study and strictly determined by the type of contract scenario. This difference arises also because our model accounts for the price sensitivity of demand, a characteristic disregarded in [36], but similar to [5]. However, in the latter work, only the manufacturer's handling strategy is examined, whereas in this study, the retailer's handling strategy is also considered. Meanwhile, Liu et al. [36] demonstrate that a buyback contract coordinates the SC. In our article, the SC is coordinated only by the CRS contract. In contrast, we prove that the RS contract with returns cannot coordinate the SC.

Although our model reflects four different channel arrangements, our results do not definitively indicate which contract is superior or should be used by managers. Specifically, our model does not determine whether the manufacturer or retailer should handle returns to maximize SC profitability. Every case of

SC setting for a specific set of model parameters should be considered individually. In [36], the conditions that point to the more beneficial strategy satisfy complicated conditions involving the exogenous price and a set of specific model parameters. Let us add that in our case the price cannot determine the constraint on which more profitable strategy is chosen because it is a decision variable. In both this work and [36] the numerical example is given to illustrate the theoretical results. Most of the recent RS articles published in 2001-2020, which is approx. 81% out of 150, illustrate their theoretical models using numerical examples. Only six works apply their model to real-life cases [3]. The statistics show that there are a very limited number of articles that rely on tangible data from case studies and laboratory analysis, which suggests that these methodologies need more attention. Therefore, one can look for actual data that can be implemented in our model. The inability to mathematically prove the sensitivity analysis of the models on changes in parameters is the limitation of our study.

In this paper, we consider the uncapacitated inventory model. Capacity and inventory management are fundamental topics of operations management as they concern the planning and control of the SC or processing side of matching supply and demand. It should be noted that only a few operations research papers mention both inventory and capacity. The reason is that studying capacity and inventory in an integrated way leads to complex problems very quickly [15, 44]. From an operations management point of view, there are some primary theoretical studies on the capacity subject. Cachon and Lariviere [9] investigate an asymmetric information setting in which a manufacturer can share its forecast demand information with a supplier who decides how much capacity to install. Later, in [51] both a manufacturer and a supplier invested in capacity. Adida and Perakis [1] study the role of capacity in the efficiency of a two-tier supply chain with two suppliers who are leaders in the first tier and a retailer who is a follower in the second tier. A model with differentiated substitutable products is considered in which the suppliers are symmetric and differ only by their production capacity. They characterize optimal prices and production amounts if suppliers compete, cooperate, or the two levels are centrally coordinated. Recently, [48] argued that for new high-tech final products, the demand is highly uncertain and the supplier's capacity for the critical component may be restricted. In their model, the supplier decides on the production of its final products and sets the wholesale price of the component and quantity allocated to its downstream manufacturer. Summarizing, by limiting the capacity, we face many additional problems: if we limit the supplier's production or retailer's storing capacity, if there is asymmetric information of the supplier's production capacity capable of satisfying the demand for the supplier's and manufacturer's products, if endogenously consider the supplier's capacity when the supplier has private capacity information, and so on. Therefore, the implementation of capacity constraints in our model is complex but can be the subject of a future study.

7. Concluding remarks

Product returns entail significant costs that affect both manufacturers and retailers. The management of product returns can either enhance or diminish a company's profitability. Our study specifically focuses on false failure returns (FFRs) and explores their handling by the manufacturer or retailer under revenue sharing (RS) or cost-revenue sharing (CRS) contracts. We investigate the reverse channel structure across various SC contract scenarios. Our research addresses an existing gap in the literature by introducing novel revenue-sharing contract models inspired by the work of Liu et al. [36], and systematically formulating, optimizing, and rigorously examining these pioneering contract structures.

In this study, the selection of a contract scenario is determined by the leader of the Stackelberg game, which is the manufacturer, and is contingent upon who handles the returns, stochastic demand, and the contract type. As a result, we established four unique contract scenarios and expressed them in mathematical terms (addressing **RQ1**). We determined optimal quantities, including the wholesale price, the retail price, and the order size, based on the specific scenario (addressing **RQ2**). Our findings revealed that SC is coordinated when both the revenue and return handling costs are shared between the parties, indicating that the CRS contract coordinates SC, while the RS contract does not (addressing **RQ3**). The numerical analysis demonstrates that the choice of the scenario and the handling costs of the returns significantly affect the optimal solutions to optimization problems. These findings contribute to the existing literature by providing strategies for returns management within RS and CRS contracts, as well as identifying the coordination conditions for these contract types.

