Vol. 34, No. 2 (2024) | DOI: 10.37190/ord240204

OPEN ACCESS

Operations Research and Decisions

www.ord.pwr.edu.pl

OPERATIONS RESEARCH AND DECISIONS QUARTERLY

A new similarity measure for rankings obtained in MCDM problems using different normalization techniques

Dariusz Kacprzak[1]

[1] *Department of Mathematics, Faculty of Computer Science, Bialystok University of Technology, Bialystok, Poland*

[*] *Corresponding author, email address: d.kacprzak@pb.edu.pl*

Abstract

The paper presents an analysis of the impact of normalization techniques on the ranking of alternatives obtained using the combined compromise solution (CoCoSo) method. Similarity measures known from the literature and a new measure called the TOPSIS similarity measure (TOPSIS-SM) are used to assess the resulting rankings. This new measure is based on the TOPSIS algorithm, where the arithmetic mean of the considered rankings is taken as the ideal solution. In contrast, the anti-ideal solution is divided into a minimum and a maximum solution, which exhibit maximum separation from the ideal solution. The results obtained by this new method are different from those obtained using other similarity measures known from the literature.

Keywords: *CoCoSo method, normalization techniques, similarity measures, a new TOPSIS similarity measure*

1. Introduction

The widespread use of multiple-criteria decision-making (MCDM) methods to solve everyday problems has resulted in their rapid development in recent decades. The MCDM methods are used in many areas, such as supply chain management [14], logistics [33], engineering [21], solar energy [8], flow control in a manufacturing system [19], gerontechnology selection [9], and many others. For more applications of MCDM methods see, e.g., [1, 2, 18].

One common application of MCDM methods is in the selection of the best alternative (decision) from a finite set under consideration. At the outset, the alternatives considered are evaluated against a finite set of criteria that often conflict with each other. Since criteria usually come from different scales or are expressed in different units, an initial and key element of MCDM methods is the normalization of ratings of alternatives against criteria. The normalization technique used in the MCDM method significantly impacts the final ranking of alternatives and the choice of the best one, as Chatterjee and Chakraborty

Received 27 April 2023, accepted 4 April 2024, published online 8 July 2024
ISSN 2391-6060 (Online)/

The costs of publishing this issue have been co-financed by the program *Development of Academic Journals* of the Polish Ministry of Education and Science under agreement No. RCN/SP/0241/2021/1

have written [6]: *while the normalization process scales the criteria values to be approximately of the same magnitude, different normalization techniques may yield different solutions and, therefore, may cause deviation from the originally recommended solutions*.

In the literature, we can find several normalization techniques. MCDM methods have dedicated normalization techniques, e.g., in the combined compromise solution (CoCoSo) method [28] we have the zero unitarization method, in the analytical hierarchy process (AHP) method [20] we have the sum method, in the simple additive weighting (SAW) method [7] we have the maximum method, while in the technique for order of preference by similarity to the ideal solution (TOPSIS) method [10] we have the vector method (these normalization techniques are often used in best-case identification analyses and are presented in Section 2.2). In this situation, the question is: which normalization technique is the most appropriate for the problem under consideration and the chosen MCDM method?

In published studies, we can mostly find analyses investigating the impact of the normalization techniques on the ranking of alternatives using classic and very popular MCDM methods, such as AHP [22, 24], SAW [5, 25] or TOPSIS [3–5, 23]. The authors use various similarity measures to select the most appropriate normalization technique, including Pearson's correlation coefficient (*PCC*), Spearman's rank correlation coefficient (*SRCC*), rank similarity index (*RSI*) or rank consistency index (*RCI*) (these measures are described in detail in Section 2.3). According to Chakraborty and Yeh [4], these measures *indicate how well a particular normalization procedure produces rankings similar to other procedures*. Vafaei et al. [22] examining the effect of the normalization technique on the ranking of alternatives in the AHP method, conclude that the most appropriate technique is the maximum method while in the next study [24], they indicate the zero unitarization method. For the SAW method, Chakraborty and Yeh [5] state that the maximum method is the most appropriate normalization technique, while Vafaei et al. [25] point to the zero unitarization method. In the TOPSIS method, Chakraborty and Yeh [4] and Celen [3] and Vafaei et al. [23] show that the vector method is the most appropriate normalization technique, while in the next study, Chakraborty and Yeh [5] demonstrate that the maximum method is the most suitable.

As the above studies have shown, various similarity measures can lead to different results of the normalization techniques in a given MCDM problem. Taking this into account, this paper proposes a new similarity measure for the rankings obtained, which can also be successfully used to select the best MCDM method, for the MCDM problem under consideration. This new similarity measure, TOPSIS-SM, is based on the algorithm of the TOPSIS method. It assumes that the most appropriate normalization technique is the one that gives the ranking closest to the mean ranking determined as the arithmetic mean of the rankings obtained for the normalization techniques considered. This means that the positive ideal solution is the average of the rankings analyzed. This is based on the following observations [29]:

- the mean value is treated as the most compromised among all values (the average method is often used in group decision-making, where the average is the final group decision),
- the mean value is the middle of the set of values and the most distant from the extreme values,
- the mean value is used in practice, e.g., in sports such as snowboard slope-style or half-pipe, where the final rating of a contender is the average of the ratings of a group of referees.

Ranking of normalization methods based on similarity to the mean ranking is more natural and intuitive than methods based on *PCC* or *SRCC* and *RSI*. In the latter, the rankings are compared in pairs and then the results are averaged across the rankings. In such a situation the ranking most similar to all others is indicated. Moreover, a single lower value of the similarity measure of the two rankings results in lower values for their average rating and lower positions in the evaluation of normalization methods.

In TOPSIS-SM as reference points, there is the positive ideal ranking (PIR), which is the average of the analyzed rankings obtained using different normalization techniques. On the other hand, there is the negative ideal ranking (NIR), which is divided into two components: the left negative ranking (LNR), composed of minimum rankings, and the right negative ranking (RNR), composed of maximum rankings. The determined coefficients of relative closeness (*RCC*) to PIR allow us to rank the normalization techniques and indicate the most appropriate technique for the problem under consideration and the chosen MCDM method (this method is described in detail in Section 3).

In this paper, the CoCoSo method and five normalization techniques are used to demonstrate the performance of the proposed similarity measure. We have chosen CoCoSo because it is a relatively new method and to the author's best knowledge has not yet been studied in terms of the impact of the normalization techniques on the ranking of alternatives. On the other hand, the selected normalization techniques (i.e., the zero unitarization method, the maximum method, the sum method, the vector normalization technique and the logarithmic normalization technique) are commonly used in the MCDM method and frequently used in analyses [3, 4, 18, 22–25].

The proposed method is based on a numerical example from the paper by Yazdani et al. [28], which presents the CoCoSo method. This numerical example is also analyzed with the other similarity measures used in the analyses and mentioned earlier to compare the results obtained with the proposed measure.

