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How to know it is "the one"? Selecting the most suitable solution from the Pareto optimal set. Application to sectorization

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Abstract

Multi-objective optimization (MOO) considers several objectives to find a feasible set of solutions. Selecting a solution from Pareto frontier (PF) solutions requires further effort. This work proposes a new classification procedure that fits into the analytic hierarchy Process (AHP) to pick the best solution. The method classifies PF solutions using pairwise comparison matrices for each objective. Sectorization is the problem of splitting a region into smaller sectors based on multiple objectives. The efficacy of the proposed method is tested in such problems using our instances and real data from a Portuguese delivery company. A non-dominated sorting genetic algorithm (NSGA-II) is used to obtain PF solutions based on three objectives. The proposed method rapidly selects an appropriate solution. The method was assessed by comparing it with a method based on a weighted composite single-objective function.

Keywords: *AHP, Pareto frontier, selection, decision making, sectorization*

1. Introduction

Sectorization is partitioning a large territory, area, or network into small parts or sectors considering one or more objectives. The application areas of sectorization problems cover political districting, maintenance operations, division of sales territories, health, schooling or police districting, forest planning etc. [see 1, 4, 5, 8, 20, 31, 39]. Moreover, the comprehensive survey of Kalcsics et al. [26] is a very important and relevant reference to the field as the authors highlight different application lines, objectives, and solution methods in a formalized way.

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Most of the sectorization problems are NP-hard, given the complexity of the multi-objective nature of these problems. Thus, the solution procedure obliges researchers to follow a proper method to obtain a solution in a feasible time. For instance, multi-objective combinatorial optimisation (MOCO) methods allow obtaining Pareto optimal solutions in theory. Nonetheless, given the difficulty of finding the real Pareto set in MOCO, the use of approximate methods based on metaheuristics increased [20]. Multi-objective optimisation (MOO) methods formed on metaheuristics have been widely applied to sectorisation in the last few years [see 18, 24, 27, 46, 49, 50]. These methods evaluate several objectives separately and simultaneously while providing a Pareto optimal set of solutions for the decision-makers (DMs). Most of the time, real-life applications (e.g., sectorization) require selecting and implementing the most appropriate solution from this set. Thus, after obtaining these viable solutions in the context of multi-objective optimisation, a fundamental step remains to be taken, selecting a suitable solution among the various solutions.

Different methods are acknowledged in the literature for selecting a solution or a reduced number of solutions from a group of trade-off solutions based on predetermined criteria. For example, in the elbow method, Thorndike [47] proposed group trade-off solutions based on their distance in the objective space. The procedure begins by aggregating the two most distant solutions into separate groups. The third cluster initialises with the solution far from either of the first two. The optimum number of clusters is determined according to the average within-cluster distances. Clusters are built up by assigning the closest solutions within the defined clusters. This method illustrates the knee approach, which extracts the knee points of a Pareto frontier as the most relevant parts for the DMs. These knees aim to find a smaller number but a more representative and interesting set of solutions [6, 12]. Likewise, Dumedah et al. [16] also used cluster analysis considering the properties of the solutions from the parameter space (parameter values), from the objective space (spread of solutions) and both spaces. Their results obtained solutions with unique properties among all the trade-off solutions in the Pareto frontier.

Das et al. [11] propose a method by calculating the marginal substitution rates for each solution's objectives. That way, they aim to provide a comparison between the two solutions. However, this method is only applicable to bi-objective optimisation problems. Moreover, Khu et al. [28] aim to select fewer solutions by preference ordering and compensation between the objectives.

Furthermore, Ferreira et al. [19] proposed a method allowing DM's integration into the selection process. This method aims to optimise all the objectives according to their importance. Another method is proposed by Crispim and Pinho de Sousa [10]. The authors first ranked the Pareto frontier solutions using the fuzzy TOPSIS algorithm and then clustered them according to the given weights to select the solution.

In the current paper, we propose a classification method that helps to select the most suitable solution from the Pareto optimal set obtained by multi-objective optimisation, considering the preferences in the context of sectorization problems. This classification method can be conceptualized under the analytical Hierarchy process (AHP), a multi-criteria decision technique proposed by Saaty in the 1970s [43, 44]. As is known, the objective weights can be defined using the pairwise comparison matrix (PCM) in AHP. In PCM, DMs compare and weigh each pair of objectives in terms of their relative importance. Besides, the alternatives are evaluated separately for each objective within the PCMs and scored according to their performance on the specific objective. Using the objective weights and the scores defined for each alternative under each objective, the DM concludes the convenience level of the alternatives regarding the preferences. When considering the set of Pareto optimal solutions as alternatives, such a comparison may be tougher due to a large number of Pareto frontier solutions, the similarity between them, and their numeric nature (i.e., each trade-off solution has a numeric performance for each objective considered). In this context, the novelty of this work is the classification method that helps to compare each solution, represented by a value, among each other for each separate objective within the fundamental scale proposed under AHP to establish PCMs. Such a method helps distinguish the significantly different solutions on the Pareto front. Following this method, the DMs can obtain the most suitable solution among several solutions and observe the ranking of the existing solutions. This method is similar to knee or elbow methods by logic with lower computational difficulty. Besides being easily implemented, it is useful to select a trade-off solution regardless of the number of objectives. Moreover, it is relevant to mention that this method can be adopted by any research seeking to select the most suitable solution among the several trade-off solutions included in the Pareto optimal set.

We implement the proposed classification method to select the most convenient solution among the trade-off solutions using artificially created instances and real-world data from a Portuguese delivery company. We follow the non-dominated sorting genetic algorithm II (NSGA-II) to obtain multiple solutions by considering three objectives: equilibrium, compactness, and contiguity.

According to Deb [14], when the preferences are certain, combining all the objectives into a composite weighted single-objective function should be enough to obtain an adequate solution. Considering this, we implement a single-objective genetic algorithm (GA) by assigning a certain weight to each objective and comparing the similarity between the solution selected by our proposed method and the solution obtained by the single-objective method.

It is essential to mention that, in sectorization or other real-world problems, following a multi-objective optimisation method may be necessary since it is tough to be certain about the importance (i.e., weights) needed for each objective when several are in question [29]. However, this does not mean the DM has no insight into them. Our selection method is more proper when there is an idea about the importance of the objectives but not certainty about them. Otherwise, it is advisable to use a single-objective method. Combining the objectives within a single-objective function would be more proper and straightforward than a MOO method.

The rest of the paper is organised as follows. Section 2 exhibits the solution approach. First, we present the method used to obtain a Pareto-optimal set of solutions, namely, NSGA-II. Second, the proposed classification method to select the most convenient solution among several trade-off solutions obtained from NSGA-II is displayed. Section 3 gives some insights into the objectives and data used to solve Sectorization problems. Section 4 includes experimental results with their instances and real data from a large delivery company, a comparative analysis, and related discussions. Finally, Section 5 concludes the article.

