Vol. 32, No. 4 (2022) | DOI: 10.37190/ord220403

OPEN ACCESS

Operations Research and Decisions

www.ord.pwr.edu.pl

OPERATIONS RESEARCH AND DECISIONS QUARTERLY

A linearisation approach to solving a non-linear shelf space allocation problem with multi-oriented capping in retail store and distribution centre

Kateryna Czerniachowska*[1] Marcin Hernes[1] Krzysztof Lutosławski[1]

[1]*Department of Process Management, Wroclaw University of Economics and Business, Wrocław, Poland*

**Corresponding author: Kateryna.Czerniachowska@ue.wroc.pl*

Abstract

Shelf space is one of the essential resources in logistic decisions. Order picking is the most time-consuming and labour-intensive of the distribution processes in distribution centres. Current research investigates the allocation of shelf space on a rack in a distribution centre and a retail store. The retail store, as well as the distribution centre, offers a large number of shelf storage locations. In this research, multi-orientated capping as a product of the rack allocation method is investigated. Capping allows additional product items to be placed on the rack. We show the linearisation technique with the help of which the models with capping could be linearised and, therefore, an optimal solution could be obtained. The computational experiments compare the quality of results obtained by non-linear and linear models. The proposed technique does not increase the complexity of the initial non-linear problem.

Keywords: *linear programming, shelf space allocation, retail store, distribution centre, order picking*

1. Introduction

Retailers and wholesalers use distribution centres to store, choose, pack, and ship items directly to customers. A distribution centre (DC) is an all-in-one storage and shipping solution that is critical for online merchants and e-commerce businesses. Storage locations may be on the rack managed by an employee, high-bay DC with stacking cranes, or pallet storage locations. Goods are often brought to a storage facility, offloaded, and stored until an item is purchased with a DC. The order is chosen, packed, and dispatched to the purchaser at that moment.

Distribution centres have long been critical to getting goods from manufacturers to their customers. Everything has moved faster as a result of the advancement of computers, information and communication technology. This means more frequent and faster deliveries of items to clients for a DC [25]. Trucks

Received 6 June 2022, accepted 12 December 2022, published online 8 February 2023
ISSN 2391-6060 (Online)/

The costs of publishing this issue have been cofinansed by the program *Development of Academic Journals* of the Polish Ministry of Education and Science under agreement No. RCN/SP/0241/2021/1

can easily unload or pick up merchandise for final delivery, as distribution centres are usually located near major roads and highways. Although a corporation can use only one DC to store merchandise, it is not common for a company's extensive network to include numerous distribution centres.

The main reasons for automating are to improve efficiency and reduce the cost of human resources. Lower management expenses, higher revenues, and satisfied consumers are all benefits of rapid freight transport. Therefore, material handling, picking, sorting, and packaging can be automated first. In a typical DC, these functions require more than half of the workforce.

Order-picking costs typically account for around 55% of logistic operating costs, and order-picking can be broken down further [20]. The list presents the order-picking time as a percentage of activity:

- travelling – 55%,
- searching – 15%,
- extracting – 10%,
- paperwork and other activities – 20% [4], [20].

Travel is the most expensive aspect of order picking, which is the most expensive part of warehouse operating costs. The order-picking method has been designed with the goal of reducing unproductive time [4]. In most DCs, order picking remains manual. Among the picker's responsibilities are:

- order picking containing various products with a mobile scanner,
- pick-to-light LEDs and barcode scanners on racks,
- order picking by voice with a headset,
- completing and sorting products in stock,
- packing products,
- other DC activities.

Therefore, products should be placed on the shelf so that one person can take orders quickly and efficiently. Products should be easy to reach despite the fact that the number of stock-keeping units (SKUs) is very high. This research proposes the capping method of allocating SKU on the shelves without expanding the shelf space. The term capping was first introduced by Czerniachowska and Hernes [12], [13], which means that additional SKUs must be placed above the main SKU on the same product on the shelf. Capping orientation could vary. Later, it was used in several shelf space allocation models [14], [15], [16], [11]. But all models were non-linear, which does not allow for obtaining an optimal solution to a problem. In this research, we propose a transformation technique which provides for rewriting the model in a linear form and solving it optimally by a commercial solver. In this transformation approach, floor and ceiling functions are analysed. This method could be used in the retail store and DC in order to place products on the rack.

The aims of the research are:

- present capping allocation method while allocation products on the rack,
- reformulate the non-linear model into a linear one using some transformation approaches,
- replace some variables in order to remove non-linearities,
- provide computational experiments which allow finding the optimal solution to the linearised model.