The implications of our research offer valuable suggestions for managers grappling with the challenges posed by the returns of new non-defective products, often bought through the online channel, i.e., shoes, apparel, accessories, electronics, food and beverages, books, toys, and baby products [4]. They guide how to select the appropriate reverse channel strategy. However, it is important to note that the analysis presented in the article does not determine which contract scenario is universally the best for all channel members, except for straightforward cases where all returns handling costs are lower in one model compared to the other, while other model parameters remain constant. Therefore, identifying the constraints under which a particular strategy should be adopted, in terms of maximizing profitability for the manufacturer, retailer, or entire SC, represents a direction for future research.

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A. Appendix. Proofs of theorems

Proof of Theorem 1. Using (2) we have

$$\frac{\mathrm{d}\Pi^{\mathrm{M}}_{\mathrm{cen}}(p,\, z)}{\mathrm{d}p} = \mu(z) + a + bc + b\alpha h_m - b\alpha h^{\mathrm{M}}_{\mathrm{cr}} - 2bp$$

which by the first order condition $\frac{\mathrm{d}\Pi^{\mathrm{M}}_{\mathrm{cen}}(p,\, z)}{\mathrm{d}p} = 0$ gives tequation (7) for optimal price $p^{\mathrm{M}}_{\mathrm{cen}}(z)$. The solution $p^{Mad}_{\mathrm{cen}}(z)$ is unique since the gradient of the linear function of p is negative which implies that it is decreasing. Therefore, $\frac{\mathrm{d}\Pi^{\mathrm{M}}_{\mathrm{cen}}(p,\, z)}{\mathrm{d}p} > 0$ for $p < p^{\mathrm{M}}_{\mathrm{cen}}(z)$ and $\frac{\mathrm{d}\Pi^{\mathrm{M}}_{\mathrm{cen}}(p,\, z)}{\mathrm{d}p} < 0$ for $p > p^{\mathrm{M}}_{\mathrm{cen}}(z)$. Let us note that $\frac{\mathrm{d}\Pi^{\mathrm{M}}_{\mathrm{cen}}(p,\, z)}{\mathrm{d}z} = \frac{\delta\Pi^{\mathrm{M}}_{\mathrm{cen}}(p,\, z)}{\delta z} + \frac{\delta\Pi^{\mathrm{M}}_{\mathrm{cen}}(p,\, z)}{\delta p}\frac{\mathrm{d}p}{\mathrm{d}z}$. Next, substituting equation (7) into equation (2), we get $\Pi^{\mathrm{M}}_{\mathrm{cen}}(p^{\mathrm{M}}_{\mathrm{cen}}(z), z) = \Pi^{\mathrm{M}}_{\mathrm{cen}}(z)$, which is a continuous and smooth function. Then, the optimal solution $z^{\mathrm{M}}_{\mathrm{cen}}$ is determined by the first order condition $\frac{\mathrm{d}\Pi^{\mathrm{M}}_{\mathrm{cen}}(z)}{\mathrm{d}z} = (p^{\mathrm{M}}_{\mathrm{cen}}(z) - v - \alpha h_m)\bar{F}(z) - (c - v) = 0$. Furthermore, $\frac{\mathrm{d}\Pi^{\mathrm{M}}_{\mathrm{cen}}(z)}{\mathrm{d}z}\,|_{z=A} = p^{\mathrm{M}}_{\mathrm{cen}}(A) - c - \alpha h_m > 0$ by assumption 1a and $\frac{\mathrm{d}\Pi^{\mathrm{M}}_{\mathrm{cen}}(z)}{\mathrm{d}z}\,|_{z=B} = -(c-v) < 0$. Moreover,

$$\frac{\mathrm{d}^2\Pi^{\mathrm{M}}_{\mathrm{cen}}(z)}{\mathrm{d}z^2} = -(p^{\mathrm{M}}_{\mathrm{cen}}(z) - v - \alpha h_m)f(z) + \frac{\bar{F}^2(z)}{2b}$$

which gives $\frac{\mathrm{d}^2\Pi^{\mathrm{M}}_{\mathrm{cen}}(z)}{\mathrm{d}z^2} < 0$ if $h(z) > \frac{1}{2b(p^{\mathrm{M}}_{\mathrm{cen}}(A) - v - \alpha h_m)}$. This is true by IFR property for sufficiently large $z \in [A, B]$. Thus, $\Pi^{\mathrm{M}}_{\mathrm{cen}}(z)$ is increasing in A and unimodal. We achieve that $z^{\mathrm{M}}_{\mathrm{cen}}$ is a unique maximum. The proof is complete.