The rest of the paper is organized as follows. Section 2 presents the CoCoSo method, selected normalization techniques, and selected similarity measures. Next, the new similarity measure, as well as a numerical example, are introduced. Section 5 compares the proposed method with other methods known from the literature. The last section contains conclusions.

2. Preliminaries

In this Section, some concepts and methods used in the paper are briefly presented such as the algorithm of the CoCoSo method, selected normalization techniques and a similarity measures.

2.1. The CoCoSo method

The CoCoSo method was proposed by Yazdani et al. [28] as a new method for solving MCDM problems. These problems are characterized by a set of alternatives $\{A_1, A_2, ..., A_m\}$ and a set of criteria $\{C_1, C_2, ..., C_n\}$. The set of criteria is additionally divided into benefit criteria denoted by B (the higher the value, the better) and cost criteria denoted by C (the lower the value, the better). Then an MCDM problem is presented as a matrix X

$$X = (x_{ij})_{m\times n} = \begin{array}{c c} & \begin{array}{cccc} C_1 & C_2 & \dots & C_n \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_m \end{array} & \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{pmatrix} \end{array}, \tag{1}$$

where x_{ij} denotes the evaluation of alternative i with regard to criterion j. In addition, the relevance of the criteria is described by a vector of criteria weights

$$w = (w_1, w_2, ..., w_n) \tag{2}$$

where $w_j \in \mathbb{R}^+$ and $\sum_{j=1}^{n} w_j = 1$. Then the algorithm of the CoCoSo method is as follows.

Step 1. The normalized decision matrix Y

$$Y = (y_{ij})_{m\times n} = \begin{array}{c c} & \begin{array}{cccc} C_1 & C_2 & \dots & C_n \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_m \end{array} & \begin{pmatrix} y_{11} & y_{12} & \dots & y_{1n} \\ y_{21} & y_{22} & \dots & y_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ y_{m1} & y_{m2} & \dots & y_{mn} \end{pmatrix} \end{array} \tag{3}$$

is determined using the zero unitarization method [15, 16] where

$$y_{ij} = \begin{cases} \dfrac{x_{ij} - \min_i x_{ij}}{\max_i x_{ij} - \min_i x_{ij}} & \text{if} \quad j \in B \\[2ex] \dfrac{\max_i x_{ij} - x_{ij}}{\max_i x_{ij} - \min_i x_{ij}} & \text{if} \quad j \in C \end{cases} \tag{4}$$

Step 2. Two characteristics are determined for each alternative: a weighted sum of normalized evaluation values is calculated, as in the SAW method

$$S_i = \sum_{j=1}^{n} y_{ij} w_j \tag{5}$$

and an exponentially weighted product, as in the weighted product model (WPM)

$$P_i = \sum_{j=1}^{n} (y_{ij})^{w_j} \tag{6}$$

Step 3. Based on the characteristics (5) and (6) determined in Step 2, three assessments are calculated

$$k_{ia} = \frac{S_i + P_i}{\sum_{i=1}^{m} (S_i + P_i)} \tag{7}$$

$$k_{ib} = \frac{S_i}{\min_i S_i} + \frac{P_i}{\min_i P_i} \tag{8}$$

$$k_{ic} = \frac{\lambda S_i + (1-\lambda) P_i}{\lambda \max_i S_i + (1-\lambda) \max_i P_i} \tag{9}$$

where $0 \leq \lambda \leq 1$ is the parameter determined by the decision maker (often selected as $\lambda = 0.5$).

Step 4. The ranking of the alternatives is determined based on the index

$$k_i = (k_{ia}k_{ib}k_{ic})^{\frac{1}{3}} + \frac{1}{3}(k_{ia} + k_{ib} + k_{ic}) \tag{10}$$

A higher k_i value indicates a higher position in the final ranking of alternative A_i.

2.2. Selected normalization techniques

To solve an MCDM problem, normalization of the decision matrix is needed. It results in data from different scales or with different units being transformed into dimensionless data allowing them to be compared and aggregated to create a ranking of alternatives. Many methods of normalization have been developed. It is difficult to assess which one is best for the problem under consideration or the MCDM method used. In this paper five commonly used normalization techniques N_i are analyzed:

- N_1. Linear scale transformation – the zero unitarization method [15, 16]

$$y_{ij} = \begin{cases} \dfrac{x_{ij} - \min_i x_{ij}}{\max_i x_{ij} - \min_i x_{ij}} & \text{if} \quad j \in B \\[2ex] \dfrac{\max_i x_{ij} - x_{ij}}{\max_i x_{ij} - \min_i x_{ij}} & \text{if} \quad j \in C \end{cases} \tag{11}$$

- N_2. Linear scale transformation – the maximum method [11]

$$y_{ij} = \begin{cases} \dfrac{x_{ij}}{\max_i x_{ij}} & \text{if} \quad j \in B \\[2ex] 1 - \dfrac{x_{ij}}{\max_i x_{ij}} & \text{if} \quad j \in C \end{cases} \tag{12}$$

- N_3. Linear scale transformation – the sum method [3, 30]

$$y_{ij} = \begin{cases} \dfrac{x_{ij}}{\sum\limits_{i=1}^{m} x_{ij}} & \text{if} \quad j \in B \\[4ex] \dfrac{x_{ij}^{-1}}{\sum\limits_{i=1}^{m} x_{ij}^{-1}} & \text{if} \quad j \in C \end{cases} \tag{13}$$

- N_4. Vector normalization [17]

$$y_{ij} = \begin{cases} \dfrac{x_{ij}}{\sqrt{\sum\limits_{i=1}^{m} x_{ij}^2}} & \text{if} \quad j \in B \\[4ex] 1 - \dfrac{x_{ij}}{\sqrt{\sum\limits_{i=1}^{m} x_{ij}^2}} & \text{if} \quad j \in C \end{cases} \tag{14}$$

- N_5. Logarithmic normalization [31]

$$y_{ij} = \begin{cases} \dfrac{\ln x_{ij}}{\ln \prod\limits_{i=1}^{m} x_{ij}} & \text{if } \ j \in B \\ \\ \dfrac{1}{m-1}\left(1 - \dfrac{\ln x_{ij}}{\ln \prod\limits_{i=1}^{m} x_{ij}}\right) & \text{if } \ j \in C \end{cases} \tag{15}$$

Other well-known normalization techniques include [12, 26, 27, 30, 32]:

- standardization

$$y_{ij} = \frac{x_{ij} - \bar{x}_j}{\sigma_j} \tag{16}$$

- unitization

$$y_{ij} = \frac{x_{ij} - \bar{x}_j}{r_j} \tag{17}$$

- quotient transformations

$$y_{ij} = \frac{x_{ij}}{\bar{x}_j} \tag{18}$$

$$y_{ij} = \frac{x_{ij}}{r_j} \tag{19}$$

$$y_{ij} = \frac{x_{ij}}{\sigma_j} \tag{20}$$

where $\bar{x}_j = (1/m)\sum\limits_{i=1}^{m} x_{ij}$ is mean for jth criterion, $\sigma_j = \sqrt{(1/m)\sum\limits_{i=1}^{m}(x_{ij} - \bar{x}_j)^2}$ standard deviation for jth criterion, $r_j = \max\limits_{i} x_{ij} - \min\limits_{i} x_{ij}$ is the range for jth criterion.