2. Solution approach

The multi-objective optimisation method studies several objectives simultaneously and separately. The results extracted from each objective are evaluated in the solution space and sequentially located in Pareto frontiers.

The generalised version of the multi-objective optimisation problem can be defined as follows:

$$\min(\max) F(x) = \big(f_1(x),\ f_2(x),\ \ldots,\ f_n(x)\big),\ x \in X \tag{1}$$

In equation (1), $f_i(x)$ represents the ith objective $(i = 1, 2, 3, \ldots, N)$. Moreover, x is a vector of decision variables in the feasible region of decision space (X). Solutions are assigned to the Pareto frontiers according to their performance on each objective. The solutions located in the Pareto front constitute the Pareto optimal set of solutions. The main difficulty in this approach starts after obtaining the Pareto optimal set. Most of the time, selecting a solution among the multiple possible solutions is complicated.

The selection method proposed in this paper aims to compete with this difficulty and offers a method to pick the most convenient solution. The remainder of this section includes the multi-objective optimization method adopted to solve sectorization problems and the AHP-based selection method proposed in this work.

2.1. Obtaining Pareto optimal solutions

There are several multi-objective optimisation methods to obtain the Pareto frontier solutions. In the current paper, we adopted NSGA-II as a solution approach, proposed by Deb et al. [15] to solve sectorization problems and to get the Pareto-optimal set of solutions.

NSGA-II is one of the most used multi-objective genetic algorithms in the literature [17, 25, 30, 48, 50]. This algorithm finds the set of non-dominated solutions, approximations to the actual Pareto front, according to their performances on several objectives.

According to NSGA-II, for a solution to be superior to another solution, it should be better than the other in at least one objective while not being worse in the rest of the objectives. That is called domination. However, commonly, some solutions do not dominate each other in all objectives. In other words, a solution can be better than the other solution in one objective while being worse than the other one in other objectives. We need to construct two values for each solution to understand which solution should be located in which Pareto frontier. These values are (i) domination count and (ii) set of dominated members. The former refers to that a given solution is dominated by how many other solutions. The latter is the set that collects the information of the solutions dominated by a given solution. The Pareto frontiers are formed according to the domination count of each solution. The first frontier is composed of the solutions with the lowest domination count. The second frontier is constructed by the solutions dominated only by the solutions in the first frontier. This process continues until all the solutions are located in their Pareto frontiers.

Figure 1 introduces the NSGA-II framework as well as where the proposed selection method interacts with the solution approach. NSGA-II algorithm starts with the creation of the initial population which includes a set of solutions obtained randomly or considering a specific order which should still allow randomisation. Some genetic encoding systems are proper for sectorization, clustering or assignment problems. We used a matrix form binary grouping genetic encoding system [34]; solutions are represented in a matrix where rows represent the basic units, nodes, etc., and columns represent the number of sectors. If a basic unit is assigned to a sector, the row cell is set to 1; otherwise, it is set to zero. That is a proper encoding system when sectorization, assignment or allocation problems are considered [13, 33, 35].

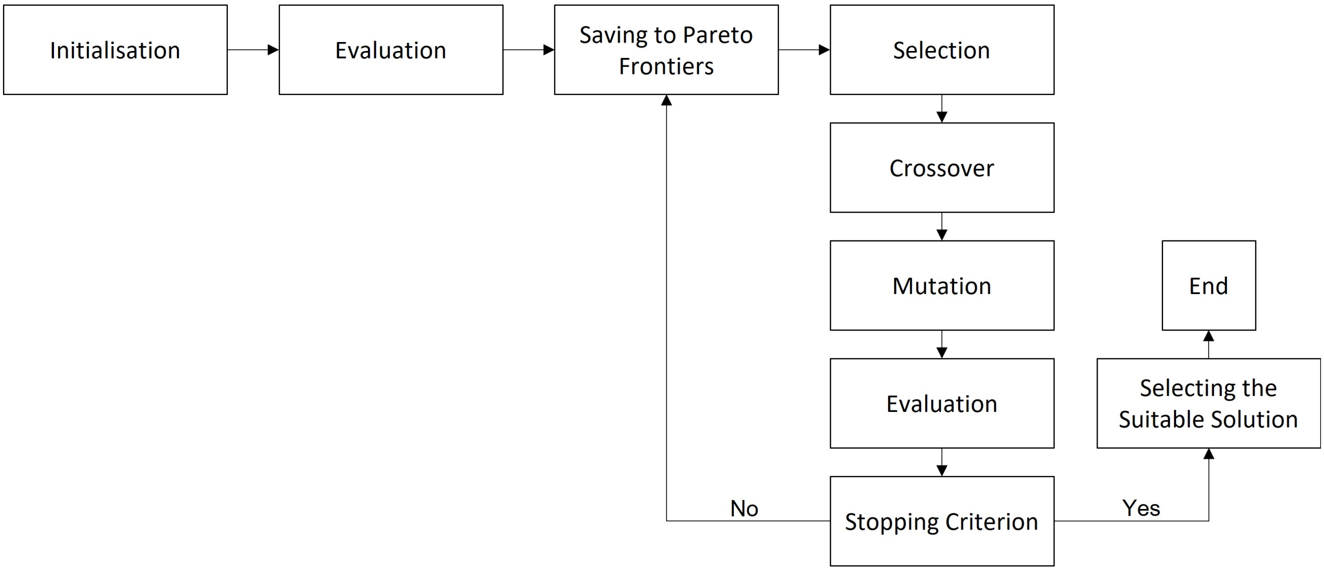

Figure 1. The integration of the proposed selection method into the NSGA-II framework

Each solution in the population is evaluated and saved in the Pareto frontiers regarding its performance on the objectives. The algorithm completes a single generation through the following steps: selection, crossover, and mutation. This loop continues until the stopping criterion occurs. The selection method intervenes when the algorithm stops and selects the most suitable solution among the multiple trade-off solutions obtained through the NSGA-II algorithm.

2.2. Finding the most suitable solution

This section includes a brief exposition of AHP, a presentation of the classification method, and an explanation of how it articulates with AHP to select the most suitable solution among the trade-off solutions.

2.2.1. Analytical hierarchy process

AHP, whose basic hierarchical structure is represented in Figure 2, is a well-established multi-criteria decision method. The focus or goal of decision-making is considered the first level. The second level comprises the objectives or criteria considered important for this goal. The DM compares each objective with each other regarding their relative importance. Finally, the last level is composed of the different alternatives to be evaluated against each objective separately. Level 2 and Level 3 comparisons eventually lead to the most convenient alternative selection. Thus, AHP is a method that assesses each alternative's suitability for the objectives' determined importance in a decision-making process defined by an expert or DM from the area [7, 23].