Proof of Theorem 2. Using (8), for a given $z \in [A, B]$ and w we have

$$\frac{\mathrm{d}\Pi^{\text{RS-M}}_{r}(p,\, z)}{\mathrm{d}p} = r(\mu(z) + a - b\alpha h^{\mathrm{M}}_{\mathrm{cr}}) + bw - 2brp$$

which by the first order condition $\frac{\mathrm{d}\Pi_r^{\text{RS-M}}(p,\, z)}{\mathrm{d}p} = 0$ gives the equation (9) for optimal price $p_{\text{dec}}^{\text{RS-M}}(z)$. The solution is unique since the gradient of a function is negative, and therefore, $\frac{\mathrm{d}\Pi_r^{\text{RS-M}}(p,\, z)}{\mathrm{d}p} > 0$ for $p < p_{\text{dec}}^{\text{RS-M}}(z)$, and $\frac{\mathrm{d}\Pi_r^{\text{RS-M}}(p,\, z)}{\mathrm{d}p} < 0$ for $p > p_{\text{dec}}^{\text{RS-M}}(z)$. Next, substituting (9) to equation (8) we get $\Pi_r^{\text{RS-M}}(p_{\text{dec}}^{\text{RS-M}}(z), z) = \Pi_r^{\text{RS-M}}(z)$, which is a continuous function of z with the first derivative equal to $\frac{d\Pi_r^{\text{RS-M}}(z)}{dz} = r(p_{\text{dec}}^{\text{RS-M}}(z) - v)\bar{F}(z) - (w - rv)$. The optimal $z_{\text{dec}}^{\text{RS-M}}(w)$ is given by the first order condition $\frac{d\Pi_r^{\text{RS-M}}(z)}{dz} = 0$. Furthermore, $\frac{\mathrm{d}\Pi_r^{\text{RS-M}}(z)}{\mathrm{d}z} |_{z=A} = rp_{\text{dec}}^{\text{RS-M}}(A) - w > 0$ by assumption 3 and $\frac{\mathrm{d}\Pi_r^{\text{RS-M}}(z)}{\mathrm{d}z} |_{z=B} = -(w - rv) < 0$. Moreover, $\frac{d^2\Pi_r^{\text{RS-M}}(z)}{dz^2} < 0$ if $h(z) > \frac{1}{2b(p_{\text{dec}}^{RS_M}(A) - v)}$ which is true for sufficiently large z by IFR property. This means that the expected profit function $\Pi_r^{\text{RS-M}}(z)$ is unimodal and $z_{\text{dec}}^{\text{RS-M}}(w) \in [A, B]$ is a unique maximum. The proof is complete.

Proof of Theorem 3. Using (2) and (4), we get

$$\Pi_r^{\text{CRS-R}}(p, q \mid w) = -(w - rv)(z + a - b(p + \alpha h_{\text{cr}}^{\text{M}})) + r(p - v - \alpha h_m)(\mu(z) + a - b(p + \alpha h_{\text{cr}}^{\text{M}}))$$

Solving the equation $(p_{\text{cen}}^{\text{M}}, z_{\text{cen}}^{\text{M}}) = (p_{\text{dec}}^{\text{CRS-R}}, z_{\text{dec}}^{\text{CRS-R}})$, we obtain that the decentralized CRS-M channel is coordinated only if $w = rc$. The proof of (1) is complete.

Using the similar consideration, we obtain that the RS-M channel could be coordinated only if $w = r(c + \alpha h_m)$ and $p = c + \alpha h_m$ but then the expected profits in the central and RS-M scenario are negative. Therefore, the coordination is impossible which ends the proof of (2).

Proofs of Theorems 4, 5 and 6 are analogous to the proof of Theorems 1, 2 and 3, respectively.