2.3. Selected similarity measures of the obtained final values of alternatives and their rankings

In this section, selected measures of similarity well-known from the literature are presented. They are used to compare the final evaluations of the alternatives obtained using the CoCoSo method and the rankings they generate for the different techniques of decision matrix normalization (equations (11)–(15)) with the results obtained using the TOPSIS-SM method. Let $R = (r_1, r_2, ..., r_m)$ and $T = (t_1, t_2, ..., t_m)$ be two sequences. Their elements, depending on the measure used, are indexes k_i (10) (when using *PCC* or *CSM*) or rankings of alternatives (when using *SRCC* or the *RCI*).

2.3.1. Pearson's correlation coefficient (PCC)

PCC is used to analyze the level of the linear relationship between two sequences R and T

$$PCC(R,T)=\frac{\sum_{i=1}^{m}(r_i-\overline{r})(t_i-\overline{t})}{\sqrt{\sum_{i=1}^{m}(r_i-\overline{r})^2}\sqrt{\sum_{i=1}^{m}(t_i-\overline{t})^2}} \tag{21}$$

where m denotes the length of the sequence, r_i and t_i elements of the sequences at position ith, $\overline{r}$ and $\overline{t}$ are average values of the elements of the sequences calculated from the formulas $\overline{r}=\frac{1}{m}\sum_{i=1}^{m} r_i$ and $\overline{t}=\frac{1}{m}\sum_{i=1}^{m} t_i$, respectively. A *PCC* value close to 1 or -1 indicates a strong linear relationship between R and T. In the particular case when $PCC=1$ or $PCC=-1$, all observations lie in a straight line. If $PCC=1$, an increase in R means an increase in T, and when $PCC=-1$, an increase in a R means a decrease in T. When $PCC=0$, there is no linear relationship between R and T. The *PCC* can be used to compare final evaluations of alternatives obtained using different techniques of normalization.

2.3.2. The cosine similarity measure (CSM)

CSM determines the similarity of two sequences R and T and is based on the Euclidean dot product. It is defined as follows

$$CSM(R,T)=\frac{\sum_{i=1}^{m} r_i t_i}{\sqrt{\sum_{i=1}^{m} r_i^2}\sqrt{\sum_{i=1}^{m} t_i^2}} \tag{22}$$

The closer the *CSM* is to 1, the more similar the sequences are. In particular, when $CSM=1$, sequences are equal.

2.3.3. Spearman's rank correlation coefficient ($SRCC$)

SRCC is a frequently used measure of the relationship between two rank sequences. *SRCC* is defined as

$$SRCC(R,T)=1-\frac{6\sum_{i=1}^{m} d_i^2}{m^3-m} \tag{23}$$

where $d_i=r_i-t_i$ is the difference in rankings at position i for $i=1,...,m$. The closer the *SRCC* is to 1, the more similar the considered rankings are. When $SRCC=1$, the rankings are identical.

2.3.4. The rank similarity index (RSI)

SRCC describes the relationship between two rank sequences. Given K different sequences, we can construct an $K\times K$ matrix $S=(s_{ij})$ with the value of *SRCC* between rankings i and j at position

ij (instead of *SRCC* we can use *PCC* or *CSM*). However, the identification of the best technique of normalization, based on this matrix, can be difficult. To solve this problem, we can use the method proposed by Chakraborty and Yeh [5], who suggest calculating *RSI* for each normalization technique as the average of the elements of the ith row of the matrix S; the method with the highest value indicates the normalization technique most preferred.

2.3.5. The ranking consistency index (RCI)

RCI is a measure to determine how similar the ranking obtained by the chosen normalization technique is to rankings obtained by other methods. The best technique of normalization is the one for which the *RCI* reaches the highest value. We present an *RCI* adapted to our example (for more details, see e.g., [4]). In the numerical example, we consider five different normalization techniques of the decision matrix so the *RCI* equation is

$$RCI(N_s) = T_{1-5} + \frac{3}{4} \sum_{\substack{i,j,k=1,\dots,5 \\ i,j,k\neq s \\ i<j<k}} T_{sijk} + \frac{2}{4} \sum_{\substack{i,j=1,\dots,5 \\ i,j\neq s \\ i<j}} T_{sij} + \frac{1}{4} \sum_{\substack{i=1,\dots,5 \\ i\neq s}} T_{si} \tag{24}$$

where N_s denotes the selected normalization technique for $s = 1, ..., 5$ and T_{1-5} – the number of times when N_1, N_2, N_3, N_4 and N_5 gave the same ranking, T_{sijk} – the number of times when N_s, N_i, N_j and N_k gave the same ranking, T_{sij} – the number of times when N_s, N_i and N_j gave the same ranking, T_{si} – the number of times when N_s and N_i gave the same ranking.

3. A new similarity measure of the rankings obtained

Suppose we have an MCDM problem with m alternatives whose rankings are obtained using K different methods. The ranking obtained for the kth method is written as

$$R^k = \begin{pmatrix} r_1^k \\ r_2^k \\ \vdots \\ r_m^k \end{pmatrix} \tag{25}$$

where $k = 1, ..., K$. Based on these rankings, we can construct a matrix R

$$R = (r_i^k)_{m\times K} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} R^1 & R^2 & \dots & R^K \end{array} \\ \begin{pmatrix} r_1^1 & r_1^2 & \dots & r_1^K \\ r_2^1 & r_2^2 & \dots & r_2^K \\ \vdots & \vdots & \ddots & \vdots \\ r_m^1 & r_m^2 & \dots & r_m^K \end{pmatrix} \end{array}, \tag{26}$$

that corresponds to the decision matrix in MCDM methods. The TOPSIS method is applied to the matrix R, which does not require normalization. The reference points are defined as follows:

- the positive ideal ranking (PIR) is the average of all rankings (average or compromise ranking)

$$R^{+} = (r_i^{+})_{m\times 1} = \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{pmatrix} \frac{1}{K}\sum_{k=1}^{K} r_1^k \\ \frac{1}{K}\sum_{k=1}^{K} r_2^k \\ \vdots \\ \frac{1}{K}\sum_{k=1}^{K} r_m^k \end{pmatrix} \tag{27}$$

- the negative ideal ranking (NIR), which is divided into two parts: the left negative ranking (LNR)

$$R^{-L} = (r_i^{-L})_{m\times 1} = \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{pmatrix} \min_k r_1^k \\ \min_k r_2^k \\ \vdots \\ \min_k r_m^k \end{pmatrix} \tag{28}$$

  and the right negative ranking (RNR)

$$R^{-R} = (r_i^{-R})_{m\times 1} = \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{pmatrix} \max_k r_1^k \\ \max_k r_2^k \\ \vdots \\ \max_k r_m^k \end{pmatrix} \tag{29}$$

  which provide maximum separation from the PIR.