The remainder of this paper is structured as follows. We start in Section 2 with the literature review, which includes shelf space allocation in a retail store as well as product placement on the rack and order picking in DC. In Section 3, the definition of the problem of supplying products with capping, non-linear problem formulation, and the linearisation technique is provided. In the following Section 4, the results of computational experiments are presented. The article is concluded in Section 5. The future application of the proposed technique is given in the last Section 6.

2. Literature review

2.1. Shelf space allocation on a rack in a retail store

A successful product-to-shelf assignment approach not only makes it easier for customers to find what they want, but also boosts retailer profits. In logistics and store management, shelf space is a valuable resource, and well-designed space allocation can attract more customers and improve sales. A shelf space allocation problem is a choice problem in a retail business where the goal is to achieve the best possible outcome given specific operational restrictions.

The end goal of shelf space planning is to maximise the profit of a retailer from the demands of the consumers realised, which are determined by the vertical and horizontal positioning of products on the shelf, product profits, inventories and operational costs. Shelf management is divided into two layers. The very first (macro) level is strategic, and it entails selecting on shelf kinds and product categories (for example, beverages and chocolate). The tactical stage (second) involves designating individual items within each category [5].

Effective shelf space management is essential to increase store profits, reduce inventories, and cultivate good vendor relationships [27]. Category managers must understand how to solve shelf space allocation problems, establish key algorithms' issues, and analyse results. There are a variety of tools for category management in a retail store [27]. The strategy category planning approach includes shelf space planning [29]. Hübner and Kuhn [29], [30], and Hübner [28] distinguish between a sequence of hierarchical planning processes in category planning:

- Assortment planning involves product listing and delisting, as well as product selection within a category. For delisted products, the demand model must account for substitution impacts.
- Shelf space planning allocates space and determines the horizontal and vertical placement of listed products on shelves, keeping in mind capacity limits. The demand for a product may be influenced by the quantity offered and its location on the shelf.
- Store replenishment planning determines refill rules. It includes topics such as in-store logistics processes and inventory review procedures. Its goal is to meet the specified on-shelf service levels using the shelf space plans that have been established [5].

There are five components of shelf space allocation that management must consider in order to optimise the store's financial success. The position of the fixture, the location of the product category, the location of items within categories, the display on the shelf, and the promotion support at the point of sale are factors to consider [6]. The sales productivity methodology and the build-up approach are two techniques that are regularly used in commercial systems to allocate shelf space [37], [53]. Examples of different models for the shelf space allocation problem are proposed by Yang and Chen [53], Frontoni et al. [21], and Huang et al. [27].

Retailers must use appropriate retail mix strategies, such as store location, product availability, price, advertising and promotion, store layout and display on shelves, services, and personal sales, to attract customers and thrive in a competitive environment. One of the most critical aspects in shaping client purchasing decisions is shelf-space allocation [27]. The study by Huang et al. [27] presented a three-stage product-to-shelf assignment approach that takes into account physical and category restrictions, as well as customer buying behaviour. It integrated data mining and network analysis to address the persistent product-to-shelf allocation problem [27].

Following an assessment of current shelf space allocation techniques, a thorough optimisation model, including the application of an integer programming of the model, as suggested by Yang and Chen [53]. The difficulty of this model due to non-linear integer programming may prevent it from being used in a retail store with a large number of displayed products [53].

For merchants and shelf out-of-stock events, product selection and the shelf space reserved for each product is a vital activity. Frontoni et al. [21] provided an integer linear programming model that best reallocates shelf space based on real-time shelf out-of-stock and sell-out data from a sensor network technology called the shelf detector system [21].

2.2. Product placement in a storage area in a distribution centre

A warehouse is a site in the supply chain where stock is stored. It acts as a buffer to meet user needs, assisting in the transition of stocks from one stage to the next in a supply chain. Raw materials or prepared commodities can be stored in a warehouse. The term distribution centres (DCs) refers to large regional warehouses. A DC's flow rate per unit time is usually quite high. The main purpose of a warehouse is to supply items to customers on time and at a low cost by combining various products according to the client's needs. For similar types of items, the distribution method varies by company. The location, capacity, and architecture of a company's warehouses and DCs are all part of the plan [49].

The appropriate allocation of shelf space in the distribution centre ensures that most of the products in the offer are available immediately. The product can be chosen, packed, and shipped to customers around the world.

