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Scenario planning as a new application area for TOPSIS

Helena Gaspars-Wieloch[1*]

[1]*Department of Operations Research and Mathematical Economics, Poznań University of Economics and Business, Poznań, Poland*
[*]*Email address: helena.gaspars@ue.poznan.pl*

Abstract

TOPSIS is a well-known approach applied to multi-criteria decision-making under certainty (M-DMC). However, recently, some analogies between this domain and scenario-based one-criterion decision-making under uncertainty (1-DMU) have been revealed in the literature. Thus, the similarities aforementioned give the possibility to adjust TOPSIS to another area. The goal of the paper is to create a new method for problems with non-deterministic parameters on the basis of TOPSIS ideas. In the suggested approach criteria weights (declared within TOPSIS) are replaced by subjective chances of occurrence which are estimated for each scenario. The novel method has an advantage over existing classical decision rules designed for 1-criterion decision-making under uncertainty since within this procedure each payoff connected with a given option is compared with the positive and negative-ideal solutions.

Keywords: *TOPSIS, scenario planning, uncertainty and certainty, one-criterion and multi-criteria decision-making, rankings*

1. Introduction

The aim of the paper was to develop a novel procedure for uncertain problems on the basis of TOPSIS ideas. TOPSIS has been already extended to the non-deterministic version (e.g., [29, 36]), however, in this article, the purpose is not to propose any TOPSIS extension but to apply the original TOPSIS method (designed for multi-criteria decision-making under certainty M-DMC) in an entirely new area – the scenario-based 1-criterion decision-making under uncertainty (1-DMU). For that topic, numerous decisions rules (e.g., Wald rule, Hurwicz rule, max-min joy criterion) have been formulated in the last 70 years, but using TOPSIS ideas in 1-DMU may contribute to creating a procedure that would better consider the decision-makers' preferences, needs and expectations. And that is why, the research described in this paper is significant for the decision-making process.

The first motivation to explore the topic above results from some analogies between M-DMC and 1-DMU which have been recently revealed in [10, 12–14]. Thanks to these similarities the use of the

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original TOPSIS algorithm in a totally new domain is possible and may facilitate the final choice under uncertainty. The analogies between both areas concern the structure of both problems and the possibility to search optimal both pure and mixed strategies. Differences between the issues aforementioned also exist, but they do not constitute an obstacle to the achievement of our goal. The second motivation is related to the attractiveness of scenario planning (SP) which is a frequent tool used in the decision-making process. It is very helpful when the decision-maker deals with issues under uncertainty. The ability to predict future economic events (e.g. sales forecasts) is undoubtedly crucial to the maintenance of successful business activities [1]. Durbach et al. [8] even prepared an impressive review of possible models, methods, and tools supporting uncertain decision-making and they get to the conclusion that uncertainties become increasingly so complex that such approaches as models with fuzzy numbers, models using probabilities, models with explicit risk measures become operationally difficult for decision-makers to comprehend and virtually impossible to validate. Therefore, they state that the construction of scenarios describing possible ways in which the future might unfold is a more appropriate way of dealing with uncertainty.

The research gap may be easily presented by means of a comparative analysis shown in Table 1. As it can be observed, for the numerous various possible assumptions made by the decision-maker (DM), there already exists a specific procedure designed for M-DMC and its equivalent method developed for 1-DMU. For instance, if the decision-maker intends to maximize the minimum payoff, he can apply the max-min approach (if he solves a multi-criteria problem under certainty) or the Wald rule (if he solves a one-criterion problem under uncertainty). On the other hand, if the decision-maker wants to consider subsequent criteria (or scenarios) according to a defined order connected with the importance (or chance of occurrence), interactive procedures have already been formulated for both issues. Nevertheless, if the DM aims to compare each alternative with the positive-ideal and the negative-ideal solution, it turns out that such assumptions may be taken into account only in the case of M-DMC, thanks to such methods as TOPSIS. For 1-DMU, these assumptions are also vital, but the existing decision rules do not give the opportunity to take them into consideration. This paper may fill the identified research gap by adapting TOPSIS ideas to one-criterion optimization under uncertainty.