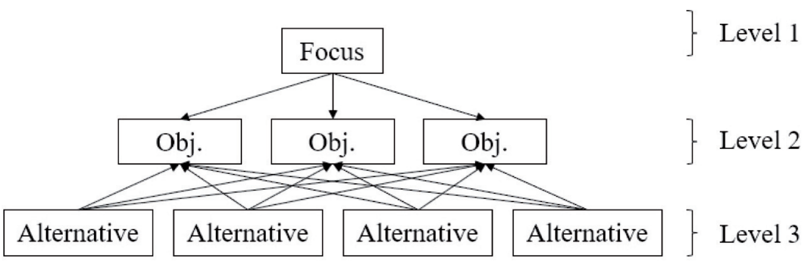

Figure 2. Levels of decision in AHP (adapted from [42])

The AHP method is essential to build the PCM for the objectives to determine the weights. That is a reciprocal matrix. The elements located in the diagonal are equal to one. Thus, $n(n-1)/2$ times comparison is required for n objectives. This comparison is practised according to the fundamental comparison scale by Saaty [44], presented in Table 1.

Table 1. The fundamental scale [42, p. 163]

Intensity value	Definition
1	equal importance
3	moderate importance of one over another
5	essential or strong importance
7	very strong importance
9	extreme importance
2, 4, 6, 8	intermediate values between the two adjacent

The fundamental scale helps to define the preferences and determine the most convenient solution regarding them. Defining the preferences indicates constructing the weights for each objective included in the problem. Besides, determining the most suitable solution implies selecting by evaluating each solution of the final solutions set, regarding the defined preferences.

The following is a step-by-step explanation of AHP. The first step consists of constructing the weighting scheme for each objective regarding the preferences (i.e., Level 2 comparison). The expression below represents the PCMs for N objectives.

$$\mathrm{PCM}_{\mathrm{obj.}} = [c_{ij}]_{i,j=1,2,3,\ldots,N}$$

$$c_{ij} = \begin{cases} 1 & \text{if } i = j \\ \dfrac{1}{c_{ji}} & \text{if } i \neq j \end{cases}$$

The above expression represents the PCM for the number of objectives. $PCM_{obj.}$ is a $N \times N$ matrix. c_{ij} shows the value that each comparison takes from the fundamental comparison scale. This matrix is normalised to construct the weights for each objective. The normalisation is performed by dividing each element of each column by the total sum of that column.

$$\mathrm{PCM}^{\mathrm{norm}}_{\mathrm{obj.}} = [c^{\mathrm{norm}}_{ij}]_{i,j=1,2,3,\ldots,N}$$

where

$$c^{\mathrm{norm}}_{ij} = \frac{c_{ij}}{\sum_{i=1}^{N} c_{ij}} \; \forall j = 1, \ldots, N$$

Weights are constructed by the average of each row using the normalised pairwise comparisons. The expression that represents weights can be followed below.

$$W = [w_{i1}]_{i=1,2,3,\ldots,N}$$

$$w_{i1} = \frac{\sum_{j=1}^{N} c_{ij}^{\text{norm}}}{N}$$

W is a $N \times 1$ matrix. The weights constructed in this step are used after the final evaluation of alternatives in AHP.

The second step is to evaluate each alternative for each objective separately (i.e., Level 3 evaluations).

$$PCM_n = [c_{kp}]_{k,\, p=1,\, 2,\, 3,\, \ldots,\, K}$$

$$c_{kp} = \begin{cases} 1 & \text{if } k = p \\ \dfrac{1}{c_{pk}} & \text{if } k \neq p \end{cases}$$

PCM_n is the PCM for an alternative where it is evaluated for objective N, in size of $K \times K$. Using PCM_n, one can calculate the score of each alternative for a specific objective in the same way as the weights were calculated. All the columns of the matrix are summed separately. Then, each element of the column is divided with its sum to normalise, as seen below.

$$PCM_n^{\text{norm}} = [c_{kp}^{\text{norm}}]_{k,\, p=1,\, 2,\, 3,\, \ldots,\, K}$$

where

$$c_{kp}^{\text{norm}} = \frac{c_{kp}}{\sum_{k=1}^{N} c_{kp}} \; \forall p = 1,\, \ldots,\, K$$

Finally, we take the average of each row to obtain the scores.

$$S_n = [\zeta_{k1}]_{k=1,\, 2,\, 3,\, \ldots,\, K}$$

$$\zeta_{k1} = \frac{\sum_{p=1}^{N} c_{kp}^{\text{norm}}}{K}$$

S_N is a $K \times 1$ matrix for a single-objective. Given that this process is followed for all objectives, the scores matrix (S) can be represented as below:

$$S = [S_1 S_2 \cdots S_N]$$

S is in the size of $K \times N$, where K is the number of alternatives (e.g., trade-off solutions) and N is the number of objectives.

The last step of AHP consists of the multiplication of the score matrix and weights extracted for each objective in the first step. This final step leads to picking the best solution among all solutions according to their scores.

$$\text{Performance} = S \times W$$

The solution with the highest performance value is selected.

2.2.2. The classification method

The proposed classification method aims at re-scaling several trade-off solutions found by the multi-objective optimisation into the fundamental comparison scale presented in Table 1. Such classification will cluster similar solutions into the same fundamental scale division, leading to equal importance. Thus, only the solutions with a significant difference will be exposed. The classification method performs level 3 evaluations where each Pareto front solution is evaluated for each objective separately. Such a method is important since it automates the level 3 comparisons which are often very complicated for the DMs given a large number of Pareto optimal solutions and the numeric projection of these solutions in each objective.

The below expression represents the performance of each solution for each objective:

$$f_N(x) = [s_k^n]$$

where $n = 1, 2, 3, \ldots, N$ (N is the number of objectives) and $k = 1, 2, 3, \ldots, K$ (K is the number of trade-off solutions). Thus, s_k^n represents the image of the solution k of objective n. $f_N(x)$ is a list of final results for each objective considered.

Considering the Pareto solutions as different alternatives, N separate matrices (for N objectives) in the size of the number of trade-off solutions (K) are required. To create these PCMs, we first need to convert the numerical values of the solutions to the fundamental comparison scale values presented in Table 1. It is possible to monitor all possible comparison values in the list below.

$$\text{Comparison list} = 1/9, 1/8, 1/7, 1/6, 1/5, 1/4, 1/3, 1/2, 1, 2, 3, 4, 5, 6, 7, 8, 9$$

To modify the numerical values of each solution in terms of that list, we first created a difference matrix for each objective. In this matrix, we subtracted the values of the solutions from each other. The matrix aims to understand which solution is superior to the other. The expression representing the differences between the solutions for each objective is as follows:

$$O_n^{\text{diff}} = [e_{kp}^n]_{k,\,p=1,\,2,\,3,\,\ldots,\,K}$$

$$e_{kp}^n = s_k^n - s_p^n$$

O_n^{diff} is the difference matrix in the size of $K \times K$ where K is the total number of trade-off solutions. e_{kp}^n is the difference between the values of the two solutions for objective n.