Next, we calculate the similarity measure of each ranking R^k $(k = 1, 2, ..., K)$ to the PIR and to each part of NIR, respectively, denoted by

$$R^{+k},\ R^{-kL},\ R^{-kR} \tag{30}$$

To this end, the Kendall rank correlation coefficient is used [13].

Note. The elements of the vector R^{+} (27) may not be integers. In that case, we can rank order them and replace the vector R^{+} (27) by this ranking (such a transformation does not affect the result).

Finally, the *RCC* to PIR (for each ranking) is calculated from the formula

$$RCC^k = \frac{(1-R^{-kL}) + (1-R^{-kR})}{(1-R^{+k}) + (1-R^{-kL}) + (1-R^{-kR})}. \tag{31}$$

Using the RCC^k $(k = 1, ..., K)$, the methods under consideration are ordered in descending order regarding values and the one with the highest RCC^k is regarded as the most appropriate.

4. Numerical example

The numerical example from the paper by Yazdani et al. [28] is used to analyze the impact of the normalization technique on the obtained values of alternatives in the CoCoSo method and on their ranking. The MCDM problem consists of seven alternatives and five criteria, of which the second is of the cost type and the others of the benefit type. The decision matrix is shown in Table 1, while the vector criteria weights are

$$w = (0.036,\ 0.192,\ 0.326,\ 0.326,\ 0.120)$$

Table 1. Decision matrix of the MCDM problem under consideration

Alternative	C_1	C_2	C_3	C_4	C_5
A_1	60	0.4	2540	500	990
A_2	6.35	0.15	1016	3000	1041
A_3	6.8	0.1	1727.2	1500	1676
A_4	10	0.2	1000	2000	965
A_5	2.5	0.1	560	500	915
A_6	4.5	0.08	1016	350	508
A_7	3	0.1	1778	1000	920

Table 2. Values of alternatives obtained by the CoCoSo method using different normalization techniques

Alternative	N_1	N_2	N_3	N_4	N_5
A_1	2.0413	1.7210	2.0087	1.8066	1.7968
A_2	2.7880	2.2606	2.2508	2.1308	1.7848
A_3	2.8823	2.2662	2.2259	2.1113	1.7818
A_4	2.4160	2.0184	1.9933	1.9204	1.7805
A_5	1.2987	1.6364	1.6054	1.5908	1.6818
A_6	1.4431	1.6817	1.6822	1.6267	1.6893
A_7	2.5191	2.0637	2.0154	1.9404	1.7466

Table 2 and Figure 1 show the values of the indexes k_i (10) obtained by the CoCoSo method in which different techniques of normalization (equations (11)-(15)) of the input data (i.e., the decision matrix shown in Table 1) were used (in the CoCoSo method it was assumed that $\lambda = 0.5$). Different techniques of normalization produce different values of k_i (Table 2). The most varied evaluations of alternatives using the CoCoSo method are obtained using method N_1. For methods N_2 through N_4, the variation is smaller, and for method N_5 the results are quite close to each other. This is perfectly illustrated in the left-hand column of Figure 1. In turn, Table 3 and Figure 2 show the rankings obtained based on the values k_i in Table 2. We can see that the best alternative depends on the method of normalization and therefore N_1 and N_2 (which generate the same ranking) indicated A_3, while N_3 and N_4 indicated A_2. However, it is worth noting when analyzing the values in Table 2 that the evaluations of alternatives A_2 and A_3 for N_1 through N_4 are very similar. Furthermore, for N_5, alternative A_1 proved to be the best. However, considering the k_i values in Table 2, the application of N_5 gives very similar evaluations for alternatives A_1 through A_4, which is perfectly visible in the left-hand column of Figure 1. On the other hand, all the normalization methods considered indicated as the weakest alternatives A_5 and A_6, as can be seen in the figures in the right-hand column of Figure 2. Furthermore, in Table 2, we can see that their k_i values significantly deviate from the evaluations of the other alternatives.

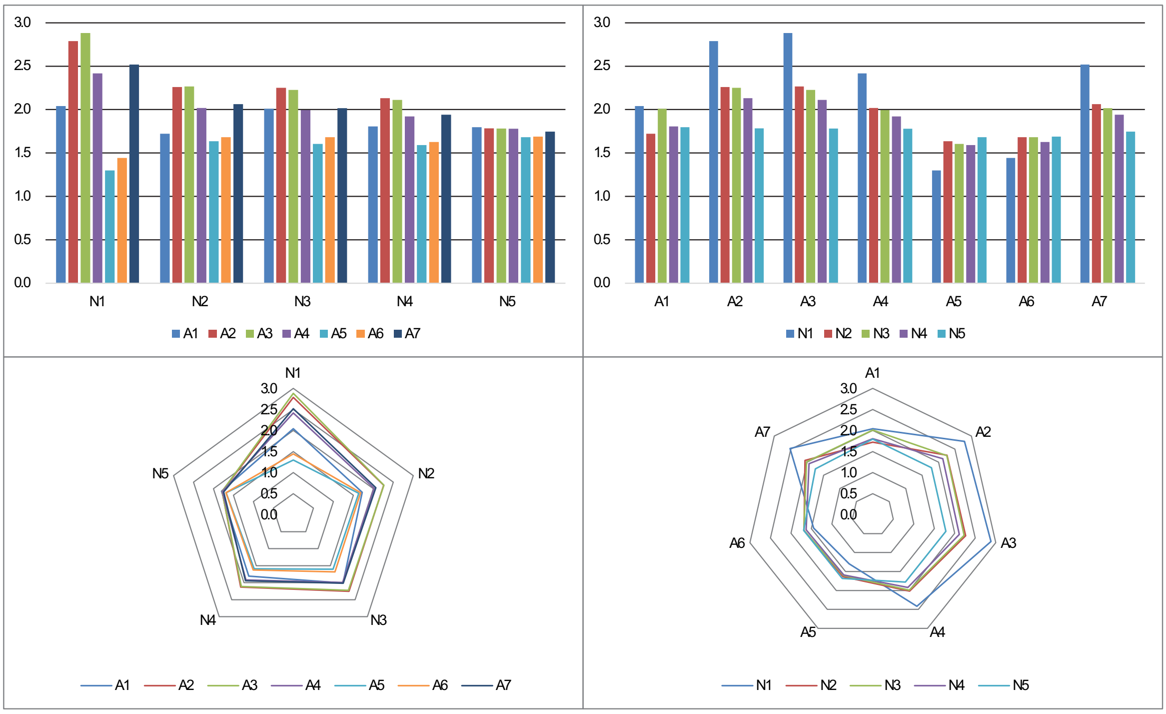

Figure 1. Values of alternatives obtained by the CoCoSo method using different normalization techniques