Table 1. Analogical (similar) methods for two different areas

Assumption	M-DMC area	1-DMU area
The worst value is important	max-min method (maximization of the minimum normalized value)	Wald rule [33]
The best value is important	max-max method	max-max rule
All the values are important par (with equal or different weights)	SAW method	Bayes rule, expected value rule
Criteria/scenarios are analyzed sequentially	interactive programming	interactive decision rule [12]
The desired value is important	goal programming [4, 5]	target decision rule [10]
The reference points are important (positive and negative ideal solutions)	TOPSIS [17]	research gap – lack of decision rules for such assumptions

To sum up, the procedure presented in the paper is vital since it allows the DM to juxtapose each option with the positive-ideal and negative-ideal solutions assuming that the decision is made within 1-DMU, i.e., on the basis of one criterion and scenario planning. Such an approach has not been developed before. The possibility to create such a method has occurred very recently thanks to some analogies discovered between M-DMC and 1-DMU.

The rest of the article is organized as follows. Section 2 reminds the main features of TOPSIS. Section 3 describes M-DMC and 1-DMU. It also discusses the analogies and differences between both areas. Section 4 contains the description of a new procedure designed for 1-DMU and based on TOPSIS. Section 5 illustrates the novel approach by means of an example. Conclusions are gathered and analyzed in the last section.

2. What are the main features of TOPSIS?

TOPSIS is one of the approaches applied to multi-criteria decision-making under certainty. This type of optimization is related to the situation where the decision-maker evaluates particular options (alternatives, decision variants, courses of action) on the basis of more than one criterion. Criteria are usually conflicting and that is why the choice is not simple in many cases. Within M-DMC two situations are explored: the discrete and continuous optimization. In the first case, the number of decision variants is known at the beginning of the decision-making process. The discrete version leads to determining the optimal (compromise) pure strategy (only one option is selected and executed). In the second case, the DM knows only the objective function and constraints which define the set of feasible solutions. The continuous version leads to setting the optimal (compromise) mixed strategy (i.e., a weighted combination of pure strategies). Nevertheless, TOPSIS can be only applied to the discrete M-DMC.

TOPSIS was originally developed by [17], but numerous further modifications and extensions are broadly presented in the literature (e.g. [18, 26, 35]).

In this paper, the following TOPSIS version will be explored:

1. Define the set of alternatives $A = \{A_1, \ldots, A_j, \ldots, A_n\}$, where n is the number of options.
2. Define the set of criteria $C = \{C_1, \ldots, C_k, \ldots, C_p\}$, where p denotes the number of criteria.
3. Estimate the outcomes for each pair (option/criterion): $b_{k,j}$ – the performance of criterion C_k if variant A_j is selected. Create a payoff matrix $(b_{k,j})_{p \times n}$ (Table 2).
4. Define the target (t_k) for each criterion. If a given target value is intermediate (i.e., lower than the maximal performance, but higher than the minimal performance), the criterion connected with this goal is considered as neutral [10]. In the remaining cases, criteria are maximized or minimized.
5. Declare the weight (w_k) for each criterion. If the objectives are equivalent, the weights are equal. The sum of weights should be equal to 1.
6. Normalize values in the payoff matrix by means of a desired normalizing scheme: $(b(n)_{k,j})_{p \times n}$

$$b(n)_{k,j} = \frac{b_{k,j} - \min_j b_{k,j}}{\max_j b_{k,j} - \min_j b_{k,j}} \tag{1}$$

$$b(n)_{k,j} = \frac{\max_j b_{k,j} - b_{k,j}}{\max_j b_{k,j} - \min_j b_{k,j}} \tag{2}$$

$$b(n)_{k,j} = \begin{cases} 1 & \text{if} \quad b_{k,j} \in [d_k^{\min}, d_k^{\max}] \\ \dfrac{\max_j b_{k,j} - b_{k,j}}{\max_j b_{k,j} - d_k^{\max}} & \text{if} \quad b_{k,j} > d_k^{\max} \\ \dfrac{b_{k,j} - \min_j b_{k,j}}{d_k^{\min} - \min_j b_{k,j}} & \text{if} \quad b_{k,j} < d_k^{\min} \end{cases} \tag{3}$$

where $d_k^{\min}$ and $d_k^{\max}$ denote the endpoints of the interval of the desired values connected with criterion C_k. Equation (1) is useful for maximized criteria, equation (2) is designed for minimized criteria and the last one can be applied to neutral objectives.