The comparison list includes 17 classes. To evaluate solutions, we established a range for the classification by looking at all the frontiers and detecting the lowest and highest values in the solution space. Let $V_{\max}$ be the highest value for objective n. This value belongs to the worst solution for objective n in the solution space for a minimisation problem. Besides, the lowest value, which belongs to the best solution for objective n, is $V_{\min}$ which must be valid in the Pareto frontier set.

In light of this information, the classification range (R) is built using the following equation

$$R = \frac{V_{\max} - V_{\min}}{9} \tag{2}$$

The value 9 in the denominator is the length of the comparison list plus 1, divided by 2.

When calculating the range dividing the difference between the maximum and minimum by 9, we create 9 equal divisions to rescale the values of the solution in each objective between 1 and 9 or 1 and $1/9$. Since all solutions are subtracted from each other, the rescaling process should also be between differences rather than the real values, namely, either between zero and V_{diff} or zero and $-V_{\text{diff}}$, where $V_{\text{diff}} = V_{\max} - V_{\min}$.

It is possible to represent the rescaling process for positive and negative cases

$$\text{Rescale} = \begin{cases} (-V_{\text{diff}} + R \times (9-\mu); -V_{\text{diff}} + R \times (10-\mu)) & \text{where } \mu = 1, 2, \ldots, 9 \\ \left(V_{\text{diff}} - R \times (9-\mu); V_{\text{diff}} - R \times \left(10-\mu\right)\right) & \end{cases}$$

These positive and negative classifications provide an interval to assign the values in the difference matrix of the objectives within the comparison list. Every possible value in the comparison list takes an equal interval except for the interval of 1. Figure 3 describes this situation.

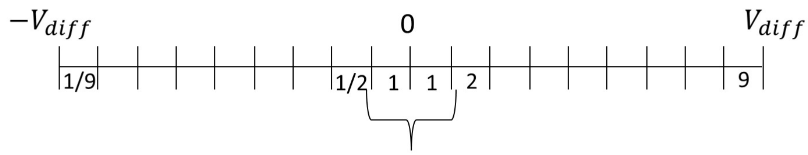

Figure 3. Interval for re-scaling procedure

The repetition in the interval of value 1 represents equal importance because the values with a minimal difference are considered equal. Following these classifications, the PCMs can be built separately for each objective. This method also provides a classification of solutions, from most to least convenient, given the assigned weights.

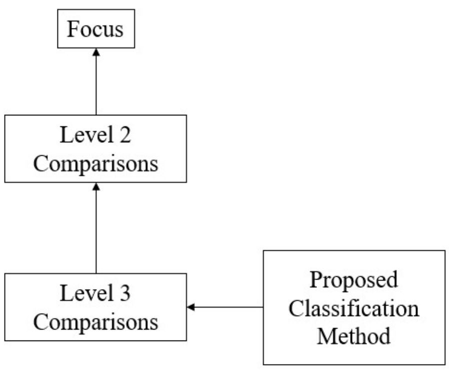

Figure 4. Nexus between AHP and the proposed classification method

2.2.3. AHP and the classification method

Given the explanations in Sections 2.2.1 and 2.2.2, the classification method helps with level 3 comparisons when several Pareto optimal solutions are in question. Figure 4 depicts where the proposed method intervenes to create a standardised classification of solutions that differ significantly.

3. Application to sectorization

Sectorization problems are diverse due to the vast fields of application. Moreover, almost every application area considers different objectives or definitions of the same objective [4, 17, 22, 31, 36, 39]. This work focused on the three most commonly used objectives in sectorization problems: equilibrium, compactness, and contiguity.

Equilibrium is an essential objective since various sectorization problems may consider it in different forms. For instance, balance in workload or work hours, equal demand or number of clients, similar distances or racial balance are all based on equilibrium in diverse concepts [3, 9, 21, 37]. Moreover, although there is no universally accepted definition, compactness is usually about creating well-shaped sectors. Several fields, namely, land-use changes, geography, waste collection, regionalisation, natural conservation or microeconomic modelling, care about compactness factor [2, 32, 38]. Finally, contiguity can be described as having a path between the two vertices in a particular zone without passing from another zone [45]. That is a common objective when routing is also in question besides sectorization.

The remainder of the section provides the specific definition of each objective, and the datasets used to test the proposed method.

3.1. Objectives

3.1.1. Equilibrium

Equilibrium aims to generate a balance or similarity between the sectors in terms of quantity, workload or demand. The measure we used for equilibrium is adopted from Rodrigues and Ferreira [40] .

$$\bar{q} = \frac{\sum_{j=1}^{K} q_j}{K} \tag{3}$$

where

$$q_j = \sum_i x_{ij} \times q_i, \forall j \text{ and } \mathrm{std}'_{\mathrm{eq}} = \sqrt{\frac{1}{K-1}\sum_{j=1}^{K}(q_j - \bar{q})^2}$$

Here, q_j is the total quantity in sector j where x_{ij} is the basic unit i with quantity q_i located in sector j. $\bar{q}$ represents the mean quantity for the number of sectors K. Then, this value is used to construct the standard deviation ($\mathrm{std}'_{\mathrm{eq}}$) from the mean value in each sector. When the standard deviation is higher, the unbalance between the sectors is larger. Thus, it is desirable to minimise the equilibrium values for the sectorization problems.

3.1.2. Compactness

Compactness represents the density within the sectors. More compact sectors are commonly more desirable since it avoids inadequate sectorization. The following equation represents the measure used for this objective

$$d = \sum_{j=1}^{K} \mathrm{dist}(o_j, x_{.j}^F) \tag{4}$$

In Equation (4), o_j shows the mass centroid of the sector j, and $x^F_{\cdot j}$ is the furthest basic unit to the centroid. When the d is larger, the compactness of the sectors is worse. Thus, the objective is for minimisation.

3.1.3. Contiguity

Contiguity means the connections between the elements located in the same sector. It stands for mobility without leaving the current sector to travel between the units located in that sector.