Tables 4–6 show the results obtained with the new TOPSIS-SM measure. Table 4 and Table 5 show the reference points determined using equations (27)–(29) and the values of the similarity measure (in this case, the Kendall rank correlation coefficient) of the obtained rankings from these reference points (30). Lastly, Table 6 and Figure 3 show the results obtained by the new measure using equation (31) and the ranking of the normalization techniques

$$N_5 \prec N_3 \prec N_4 \prec N_2 \approx N_1$$

where $\prec$ means more suitable and $\approx$ means equivalent. Considering the similarity measures in Table 5, we can see that N_1–N_4 give a most similar rating (the same value) to the PIR. On the other hand, N_3 gives a rating that is most similar to both parts of the NIR, so it is inferior to N_1, N_2 and N_4. Moreover, the similarity measures to the NIR for N_1 and N_2 are lower than those for N_4, so N_4 should be lower in the ranking of methods as confirmed in Table 6. N_5 has the lower similarity measure to the PIR, so it should be the worst method, as seen in the left column of Figure 3.

Table 3. Rankings of alternatives obtained by the CoCoSo method using different normalization techniques

Alternative	N_1	N_2	N_3	N_4	N_5
A_1	5	5	4	5	1
A_2	2	2	1	1	2
A_3	1	1	2	2	3
A_4	4	4	5	4	4
A_5	7	7	7	7	7
A_6	6	6	6	6	6
A_7	3	3	3	3	5

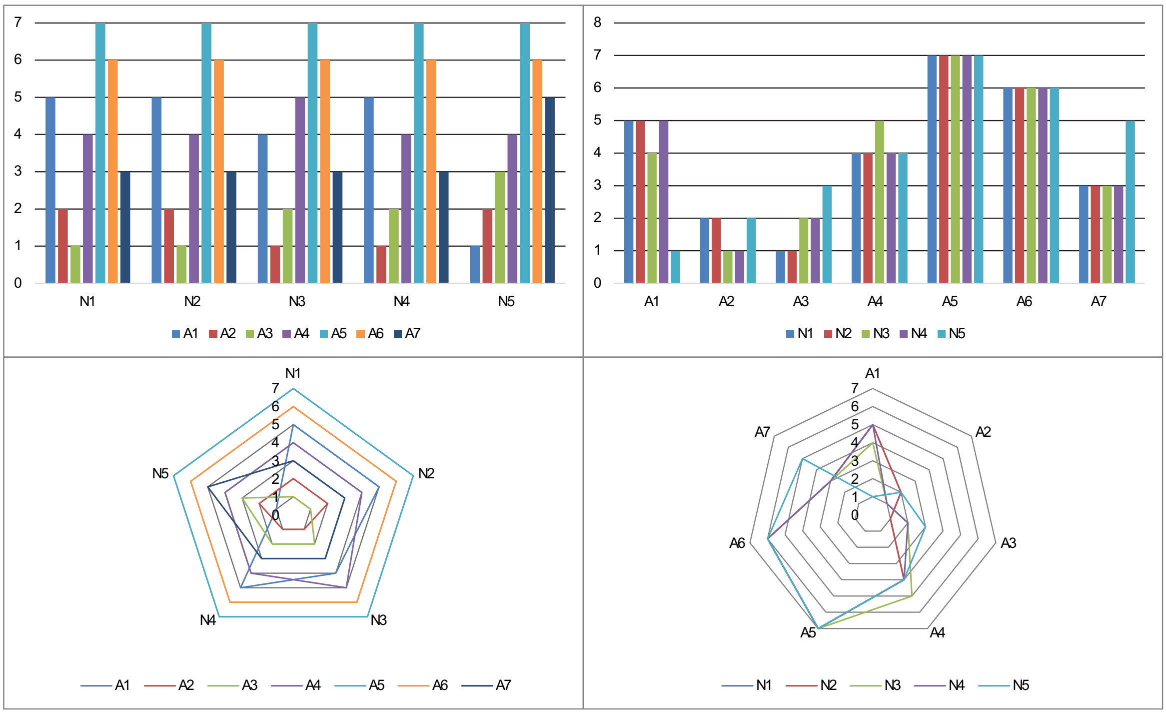

Figure 2. Rankings of alternatives obtained by the CoCoSo method using different normalization techniques

Table 4. The reference points of TOPSIS-SM

Ranking	A_1	A_2	A_3	A_4	A_5	A_6	A_7
R^+	4.0	1.6	1.8	4.2	7.0	6.0	3.4
R^{-L}	1	1	1	4	7	6	3
R^{-R}	5	2	3	5	7	6	5