7. Generate the weighted normalized decision matrix: $(t_{k,j})_{p \times n}$ where $t_{k,j} = w_k b(n)_{k,j}$.
8. Determine the worst alternative (the negative-ideal solution – NIS) and the best alternative (the positive-ideal solution – PIS)

$$\begin{aligned} \text{NIS} = \{ & \langle \max(t_{k,j}) | j = 1, \ldots, n | k \in J_- \rangle, \\ & \langle \min(t_{k,j}) | j = 1, \ldots, n | k \in J_+ \rangle \} = \{ t_{k,\text{NIS}} | k = 1, \ldots, p \} \end{aligned} \tag{4}$$

$$\begin{aligned} \text{PIS} = \{ & \langle \min(t_{k,j}) \,|\, j = 1, \ldots, n | k \in J_- \rangle, \\ & \langle \max(t_{k,j}) | j = 1, \ldots, n | k \in J_+ \rangle \} = \{ t_{k,\text{PIS}} | k = 1, \ldots, p \} \end{aligned} \tag{5}$$

where J_- and J_+ denote the sets of minimized and maximized criteria, respectively.

9. Compute the distance between particular alternatives and NIS

$$d_{\text{NIS},j} = \sqrt{\sum_{k=1}^{p} (t_{k,j} - t_{k,\text{NIS}})^2}, \quad j = 1, \ldots, n \tag{6}$$

10. Compute the distance between particular alternatives and PIS

$$d_{\text{PIS},j} = \sqrt{\sum_{k=1}^{p} (t_{k,j} - t_{k,\text{PIS}})^2}, \quad j = 1, \ldots, n \tag{7}$$

11. Calculate the similarity to the worst solution where $s_{\text{NIS},j} \in [0, 1]$

$$s_{\text{NIS},j} = \frac{d_{\text{NIS},j}}{d_{\text{NIS},j} + d_{\text{PIS},j}}, \quad j = 1, \ldots, n \tag{8}$$

12. Rank the options according to $s_{\text{NIS},j}$ from the largest value to the smallest.

As is seen, the selected option should have the shortest geometric distance from the positive-ideal solution and the longest geometric distance from the negative-ideal solution (Step 12). PIS is the solution that maximizes the benefit criteria and minimizes the cost criteria whereas NIS maximizes the cost criteria and minimizes the benefit criteria [24].

In the described algorithm, it is assumed that all the decision data are represented by crisp numbers. TOPSIS extensions [36] consider *interval or fuzzy criteria and interval or fuzzy weights to model imprecision, uncertainty, lack of information or vagueness* [24]. TOPSIS and its extensions are often applied to diverse areas [3, 19, 23, 27–29].

3. What do M-DMC and 1-DMU have in common?

As was already mentioned, the structure of multi-criteria optimization under certainty is extremely similar to the structure of 1-criterion optimization under uncertainty. Table 2 represents M-DMC with the set of alternatives, the set of criteria and payoffs for each pair: alternative/criterion. Table 3 refers to 1-DMU. The second table indicates the set of alternatives, the set of scenarios $S = \{S_1, \ldots, S_i, \ldots, S_m\}$ where m is the number of scenarios, and payoffs for each pair alternative/scenario $a_{i,j}$ denotes the outcome if variant A_j is selected and scenario S_i occurs.

Table 2. Payoff matrix for the discrete M-DMC [10, 13]

Criterion	Alternative				
	A_1	$\cdots$	A_j	$\cdots$	A_n
C_1	$b_{1,1}$	$\cdots$	$b_{1,j}$	$\cdots$	$b_{1,n}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
C_k	$b_{k,1}$	$\vdots$	$b_{k,j}$	$\vdots$	$b_{k,n}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
C_p	$b_{p,1}$	$\cdots$	$b_{p,j}$	$\cdots$	$b_{p,n}$

Table 3. Payoff matrix for the scenario-based 1-DMU [10, 13]

Scenario	Alternative				
	A_1	$\cdots$	A_j	$\cdots$	A_n
S_1	$a_{1,1}$	$\cdots$	$a_{1,j}$	$\cdots$	$a_{1,n}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
S_i	$a_{i,1}$	$\vdots$	$a_{i,j}$	$\vdots$	$a_{i,n}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
S_m	$a_{m,1}$	$\cdots$	$a_{m,j}$	$\cdots$	$a_{m,n}$