It is possible to represent this situation using the matrix M^j. This is a symmetric square matrix and takes the value of 1 if there is a link (direct or indirect) between the two points and zero otherwise.

$$M^j = \begin{pmatrix} 0 & m^j_{12} & \cdots & m^j_{1n} \\ m^j_{21} & 0 & \cdots & m^j_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ m^j_{n1} & m^j_{n2} & \cdots & 0 \end{pmatrix}$$

$$m^j_{wi} = \begin{cases} 1 & \text{if there is a path between point } w \text{ and point } i \text{ in sector } j \\ 0 & \text{otherwise} \end{cases}$$

Equation (5) shows how the contiguity is calculated. Here, the numerator counts the existing paths in sector j. The n_j represents the total number of basic units $\left(\sum_{i=1} x_{ij}\right)$ in sector j. Thus, the denominator shows the the maximum number of paths in sector j. c_j takes the value of 1 when numerator is equal to denominator or when all the points are connected.

$$c_j = \frac{\sum_{w=1}^{n_j} \sum_{i=1}^{n_j} m^j_{wi}}{n_j(n_j - 1)} \tag{5}$$

Then, we use this information to calculate the mean contiguity by dividing it by the total number of basic units $N = \sum_{j=1} n_j = \sum_{j=1} \sum_{i=1} x_{ij}$ after multiplying it by the number of basic units in sector j .

$$\bar{c} = \frac{\sum_{j=1}^{K} c_j \times n_j}{N} \tag{6}$$

$\bar{c}$ takes the value between 0 and 1. The higher value shows better connectivity in the sectors. We consider contiguity also as minimisation by subtracting it by one $(1 - \bar{c})$.

3.2. Data

3.2.1. Instances created for sectorization

We created instances and made them available to test the classification method on sectorization problems. The reason for that is the data scarcity in the literature on sectorization or similar problems. It is possible to find these instances in the link here. This link includes three files: (i) contiguity maps, (ii) coordinates, and (iii) quantity. Each of these files provides information for 100 instances created with a size of a minimum 25 and a maximum 1000 nodes. For comparative reasons, a real-world instance is around 8,000 up to 16,000 nodes.

Figure 5 represents an example instance, which can be found in the above link as Gamma 5. More precisely, the figure represents basic units (e.g., pick-up or delivery locations, demand points, etc.) in an area with different demands (symbolised by circle points with different radii proportional to their specific demand) and their connections (between each pair of points symbolised by the links).

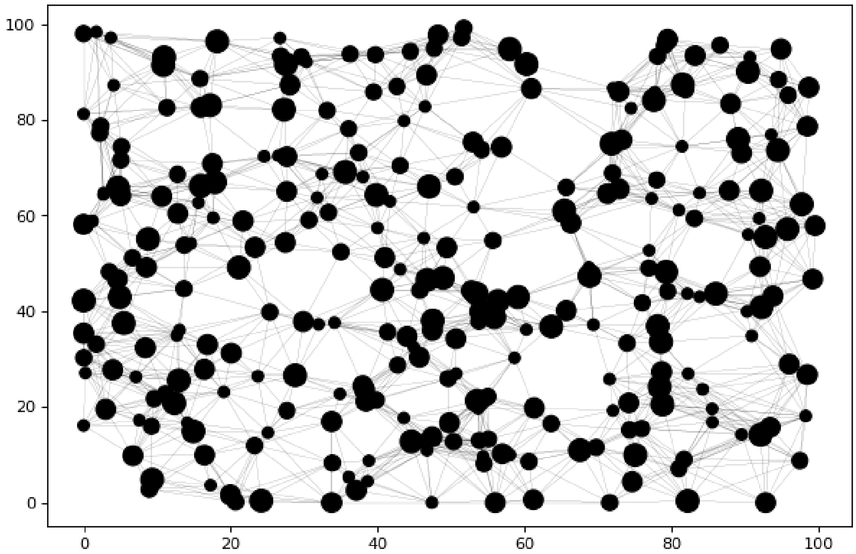

Figure 5. A visual representation of a test instance

The contiguity maps are in the form of a matrix. These matrices are square, symmetric, and binary, where an entrance takes the value of one if there is a link between two nodes. The coordinates are designed to provide latitude and longitude information for each node. We adopted gamma distribution to generate half of the instances and normal distribution for the other half. In the first half of the instances, each instance is generated by a gamma distribution with different parameters of shape and scale, where the respective group of nodes are points of that distribution. In the second half of the instances, each instance is generated by a normal distribution with different mean and variance, where the respective group of nodes are points of that distribution. Finally, quantities are generated using the uniform distribution for each node in each instance.

As already mentioned, we also used real data to test the proposal's validity in addition to the designed test instances. By doing so, we intend to make a significant contribution to the resolution of multi-objective problems, specifically focusing on sectorization. In this way, we hope to contribute to resolving multi-criteria problems and, in particular, sectorization problems.

3.2.2. Data from a delivery company

The confidential real data was taken from a large Portuguese pick-up and delivery company in the district of Porto. The points are filtered to the subset of points consisting of daily demand for pick-up operations. Demand for each point represented in Figure 6 is considered one due to insufficient data. The original dataset is simplified due to its size to get results in an adequate time. That made the instance simpler to address while preserving its original characteristics. The original dataset was 15 times larger. The distances between the points are Euclidean distances and the links are generated according to Delaunay triangulation.

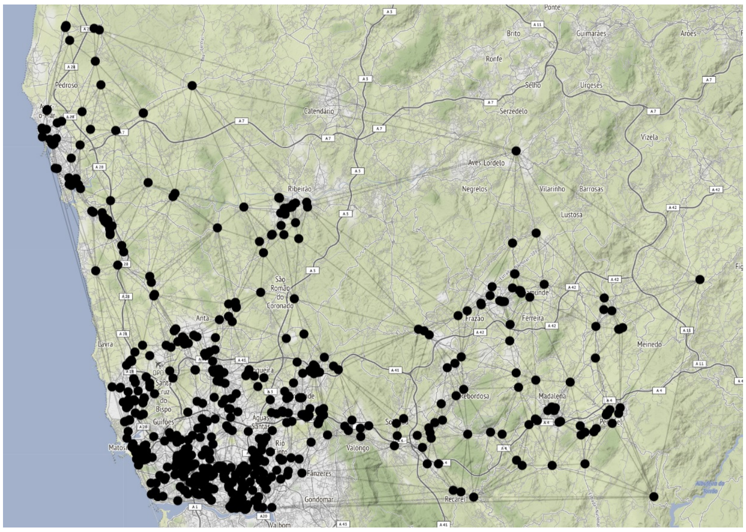

Figure 6. A visual representation of a real instance from Porto, Portugal

4. Results and discussion

The main concern of this section is verifying how the selection method behaves in practice. It starts with some experimental results implemented in the artificially created instances, and then it presents a discussion and a comparative analysis to examine the method's quality using real-world data.

4.1. Experimental results

To select an instance that is more visually descriptive and of reasonable complexity to show the performance of the proposed method, a sectorization problem with 873 nodes is considered. This instance is divided into 30 sectors, which can be found in the link presented in Section 3.1.1 as Gamma 8.

An example is presented below to observe how the selection method picks a solution and ranks all the solutions in the Pareto frontier set. First, we introduce the weights assigned to the three objectives, according to the pairwise comparisons. In this example, equilibrium is considered two times more preferred than compactness and contiguity. Moreover, contiguity and compactness are considered equally important. Following these comparisons, the weighting scheme of the three objectives is calculated through AHP and represented in Table 2.