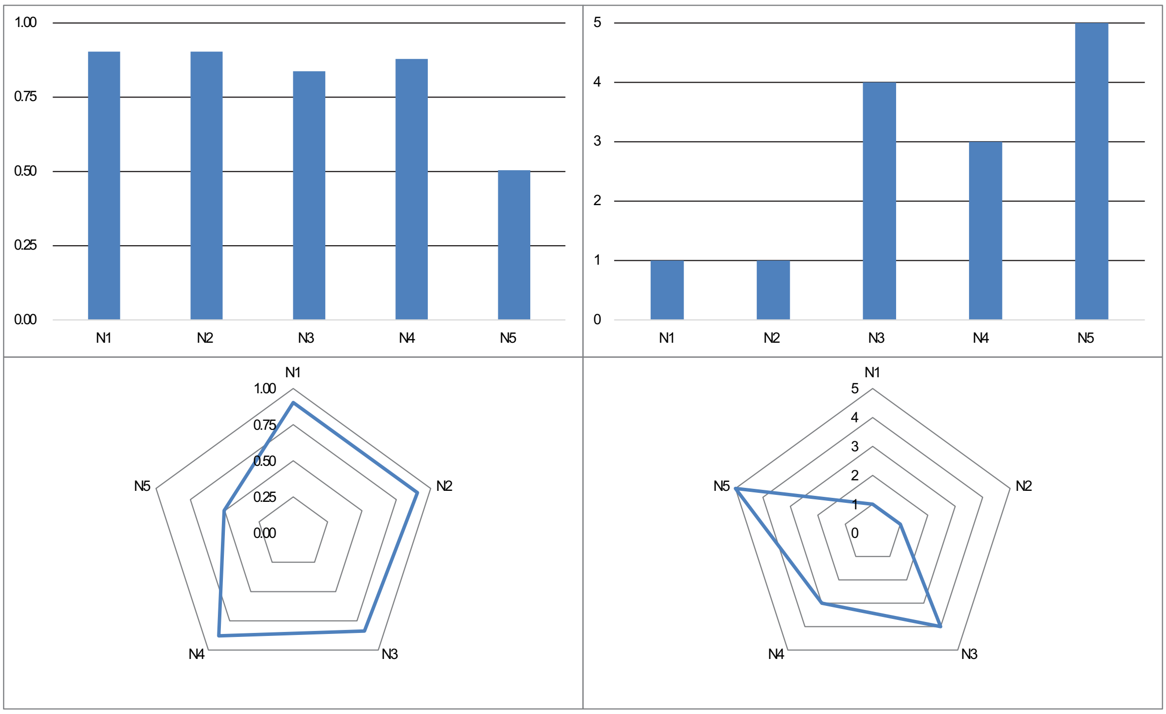

Figure 3. Values of TOPSIS-SM and rankings obtained by the CoCoSo method using different normalization techniques and their ranking

Table 5. The similarity measures of each ranking from PIR and NIR

Ranking	N_1	N_2	N_3	N_4	N_5
R^{+k}	0.95119	0.95119	0.95119	0.95119	0.55069
R^{-kL}	0.72008	0.72008	0.82295	0.72008	0.82295
R^{-kR}	0.82295	0.82295	0.92585	0.92582	0.72008

Table 6. Values of TOPSIS-SM and rankings by the CoCoSo method using different normalization techniques and their ranking

Normalization technique	TOPSIS-SM	Rank
N_1	0.9035	1
N_2	0.9035	1
N_3	0.8373	4
N_4	0.8789	3
N_5	0.5042	5

5. Comparison of the TOPSIS-SM method with other similarity measures

In this section, based on the numerical example presented in Section 4, the similarity measures described in Section 2.3 are calculated and the results are compared with those obtained using TOPSIS-SM. Tables 7–13 show the results of the similarity measures analysis of the obtained values and rankings using CoCoSo and various normalization techniques.

Table 7. Values of *PCC* obtained by the CoCoSo method using different normalization techniques

Normalization technique	N_1	N_2	N_3	N_4	N_5
N_1	1.0000	0.9548	0.9682	0.9880	0.8203
N_2	0.9548	1.0000	0.8965	0.9703	0.6408
N_3	0.9682	0.8965	1.0000	0.9762	0.8876
N_4	0.9880	0.9703	0.9762	1.0000	0.7968
N_5	0.8203	0.6408	0.8876	0.7968	1.0000

Table 8. Values of *RSI* for *PCC* by the CoCoSo method using different normalization techniques and their ranking

Normalization technique	*RSI*(*PCC*)	Rank
N_1	0.9462	1
N_2	0.8924	4
N_3	0.9457	3
N_4	0.9462	1
N_5	0.8291	5

Table 7 shows the *PCC* values for the normalization techniques considered, while Table 8 shows their *RSI* and ranking. Analysing the *PCC* values (Table 7) we can see that they are well above 0.95 except for the coefficients of the normalization technique N_5 with all the other methods. This means that after applying the *RSI*, N_5 is rated lowest as confirmed by Table 8. In addition, a single, lower value of

$PCC(N_2, N_3)$ results in N_2 and N_3 being ranked lower than N_1 and N_4, which are ranked highest by applying the *RSI* (with the same value of *RSI*). In turn, low $PCC(N_2, N_5)$ results in a lower ranking of N_2 as compared with N_3. It is also worth noting that N_3 has a slightly lower *RSI* value as compared to N_1 and N_4, due to its higher $PCC(N_3, N_5)$ value as compared to the others. Taking into account Table 8, we can conclude that the preferred normalization techniques in this numerical example are N_1 (dedicated to CoCoSo) and N_4, and their ranking is

$$N_5 \prec N_2 \prec N_3 \prec N_4 \approx N_1$$

Table 9. Values of *CSM* obtained by the CoCoSo method using different normalization techniques

Normalization technique	N_1	N_2	N_3	N_4	N_5
N_1	1.0000	0.9899	0.9888	0.9879	0.9717
N_2	0.9899	1.0000	0.9984	0.9994	0.9936
N_3	0.9888	0.9984	1.0000	0.9997	0.9956
N_4	0.9879	0.9994	0.9997	1.0000	0.9963
N_5	0.9717	0.9936	0.9956	0.9963	1.0000

Table 10. Values of *RSI* for *CSM* obtained by the CoCoSo method using different normalization techniques and their ranking

Normalization technique	*RSI*(*CSM*)	RANK
N_1	0.9876	5
N_2	0.9962	3
N_3	0.9965	2
N_4	0.9966	1
N_5	0.9914	4

Table 9 shows the results obtained with *CSM* for the values obtained with CoCoSo and the different normalization techniques, while Table 10 shows their *RSI* and rankings. Analysing the *CSM* values (Table 9) we can see that most of them have a value of around 0.99. A single low $CSM(N_1, N_5)$ value results in N_1 and N_5 being rated lowest. Furthermore, for N_5 the remaining values are higher than for N_1, resulting in N_1 being rated worst, as confirmed in Table 10. The remaining methods have values close to each other, making it difficult to rank them based on *CSM* values. This is confirmed by Table 10 where the *RSI* values for N_2 through N_4 are almost identical. Finally, *RSI*(*CSM*) generates the following ranking of normalization techniques

$$N_1 \prec N_5 \prec N_2 \prec N_3 \prec N_4$$

and indicates N_4 (vector normalization) as the most preferred one.

Table 11 shows the results obtained with *SRCC* for the rankings obtained with CoCoSo and the different normalization techniques while Table 12 shows their *RSI* and rankings. In Table 11, we can see that the *SRCC* values are quite high, above 0.9, but when N_5 is considered, they are quite low, which causes N_5 to be rated worst.