The first analogy between both problems is quite visible but note that we can observe this similarity only if uncertainty is modeled by means of scenario planning [2]. If another method is used (for instance fuzzy numbers), the demonstrated analogy will not appear. A scenario is the way the future might unfold. Other definitions can be found for instance in [6, 22]. Numerous guidelines concerning a correct scenario construction are provided, e.g., in [7] and [21]. Scenarios are usually defined by experts. Practitioners frequently apply scenario planning since within this approach the set of scenarios does not need to be exhaustive. Additionally, SP facilitates the identification of uncertain and uncontrolled factors influencing the consequences of chosen strategies. That is why scenario planning minimizes surprises and helps organizations to be better prepared to handle new situations. Scenario planning is eagerly used by project managers since it is comfortable and allows one to analyze a given problem in a more deterministic way [31, 32] than, for example, fuzzy numbers or continuous probability distributions.

The second analogy between 1-DMU and M-DMC results from the fact that for both domains the search of pure and mixed strategies is possible, but TOPSIS is only designed for the discrete version of M-DMC, so the continuous problems are not discussed in this article.

Despite the apparent similarities, it is also worth briefly discussing the differences between M-DMC and 1-DMU. First, if A_j *is chosen, the final outcome* $(a_{i,j})$ *is single and depends on the real scenario which will occur. Within M-DMC, if* A_j is selected, there are p final payoffs, i.e., $b_{1,j},\ \ldots,\ b_{k,j},\ \ldots,\ b_{p,j}$, *because the decision variants are assessed in terms of* p *essential objectives. Second, in the case of M-DMC initial values usually have to be normalized as they represent the performance of diverse criteria. In 1-DMU the problem is related to one objective. That is why the normalization is redundant* [10, 13]. The observed differences do not cancel the possibility to develop new procedures for 1-DMU on the basis of approaches already invented for M-DMC. However, their existence should be taken into consideration in the next subsection.

4. How can the TOPSIS-based approach designed for 1-DMU be constructed?

Now let us use TOPSIS ideas in 1-criterion decision-making under uncertainty. The TOPSIS 1-DMU algorithm can contain the following steps:

1. Define the set of alternatives $A = \{A_1,\ \ldots,\ A_j,\ \ldots,\ A_n\}$, where n is the number of options.
2. Define the set of scenarios $S = \{S_1,\ \ldots,\ S_i,\ \ldots,\ S_m\}$, where m denotes the number of scenarios.
3. Estimate the outcomes for each pair (option/scenario). Create a payoff matrix $(a_{i,j})_{m\times n}$ (Table 3). If the payoffs are related to a neutral criterion, transform initial values according to formula 9. In other cases (maximized criterion or minimized criterion) the transformation is not necessary

$$a(n)_{i,j} = \begin{cases} 1 & \text{if} \quad a_{i,j} \in [d_i^{\min}, d_i^{\max}] \\ \dfrac{\max_j a_{i,j} - a_{i,j}}{\max_j a_{i,j} - d_i^{\max}} & \text{if} \quad a_{i,j} > d_i^{\max} \\ \dfrac{a_{i,j} - \min_j a_{i,j}}{d_i^{\min} - \min_j a_{i,j}} & \text{if} \quad a_{i,j} < d_i^{\min} \end{cases} \tag{9}$$

where $d_i^{\min}$ and $d_i^{\max}$ denote the endpoints of the interval of the desired values connected with scenario S_i.