As mentioned, NSGA-II is used to obtain Pareto frontier solutions. The population number is selected as 50 for three objectives to keep the complexity manageable. The mutation rate is chosen as 0.05 and implemented only in the offspring population every three generations. The algorithm stopped in 100th generation according to the stopping criterion. We follow the NSGA-II specific stopping criterion presented by Rudenko and Schoenauer [41][1].

[1] For stopping criterion see Rudenko and Schoenauer [41].

Table 2. Weighting scheme of the three objectives (calculated through AHP)

	Equilibrium	Compactness	Contiguity
Weight	0.5	0.25	0.25

Eight solutions appeared in the Pareto frontier. Table 3 represents the numeric values of these solutions for each objective. Moreover, given the preference factors assigned for each objective (i.e. weights), the suitability of each solution is evaluated following the proposed selection method. Thus, the performance and the ranking of each solution regarding their suitability are presented in the table.

Table 3. Pareto frontier solutions and their suitability

Solution	Equilibrium	Compactness	Contiguity	Performance	Ranking
1	6.354	450.848	0.006	0.1037	5
2	5.461	456.024	0.0	0.1148	3
3	5.892	453.404	0.032	0.1074	4
4	2.385	453.945	0.035	0.2185	**1**
5	3.0	454.205	0.004	0.2023	2
6	7.411	452.341	0.006	0.0848	6
7	8.049	442.214	0.009	0.0833	7
8	7.663	439.866	0.012	0.0848	6

Figure 7 represents these Pareto frontier solutions. The blue point is the selected solution.

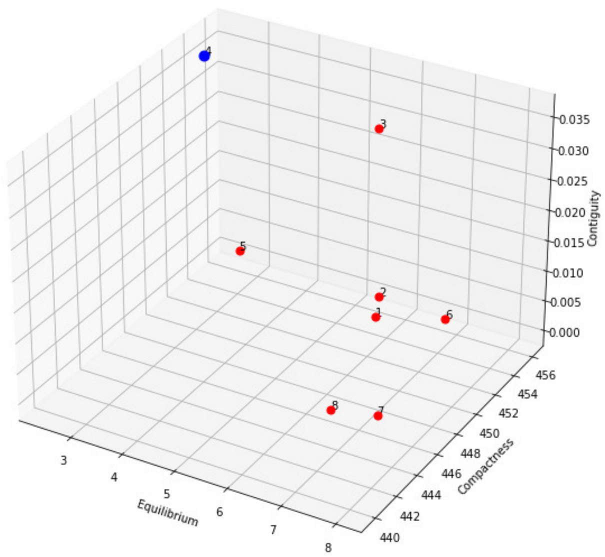

Figure 7. The representation of the trade-off solutions

The projection of the selected solution is in Figure 8. After obtaining the Pareto frontier solutions, the selection procedure took 2049 microseconds.

We present some results regarding the computational time of the proposed selection method in Table 4. Here, the first, second and third columns show the size of the test instance, the number of final solutions in the Pareto front, and the computational time of the selection method in microseconds, respectively. As is seen, the computation time depends on the number of solutions that appeared on the Pareto frontier.

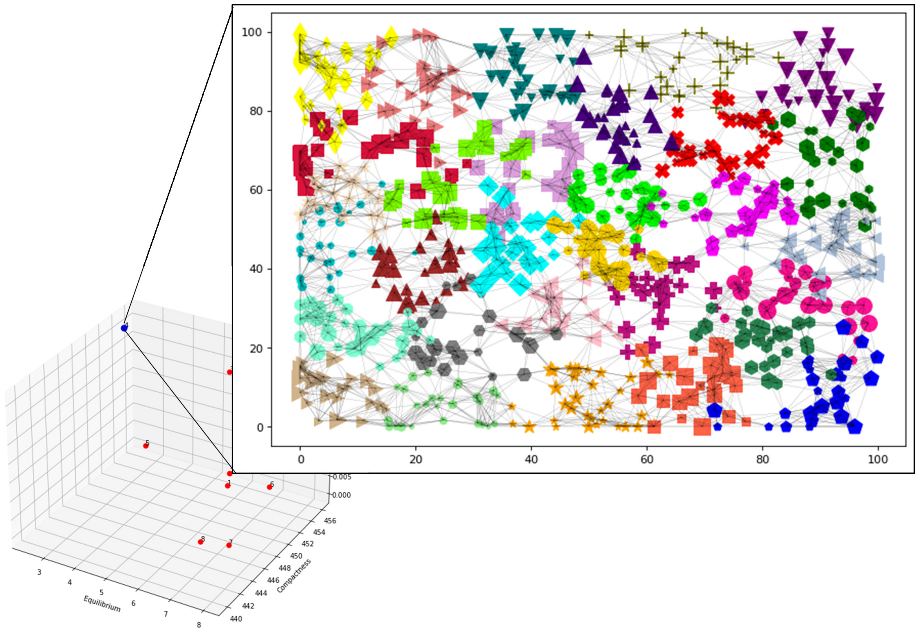

Figure 8. The projection of the selected solution

Table 4. Computational time of the selection method in microseconds

Instance	No. of nodes	No. of trade-off Solutions	Selection time in [μs]
Gamma 3	56	50	13,963
Gamma 14	204	50	10,962
Gamma 28	350	50	10,966
Gamma 9	432	50	18,949
Gamma 18	528	50	12,963
Gamma 2	690	5	998
Gamma 8	873	8	2049
Gamma 49	1000	50	16,954

In all cases, the selection method can attain the most convenient solution in less than 1 s (or 10^6 μs). When the number of trade-off solutions is higher, the selection time is longer, as is expected. Thus, the experimental results show that the proposed selection method rapidly picks the most convenient solution and provides the ranking of the solutions regarding the DM's preferences.

4.2. Discussion and comparative analysis

When the weights are more guaranteed, following a composite single-objective function gives adequate results and avoids further effort necessary to select a trade-off solution or solutions [14]. Multi-objective optimisation evaluates each solution for each objective simultaneously and separately. Thus, when a given objective is more important than the others, it is likely that the results of multi-objective optimisation may not integrate some of the best possible solutions that can be obtained by single-objective optimisation.

Given this argumentation, we evaluate the proposed selection method's credibility by comparing the similarity between the selected solution and the solution obtained by the weighted single objective optimisation function. We use a genetic algorithm (GA) to get a single solution. The fitness value of each solution is calculated according to their performance in the weighted composite single-objective function, and solutions are evaluated regarding their fitness values over generations.

To make the solutions comparable, we use the same population, crossover, mutation, and same number of generations as a stopping criterion for both GA and NSGA-II. In the single-objective method, values for the objective are normalised before being composed in the fitness function. We used min-max normalisation to build the composite single-objective function. On the other hand, no normalisation is applied in multi-objective optimisation.