Although the differences in *RSI* for N_1 through N_4 are small, they result in the following ranking

$$N_5 \prec N_1 \approx N_2 \prec N_4 \prec N_3$$

Table 11. Values of *SRCC* obtained by the CoCoSo method using different normalization techniques

Normalization technique	N_1	N_2	N_3	N_4	N_5
N_1	1.0000	1.0000	0.9286	0.9643	0.5714
N_2	1.0000	1.0000	0.9286	0.9643	0.5714
N_3	0.9286	0.9286	1.0000	0.9643	0.7143
N_4	0.9643	0.9643	0.9643	1.0000	0.6071
N_5	0.5714	0.5714	0.7143	0.6071	1.0000

Table 12. Values of *RSI* for *SRCC* obtained by the CoCoSo method using different normalization techniques and their ranking

Normalization technique	*RSI* (*SRCC*)	Rank
N_1	0.8929	3
N_2	0.8929	3
N_3	0.9072	1
N_4	0.9000	2
N_5	0.6928	5

Table 13 shows the results obtained with *RCI* for the rankings obtained with CoCoSo and the different normalization techniques. The values obtained are varied and result in the following ranking

$$N_3 \prec N_5 \prec N_4 \prec N_1 \approx N_2.$$

Table 13. Values of *RCI* obtained by the CoCoSo method using different normalization techniques and their ranking

Normalization technique	*RCI*	Rank
N_1	24.25	1
N_2	24.25	1
N_3	19.50	5
N_4	23.50	3
N_5	20.00	4

Finally, Table 14 and Figure 4 present rankings of the normalization techniques considered obtained through different similarity measures. Analysing the resulting rankings, we can notice that the different similarity measures indicate different normalization techniques as the most appropriate for the MCDM problem under consideration. In such a situation, we can use the suggestions of Vafaei et al. [24] and use the plurality voting (*PV*) method to indicate the most appropriate normalization techniques, which counts the number of times each method was the best. The results of the *PV* method are presented in Table 15. This means that the most appropriate normalization technique in this numerical example is N_1. We can write this down in the form of the following ranking

$$N_5 \prec N_3 \prec N_4 \approx N_2 \prec N_1.$$

Table 14. Final rankings of the normalization techniques considered obtained through different similarity methods

Normalization technique	*RSI* (*PCC*)	*RSI* (*CSM*)	*RSI* (*SRCC*)	*RCI*	TOPSIS-SM
N_1	1	5	3	1	1
N_2	4	3	3	1	1
N_3	3	2	1	5	4
N_4	1	1	2	3	3
N_5	5	4	5	4	5

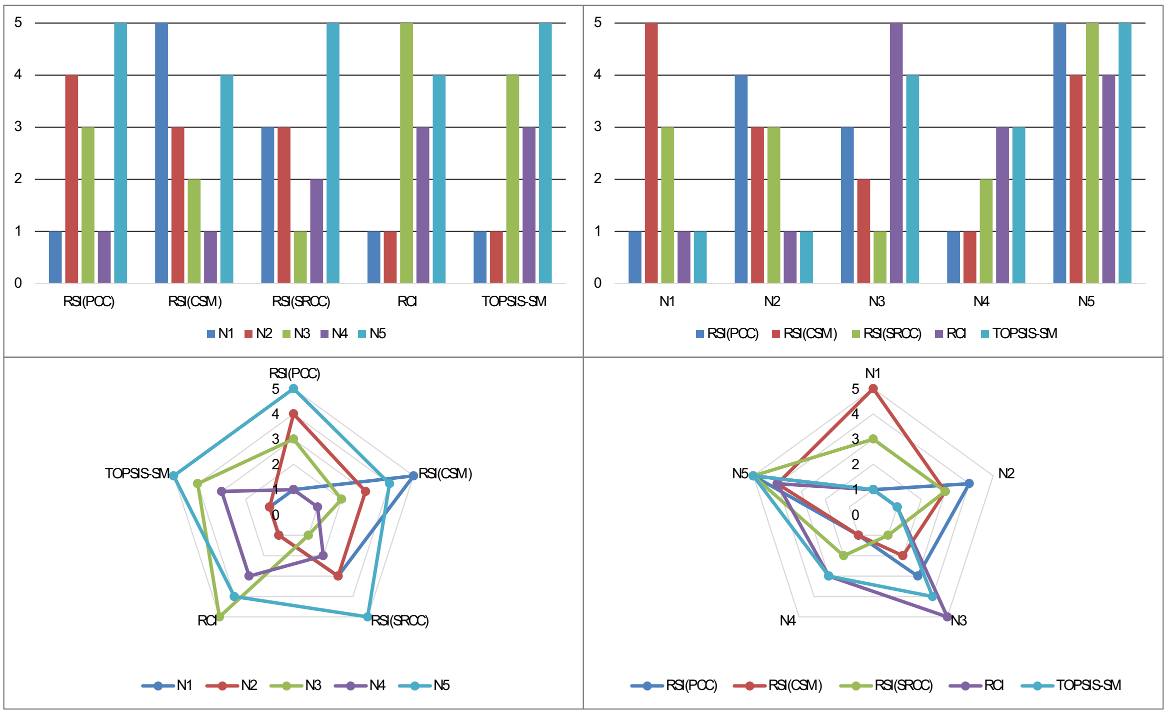

Figure 4. Final rankings of the normalization techniques considered obtained through different similarity methods

Table 15. The results of the *PV* method

Normalization technique	*PV*	Rank
N_1	3	1
N_2	2	2
N_3	1	4
N_4	2	2
N_5	0	5

6. Conclusions

Using a numerical example, the paper analyses which of the selected normalization techniques is most suitable for the CoCoSo method. In addition to the dedicated CoCoSo zero unitarization method, the maximum method, the sum method, vector normalization, and logarithmic normalization were also investigated. To compare the results obtained by different normalization techniques, selected similarity measures well-known from the literature were used, such as Pearson's correlation coefficient, Spearman's rank correlation coefficient, cosine similarity measure, and ranking consistency index. In addition, a new method for assessing the similarity of rankings, called TOPSIS-SM, was developed, based on the algorithm of the TOPSIS method.

Based on the numerical example from the paper of Yazdani et al. [28], the authors of the CoCoSo method, it was found that the most suitable normalization techniques for this example are the zero unitarization method and the vector normalization. The similarity measures *RSI* (*PCC*) and *RCI* indicated the zero unitarization method, while vector normalization is indicated by *RSI* (*PCC*) and *RSI* (*CSM*). The proposed TOPSIS-SM similarity measure indicated the zero unitarization method and maximum method as the most appropriate normalization method in this numerical example.

This makes it difficult to say unequivocally which normalization technique is the most suitable for the MCDM problem under consideration. Further research is needed in this area and perhaps even the development of new similarity measures that will unambiguously indicate which method of normalization, or which MCDM method, is the most appropriate for the MCDM problem being analyzed. It will be an area of further research for the author.

Acknowledgement

The author acknowledges the editor of the Operations Research and Decisions journal and the two anonymous reviewers for their valuable comments and suggestions.

Funding

The work has been done in the framework of project WZ/WI-IIT/2/2023 in the Bialystok University of Technology and financed by the Ministry of Science and Higher Education.

References

[1] Abdullah, L., and Adawiyah, C. W. R. Simple additive weighting methods of multi criteria decision making and applications: A decade review. *International Journal of Information Processing and Management 5*, 1 (2014), 39–49.

[2] Behzadian, M., Otaghsara, S. K., Yazdani, M., and Ignatius, J. A state-of the-art survey of TOPSIS applications. *Expert Systems with Applications 39*, 17 (2012), 13051–13069.