4. Declare the subjective chance of occurrence SCO_i of each scenario. The sum of SCO_i does not need to be equal to 1.
5. Generate the weighted decision matrix $(t_{i,j})_{m\times n}$ where $t_{i,j} = \mathrm{SCO}_i \cdot a_{i,j}$ or $t_{i,j} = SCO_i \cdot a(n)_{i,j}$.
6. Determine the worst alternative (the negative-ideal solution: NIS) and the best alternative (the positive-ideal solution: PIS)

$$\mathrm{NIS} = \{\min(t_{i,j})|j = 1,\ \ldots,\ n|i = 1,\ \ldots,\ m\} = \{t_{i,\mathrm{NIS}}|i = 1,\ \ldots,\ m\} \tag{10}$$

$$\text{PIS} = \{\max(t_{i,j})|j = 1, \ldots, n|i = 1, \ldots, m\} = \{t_{i,\text{PIS}}|i = 1, \ldots, m\} \tag{11}$$

The equations given above are applicable to benefit and neutral criteria. In the case of a cost criterion formulas are different

$$\text{NIS} = \{\max(t_{i,j})|j = 1, \ldots, n|i = 1, \ldots, m\} = \{t_{i,\text{NIS}}|i = 1, \ldots, m\} \tag{12}$$

$$\text{PIS} = \{min(t_{i,j})|j = 1, \ldots, n|i = 1, \ldots, m\} = \{t_{i,\text{PIS}}|i = 1, \ldots, m\} \tag{13}$$

7. Compute the distance between particular alternatives and NIS

$$d_{\text{NIS},\,j} = \sqrt{\sum_{i=1}^{m} (t_{i,j} - t_{i,\text{NIS}})^2}, \quad j = 1, \ldots, n \tag{14}$$

8. Compute the distance between particular alternatives and PIS

$$d_{\text{PIS},\,j} = \sqrt{\sum_{i=1}^{m} (t_{i,j} - t_{i,\text{PIS}})^2}, \quad j = 1, \ldots, n \tag{15}$$

9. Calculate the similarity to the worst solution where $s_{\text{NIS},\,j} \in [0, 1]$

$$s_{\text{NIS},\,j} = \frac{d_{\text{NIS},\,j}}{d_{\text{NIS},\,j} + d_{\text{PIS},\,j}}, \quad j = 1, \ldots, n \tag{16}$$

10. Rank the options according to $s_{\text{NIS},\,j}$ from the largest value to the smallest.

Some steps of the algorithm require additional comments.

First, in Step 4, the DM should not declare objective probabilities, because such measures are allowed only if the set of scenarios is replaced by an exhaustive set of states of nature. In scenario planning scenarios do not need to be disjoint. Therefore, the sum of subjective chances of occurrence does not have to be equal to 1. The aforementioned parameters ought to be estimated according to the DM's attitude toward risk, expectations, and predictions.

Second, the novel approach is even simpler and less time-consuming than the original TOPSIS since this time the normalization is desired only in the case of a neutral criterion.

Third, if numerous neutral criteria are considered in the classical TOPSIS version, the interval of desired values can be different for each criterion, however in the TOPSIS-based 1-DMU algorithm the interval is usually the same for each scenario since data represent only one criterion. Nevertheless, if payoff ranges for particular scenarios are significantly different, the intervals may be different as well (see Step 3).

5. Illustrative example

In this section, an illustrative example will be discussed. The analyzed case will be solved by means of the TOPSIS 1-DMU algorithm.

Let us assume that a company considers one out of five investment strategies (IS1-IS5) (Step 1). The goal of the company is to maximize the total annual revenue which depends on the selected strategy. Unfortunately, the company is not able to compute exact future values. Experts, on the basis of overall analysis and the use of different econometric models, have just stated that there were four significant scenarios worth taking into consideration (Step 2). The scenarios depend on numerous factors – they are affected by economic, social, political, demographic, epidemiological, and climatic situations. Therefore, each scenario is a potential product of the occurrence of a set of strictly defined phenomena. After a complex analysis, the experts have estimated the whole payoff matrix (Table 4, Step 3). Normalization is not required since the table contains data connected with only one criterion which is maximized.

Table 4. Future annual revenues (in million dollars)

Scenario	Alternative				
	IS_1	IS_2	IS_3	IS_4	IS_5
S_1	2.00	1.60	3.00	0.00	3.20
S_2	1.00	1.80	0.50	3.40	0.70
S_3	3.50	5.00	1.40	2.00	1.00
S_4	4.00	1.00	2.00	1.00	0.00

Table 5. Weighted annual revenues (in million dollars)

Scenario	Alternative				
	IS_1	IS_2	IS_3	IS_4	IS_5
S_1	0.80	0.64	1.20	0.00	1.28
S_2	0.30	0.54	0.15	1.02	0.21
S_3	1.75	2.50	0.70	1.00	0.50
S_4	0.80	0.20	0.40	0.22	0.00