As mentioned earlier in this chapter, the comparative analysis of the similarity between the selected multi-objective solution and the single-objective solution is conducted with a real-life application using the data from a delivery company. We considered two different scenarios for testing the similarity between the two solutions. In the first scenario, the highest priority was equilibrium, followed by contiguity and compactness. On the other hand, contiguity was considered the most desired objective in the second scenario, where lower importance was given to equilibrium and compactness. For both scenarios, a notable similarity is observed between the selected multi-objective solution and the single solution obtained through GA.

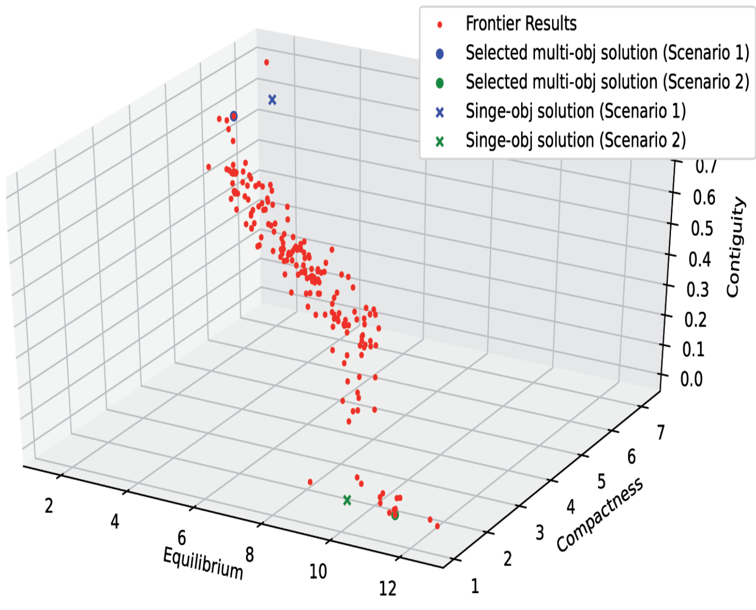

Figure 9. Pareto frontier, selected multi-objective and single-objective solutions

We present the Pareto frontier solutions, selected multi-objective solutions under both scenarios and single-objective solutions in Figure 9. The red points represent solutions on the Pareto front when the stopping criterion occurs in the NSGA-II. A blue point represents the selected multi-objective solution from the Pareto optimal set under the first scenario, where equilibrium carries the highest importance. The green point stands for the selected multi-objective solution under the second scenario, where contiguity is the most important. The blue and green x's point out the two scenarios' single-objective solutions found by GA. This figure clearly shows that the proposed selection method works well and can find the most suitable solution for the preferences.

The projections of the selected multi-objective solution and the single-objective solution are presented in Figures 10 and 11 for the first and the second scenarios, respectively. Figure 10 first represents the selected weights for three objectives. Then, the parameters and the performance of the solutions for the three objectives are introduced. Due to the data complexity, a large population with 200 solutions are

Scenario	W_{eq}	W_{comp}	W_{cont}
1	0.63	0.07	0.30

Node	Pop_{size}	Gen	Sector	P_{mut}	Type	Eq.	Comp	Cont	μ[s]
660	200	500	30	0.05	multi	1.681	6.138	0.696	403 622
			30		single	1.761	7.201	0.679	

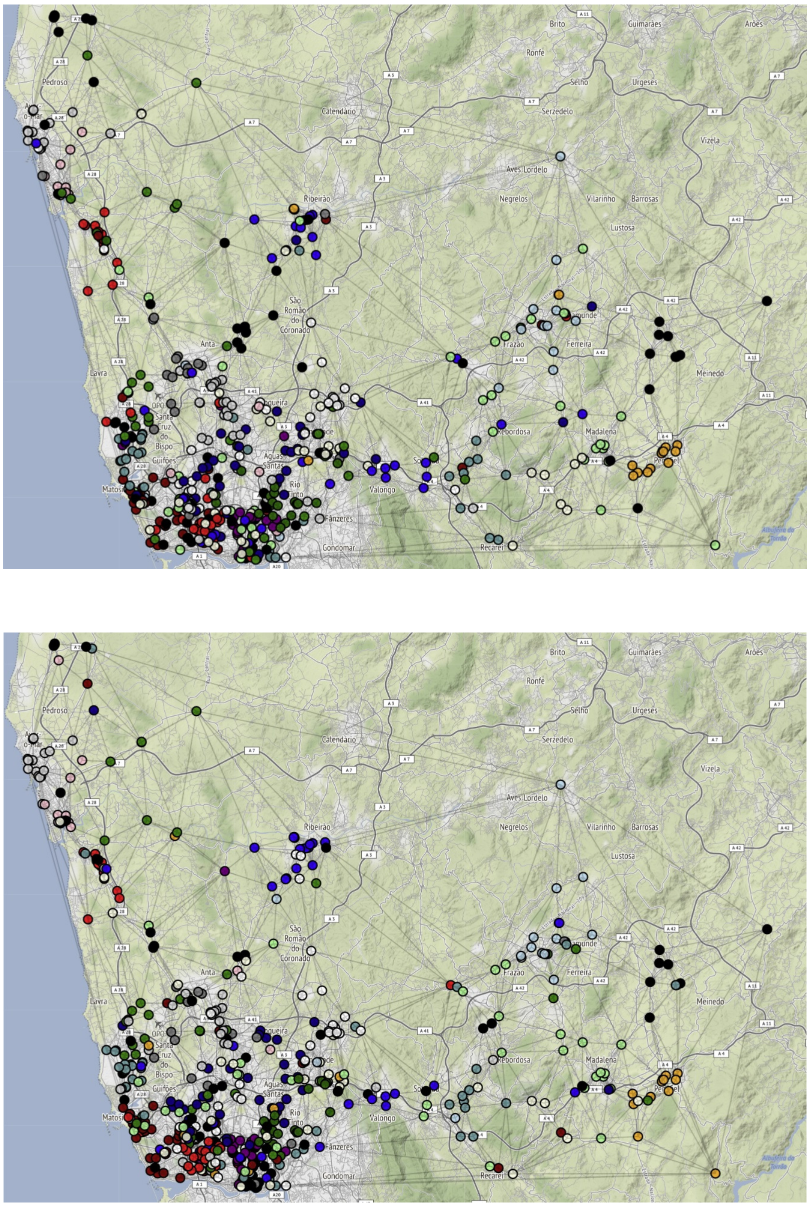

Figure 10. Projection of the selected multi-objective and single-objective solutions under first scenario: a) upper picture – selected multi-objective solution, b) lower picture – single-objective solution

used to keep the diversity in the solution space. The number of generations is kept at 500 as a large population compensates for the smaller number of generations.

The mutation probability is kept at 0.05 and implemented only on the off-springs in every three generations. It appears that the performance of the selected multi-objective solution and the single-objective solution for the three objectives are very similar. The last column shows the selection time from the Pareto frontier solutions. The projection of the two solutions is presented in Figures 10a, and 10b. As is seen, the sectors are not dense or compact. However, these are good solutions in terms of equilibrium in each sector.