[3] Çelen, A. Comparative analysis of normalization procedures in Topsis method: With an application to Turkish deposit banking market. *Informatica 25*, 2 (2014), 185–208.

[4] Chakraborty, S., and Yeh, C.-H. A simulation comparison of normalization procedures for Topsis. In *2009 International Conference on Computers & Industrial Engineering* (Troyes, France, 2009), IEEE, pp. 1815–1820.

[5] Chakraborty, S., and Yeh, C.-H. Rank similarity based MADM method selection. In *2012 International Conference on Statistics in Science, Business and Engineering (ICSSBE)* (Langkawi, Malaysia, 2012), pp. 1–6.

[6] Chatterjee, P., and Chakraborty, S. Flexible manufacturing system selection using preference ranking methods: A comparative study. *International Journal of Industrial Engineering Computations 5*, 2 (2014), 315–338.

[7] Churchman, C. W., and Ackoff, R. L. An approximate measure of value. *Journal of the Operations Research Society of America 2*, 2 (1954), 172–187.

[8] Ghasempour, R., Nazari, M. A., Ebrahimi, M., Ahmadi, M. H., and Hadiyanto, H. Multi-criteria decision making (MCDM) approach for selecting solar plants site and technology: A review. *International Journal of Renewable Energy Development 8*, 1 (2019).

[9] Halicka, K., and Kacprzak, D. Linear ordering of selected gerontechnologies using selected MCGDM methods. *Technological and Economic Development of Economy 27*, 4 (2021), 921–947.

[10] Hwang, C.-L., and Yoon, K. *Multiple Attribute Decision Making. Methods and Applications. A State-of-the-Art Survey*. Springer, Berlin, 1981.

[11] Jahan, A., and Edwards, K. L. A state-of-the-art survey on the influence of normalization techniques in ranking: Improving the materials selection process in engineering design. *Materials & Design (1980-2015) 65* (2015), 335–342.

[12] Jarocka, M. Choice of standardisation formula in the comparative analysis of multivariate objects. *Ekonomia i Zarządzanie 7*, 1 (2015), 113–126, (in Polish).

[13] Kendall, M. G. A new measure of rank correlation. *Biometrika 30*, 1/2 (1938), 81–93.

[14] Koganti, V. K., Menikonda, N., Anbuudayasankar, S. P., Krishnaraj, T., Athhukuri, R. K., and Vastav, M. S. Grahp top model for supplier selection in supply chain: A hybrid MCDM approach. *Decision Science Letters 8*, 1 (2019), 65–80.

[15] Kukuła, K. Zero-based unitarisation method against the background of selected methods for the normalisation of diagnostic characteristics. *Acta Scientifica Academiae Ostroviensis 4* (1999), 5–31 (in Polish).

[16] Kukuła, K. *Zero-based unitarisation method*. Wydanictwo Naukowe PWN, 2000 (in Polish).

[17] Nijkamp, P., and van Delft, A. *Multi-Criteria Analysis and Regional Decision-Making*. Vol. 8. Springer Science & Business Media, 1977.

[18] Palczewski, K., and Sałabun, W. The fuzzy topsis applications in the last decade. *Procedia Computer Science 159* (2019), 2294–2303.

[19] Rudnik, K., and Kacprzak, D. Fuzzy Topsis method with ordered fuzzy numbers for flow control in a manufacturing system. *Applied Soft Computing 52* (2017), 1020–1041.

[20] Saaty, T. L. *The Analytic Hierarchy Process*. McGraw-Hill, New York,1980.

[21] Stojčić, M., Zavadskas, E. K., Pamučar, D., Stević, Ž., and Mardani, A. Application of MCDM methods in sustainability engineering: A literature review 2008–2018. *Symmetry 11*, 3 (2019), 350.

[22] Vafaei, N., Ribeiro, R. A., and Camarinha-Matos, L. M. Normalization techniques for multi-criteria decision making: analytical hierarchy process case study. In *Technological Innovation for Cyber-Physical Systems: 7th IFIP WG 5.5/SOCOLNET Advanced Doctoral Conference on Computing, Electrical and Industrial Systems, DoCEIS 2016, Costa de Caparica, Portugal, April 11–13, 2016, Proceedings* (Cham 2016), L. M. Camarinha-Matos, A. J. Falcão, N. Vafaei and S. Najdi, Eds., Springer, pp. 261–269.

[23] Vafaei, N., Ribeiro, R. A., and Camarinha-Matos, L. M. Data normalisation techniques in decision making: case study with TOPSIS method. *International Journal of Information and Decision Sciences 10*, 1 (2018), 19–38.

[24] Vafaei, N., Ribeiro, R. A., and Camarinha-Matos, L. M. Selecting normalization techniques for the analytical hierarchy process. In *Technological Innovation for Life Improvement: 11th IFIP WG 5.5/SOCOLNET Advanced Doctoral Conference on Computing, Electrical and Industrial Systems, DoCEIS 2020, Costa de Caparica, Portugal, July 1–3, 2020, Proceedings* (Cham, 2020), L. M. Camarinha-Matos, N. Farhadi, F. Lopes, H. Pereira , Eds., Springer, pp. 43–52.

[25] Vafaei, N., Ribeiro, R. A., and Camarinha-Matos, L. M. Assessing normalization techniques for simple additive weighting method. *Procedia Computer Science 199* (2022), 1229–1236.

[26] Walesiak, M. Data Normalization in Multivariate Data Analysis. An Overview and Properties *Przegląd Statystyczny 61*, 4 (2014), 363–372, (in Polish).

[27] Walesiak, M. The choice of normalization method and rankings of the set of objects based on composite indicator values. *Statistics in Transition. New Series 19*, 4 (2018), 693–710.

[28] Yazdani, M., Zarate, P., Zavadskas, E. K., and Turskis, Z. A combined compromise solution (CoCoSo) method for multi-criteria decision-making problems. *Management Decision 57*, 9 (2019), 2501–2519.

[29] Yue, Z. An extended TOPSIS for determining weights of decision makers with interval numbers. *Knowledge-Based Systems 24*, 1 (2011), 146–153.

[30] Żabicki, P., and Gardziejczyk, W. Issues of criteria normalization in the multicriteria analyzes in the design of roads. *Budownictwo i Architektura 13*, 4 (2014), 325–333 (in Polish).

[31] Zavadskas, E. K., and Turskis, Z. A new logarithmic normalization method in games theory. *Informatica 19*, 2 (2008), 303–314.

[32] Zeliaś, A. Some notes on the selection of normalization of diagnostic variables. *Statistics in Transitions 5*, 5 (2002), 787–802.

[33] Zhang, X., Lu, J., and Peng, Y. Hybrid MCDM model for location of logistics hub: A case in China under the belt and road initiative. *IEEE Access 9* (2021), 41227–41245.