Table 6. Distances, similarity, and rank (TOPSIS 1-DMU)

Measure	Alternative				
	IS_1	IS_2	IS_3	IS_4	IS_5
d_{NIS}	1.693	2.145	1.281	1.027	1.281
d_{PIS}	1.145	1.000	2.040	2.055	2.301
s_{NIS}	0.596	0.682	0.386	0.333	0.358
Rank	II	I	III	V	IV

We assume that the company has declared the following subjective chances of occurrence: 0.4, 0.3, 0.5, and 0.2 (Step 4). Now, the computation of the weighted payoff matrix is possible (Step 5, Table 5). The worst alternative can be described as follows: NIS = {0.00, 0.15, 0.50, 0.00} (Step 6). The best alternative is PIS = {1.28, 1.02, 2.50, 0.80} (Step 7).

The distances between the alternatives and the ideal solutions are computed in Table 6 (Steps 8–9). They allow us to define the similarity to the worst solution (third row, Table 6). According to the results, the second investment strategy is recommended (Step 10).

Table 7 shows rankings obtained after applying existing classical decision rules (α is the pessimism coefficient). The essence of classical procedures is explained for instance in [11]. The results obtained in the research are discussed in the next section.

6. Discussion and conclusions

TOPSIS ideas were used, originally formulated for multi-criteria decision-making under certainty, in a new domain, i.e., 1-criterion decision-making under uncertainty. It is possible thanks to a similar structure of both problems provided 1-DMU is modeled by means of scenario planning. With such assumptions, the set of alternatives in M-DMC corresponds to the set of alternatives in 1-DMU; the set of payoffs in M-DMC may correspond to the set of outcomes in 1-DMU; and the set of criteria in M-DMC may correspond to the set of scenarios in 1-DMU. However, in the last two cases, the interpretation is different.

In Section 6, an example illustrating the TOPSIS 1-DMU approach has been presented. Additionally, existing classical decision rules have been applied to the same problem in order to conduct a comparative analysis. Let us briefly remind the essence of the existing methods. Within the Wald rule [33], we choose the alternative with the highest minimum value. The Wald approach is designed for extreme pessimists. Within the Hurwicz rule [16], after defining the pessimism coefficient, we select the option with the highest weighted average where the worst payoff is multiplied by the pessimism parameter and the best outcome is multiplied by the optimism coefficient. The Hurwicz approach has been rather formulated for moderate decision-makers. The max-max rule consists in finding the decision variant with the highest maximum outcome. It is a procedure applied by extreme optimists. The Bayes procedure assumes that (due to the lack of the DM's knowledge) each scenario may have the same probability of occurrence. That is why, according to this method, the alternative with the highest average of payoffs is the best. The Savage [30] and the max-min joy rules [15] have totally different assumptions. In this case, we compare the options on the basis of a relative losses matrix or a relative profits matrix, respectively. These values are computed by comparing a given payoff with the best value in a specific scenario (Savage rule) or the worst value in this scenario (max-min joy rule). Then the Savage rule recommends the variant with the smallest maximal relative loss and the max-min joy rule suggests selecting the alternative with the highest minimal relative profit. The Savage and the max-min joy rules are the only approaches where the ranking is determined by the sequence of the payoffs connected with a given option. For the remaining classical decision rules the order of outcomes related to an alternative does not affect the ranking.

Table 7. Ranks (classical decision rules)

Rule	Alternative				
	IS_1	IS_2	IS_3	IS_4	IS_5
Wald [32]	I	I	II	III	IV
Hurwicz (α = 0.75)	II	I	III	IV	V
Hurwicz (α = 0.50)	II	I	III	IV	V
Hurwicz (α = 0.25) [15]	II	I	V	III	IV
Max-max	II	I	V	III	IV
Bayes	I	II	III	IV	V
Savage [29]	I	II	IV	III	V
Max-min joy [14]	II	I	III	III	III
Expected value	II	I	III	IV	V

Conclusions concerning the comparative analysis (Table 7) are as follows:

- Ranks are different depending on the method applied.
- Investment strategies IS_1–IS_2 get always the first two ranks in the ranking.