Scenario	W_{eq}	W_{comp}	W_{cont}
2	0.25	0.15	0.60

Node	Pop_{size}	Gen	Sector	P_{mut}	Type	Eq	Comp	Cont	μ [s]
660	200	500	30	0.05	Multi	11.436	1.121	0.026	392 297
			30		Single	9.812	1.369	0.009	

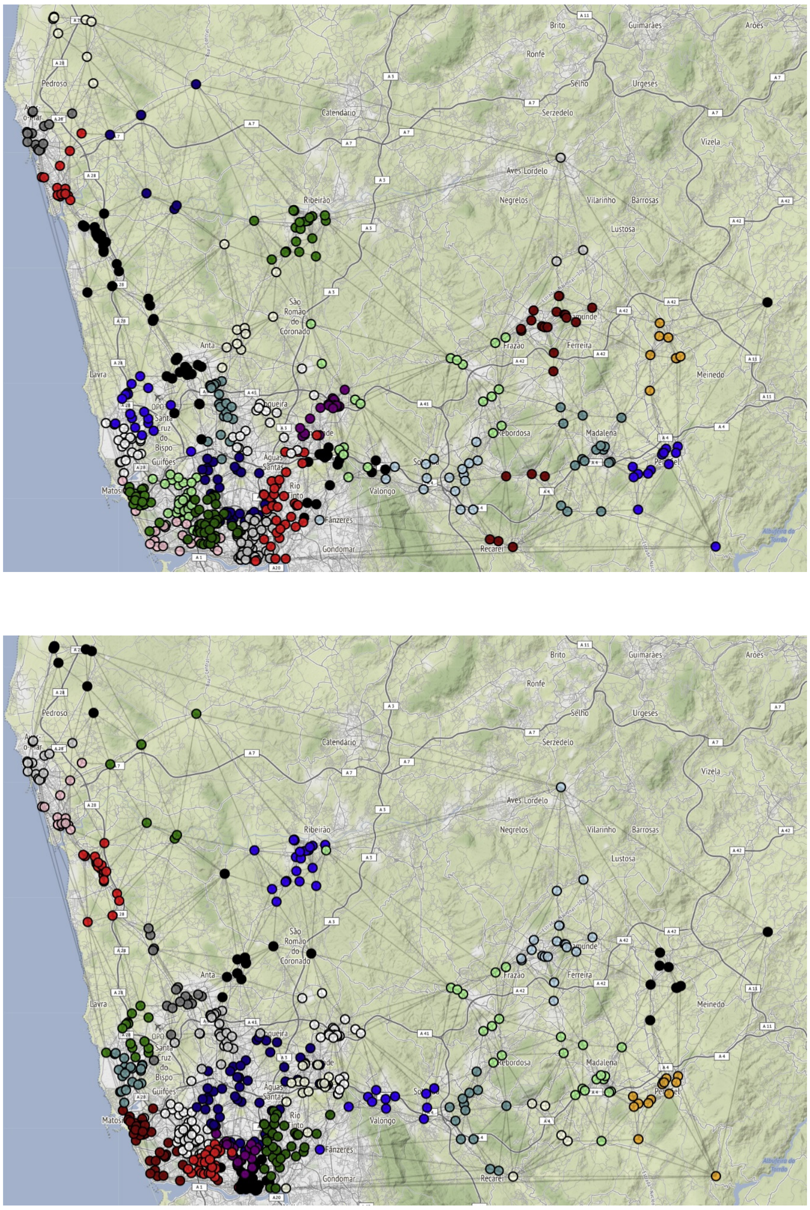

Figure 11. Projection of the selected multi-objective and single-objective solutions under the second scenario a) upper picture – selected multi-objective solution, b) lower picture – single-objective solution

Figure 11 also shows the weights considered under scenario two in the first place. Then the parameters and the performance of the algorithms are represented. As we used the same Pareto frontier solutions, the parameters are the same as in scenario one. The performance of the solution methods for the three objectives is again similar. However, it is possible to see the superiority of single-objective solution on equilibrium and contiguity over the selected multi-objective solution. The two solutions are projected in Figures 11a and 11b. Both solutions look very well shaped and compact regarding the preferences considered under the second scenario.

In some cases, Pareto front results may not be able to catch the performance of single-objective optimisation. The results from single-objective optimisation may end up being better since the algorithm searches the population and evaluates the solutions considering the predefined preference factors (or weights) over generations. Thus, the DM should only consider the MOO methods if the preferences regarding the objectives are vague. It is also important to mention that although the results presented in the current section are for three objectives, the proposed selection method does not have specific restrictions regarding the number of objectives included in the analysis. All the experiments in this section are executed using Python 3.7 in PC Intel Core i7-8550U at 1.8 GHz and Win X64 operating system.

5. Conclusions

Sectorization refers to dividing a whole into smaller parts, considering different objectives. Sectorization problems appear in many real-life situations, such as sales territories, delivery and pick-up operations, maintenance services, schooling, or health services. Multi-objective optimisation methods are adequate to deal with Sectorization problems due to the complexity and the several objectives.

Following multi-objective optimisation, several Pareto optimal solutions can be obtained. This work contributes to choosing a suitable solution, as this is a crucial and complicated task.

In the current paper, we proposed a new classification method to select the most convenient solution among the multiple Pareto-optimal solutions. This classification method fits into the well-known AHP method at the level 3 comparisons, where the different alternatives are judged based on how well they meet the objectives. Each Pareto-optimal solution is classified according to its superiority in the proposed classification method. Such evaluation allowed us to build pairwise comparison matrices (PCMs). Afterwards, the AHP procedure is followed to select the most suitable solution.

The proposed classification method was applied and tested in sectorization problems. We created test instances and employed real data from a Portuguese delivery company. First, the Pareto-optimal set of solutions was obtained by NSGA-II considering three objectives: equilibrium, compactness, and contiguity. Then, the most suitable solution was selected using the proposed method.

According to the results, the selection method can quickly select the most suitable solution after obtaining the Pareto front. Moreover, the quality and credibility of the proposed method were verified. Assuming each objective has accurately given weights, we can resort to optimisation through a weighted composite single-objective function, providing an optimal solution. The credibility of the proposed method was thus evaluated by comparing the selected solution by the proposed method and the solution obtained by a single-objective function.

Since the Pareto-optimal set of solutions was obtained with NSGA-II, we used GA to get the single-objective solution. To ensure the comparability of the results, we used the same population and parameters for the two methods. Significantly similar results between the two solutions were reached. Selecting a solution among the multiple trade-off solutions is the most relevant and challenging task. This work is intended to contribute to multi-objective optimisation and, namely, to the solution of sectorization problems. The proposed methodology for selecting the most suitable solution is quick and easy to implement, reliable and useful, regardless of the number of objectives.

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