- Strategies IS_3-IS_5, regardless of the procedure, get the last three positions in the sequence.
- The above observations cannot be generalized since the rankings for subsequent decision rules are determined by the structure of the payoff matrix.

Other interesting observations concern the following aspects:

- In decision-making under uncertainty, regardless of the approach applied, the first reason for using a given method is not to get the maximal profit but to take the DM's preferences into account in the best possible way. Therefore, as a matter of fact, the comparative analysis of the rankings is less important than the analysis of the assumptions made within particular methods.
- The Wald, max-max, and Hurwicz rules consider the extreme payoffs only, thus the intermediate outcomes are not included in the analysis. Such an attitude towards data means that the indices on the basis of which rankings are generated are not comprehensive. The same phenomenon can be observed in the case of the Savage and the max-min joy rule: they are only based on the lowest maximal relative loss or the highest minimal relative profit. Actually, the only classical existing decision rules which take all the payoffs into account are the Bayes rule and the expected value rule. However these procedures (similarly to other approaches) do not give the opportunity to investigate the position of particular outcomes related to an alternative with outcomes coming from other decision variants but connected with the same scenarios. The novel approach based on TOPSIS allows simultaneously the analysis of the attractiveness of a given option compared with the positive-ideal and negative-ideal solution. Such tools have not been used before in one-criterion decision-making under uncertainty.
- On the one hand, the new procedure (TOPSIS 1-DMU) has been developed for one-criterion indeterministic problems based on scenario planning, which means that the decision-maker may consider only one criterion in the decision-making process. But on the other hand, as was stressed in the previous section, the scenarios identified by the experts are not found on the basis of one factor. Each scenario is a result of the occurrence of a set of strictly defined events concerning diverse aspects (economic, political, climatic, etc.). And that is why, although TOPSIS 1-DMU is theoretically created for one-criterion problems, it allows the decision-maker to consider very complex processes within scenario planning, which makes the novel approach useful both from the microeconomic and macroeconomic point of view.
- TOPSIS 1-DMU enables exploring maximized, minimized, and neutral criteria. Additionally, in the case of neutral criteria, the desired levels may be presented as crisp numbers or intervals, which is certainly a significant benefit.
- In connection with the fact that the TOPSIS ideas are applied in this research to solve a new decision problem (i.e., 1-DMU), the importance of criteria is replaced by the subjective chance of occurrence of scenarios.
- Given the fact that the original TOPSIS is comprehensive and is one of the most popular methods applied to mul-ti-criteria decision-making under certainty, there is a chance that the TOPSIS 1-DMU will gain followers within scenario-based one-criterion uncertain problems.
- The novel approach is even simpler and less time-consuming than the original TOPSIS because in the suggested method the normalization is necessary only in the case of a neutral criterion.

- In the research we assumed that the payoff matrix is already given, i.e., estimated by experts – we have only focused on the second stage of the decision-making process. Nevertheless, the first stage related to the payoff matrix generation is also extremely vital and can affect the final rankings. One of the ways to estimate future outcomes consists in applying econometric models (such a tool has been mentioned in the previous section), but other fore-casting methods are also appropriate.
- The aspect discussed in the previous point may be treated as a disadvantage for TOPSIS 1-DMU and all other procedures developed for 1-DMU, because each 1-DMU decision rule based on scenario planning requires the payoff matrix estimation and this stage is often troublesome due to the lack of relevant data. However, the number of tools supporting the estimation of future factors increases. Therefore the aforementioned weakness may be sometimes treated as a negligible impediment.
- Another potential weakness of TOPSIS 1-DMU is connected with some limitations of the original TOPSIS formulated for M-DMC. The use of the classic TOPSIS approach may lead to a problem of rank reversal in the addition, deletion or replacement of the set of alternatives [9, 20]. And the same phenomenon can occur when applying TOPSIS 1-DMU.
- Therefore, in the future it would be desirable to develop a TOPSIS 1-DMU approach resistant to the aforementioned changes in the set of options. In this case, the ideas presented for instance in [34] could be adopted.
- In the future it would be also advisable to check how other M-DMC methods based on reference points can be helpful for 1-DMU problems. The analysis of both classic (VIKOR, BIPOLAR) and new (e.g. DARP, [25]) procedures might be fruitful.

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