Due to the desire of distribution centres to lower costs and increase productivity, order retrieval in storage procedures is currently a burgeoning research topic. One of their most labour-intensive and cost-intensive operations is the retrieval of products from storage areas [46], [42].

Stock goes from factories via DCs to retail outlets in a general retail distribution network. The cost of commodity transport and handling increases when the stock flows unevenly through a distribution system. These expenses increase because additional labour and vehicles must be recruited on a transitory basis to handle peak stock flows [50].

The management of a DC beverage company was concerned that a fast pick-up tunnel was very clogged during the peak pick-up season, which led to this enquiry. They wanted to see whether the order in which the SKUs are placed has an effect on throughput and performance. Therefore, Venkatadri et al. [49] tried to optimise the order of product placement in a fast pick-up tunnel in a DC to reduce congestion and increase performance. The number of pick locations for each picked product was initially optimised to reduce the total restocking effort [49].

The DC serves as an important link between suppliers and customers in supply chains. The capacity of supply chains is influenced by the performance of DCs. In DCs, order picking is the most time-consuming and labour-intensive activity. Order selection has been increasingly challenging in recent years as DCs had to hold more types of product to fulfil more small orders in a shorter period of time [10].

2.3. Order picking

In a DC, order picking refers to the process of retrieving products to be sent from their storage site in response to customer purchase orders. A customer's purchase order usually consists of a number of products with specific amounts. Order picking is the essential DC operation, as it is labour intensive and accounts for half of the overall cost of warehouse operations. Consumers will be disappointed as a result of order processing inefficiencies and the supply chain will be less efficient. As a consequence, order picking could constitute a bottleneck, hindering DCs from optimising their storage operations. Although storage site assignment (or product allocation) is a tactical choice, it has a significant impact on order picking efficiency [10].

There are numerous review papers available on order selection [17], [19], [24], [48], [2]. Storage assignment [33], [51], [10], order batching, and picker routing [26], [43], [7], [36], and zone picking are all part of the decision-making process in order-picking systems.

Bahrami et al. [2] studied a storage assignment strategy whose goal was to improve the performance of the order picking process in warehouses. They concentrated on the manual picker-to-parts order picking system, in which workers go along an aisle while driving or walking (in a suitable vehicle) to pick products at various stops. Ho and Lin [24] explored how picking performance using various order picking approaches is influenced by the task compensation plan and the regulatory attention of pickers.

Hong et al. [26] investigated order batching methods for parallel-aisle picking systems and provided a novel order batching formulation and relevant relaxation models utilizing a bin-packing problem. Cergibozan and Tasan [7] concluded that the efficiency of the batching operation depends on the storage and batching of related items. The choices about storage placement and layout design can be viewed as components of the whole problem.

Warehouse or DC efficiency can be significantly improved by synchronising order picking processes, incentive systems, and regulatory focus, according to the findings of de Vries et al. [19]. Warehouses or DCs can obtain great benefits by combining storage, batching, zone picking, and routing procedures [48], [2]. De Koster et al. [18] took into consideration a pick-and-sort order picking system, in which a team of order pickers simultaneously picks batches of orders from several (work) zones. In their study, the issue of determining the optimal number of zones is investigated to reduce the system throughput time for batch order picking and subsequent order sorting and packing (synchronised zone picking). Kostrzewski [31] investigated high–bay warehouse design and proposed to take into account a variety of elements that affect the long-term, effective and safe operation of this facility. Furthermore, Kostrzewski [32] examined and clarified the use of simulation techniques in order-picking in high-rack warehouses.

The most common order selection methods are parallel picking, zone picking, and dynamic zone picking [19]. By focussing on cooperation-based and competition-based incentive systems, as well as their performance under parallel, zone, and dynamic zone picking approaches, de Vries et al. [19] presented a new model to align incentive systems with order picking assignments.

Total picking is a method of picking up many customer orders at the same time. For each customer order, products must be sorted, usually using a pick-and-sort or sort-while-pick technique [40]. Single picking methods, on the other hand, are those in which only one customer order is picked at a time [45].

For smaller units, such as separate boxes and loose goods, order picking procedures are executed simply, that is, products removed from boxes. To put it another way, picking and the many order picking strategies:

- Person-to-goods, which requires the picker to go through the warehouse to select each of the products for each of the lines that require preparing.
- Goods-to-person, in which the operator stays at the picking post while a machine transports the items to that location, allowing the operator to reach them.
- Hybrid of the first two strategies, in which several storage systems with various working principles, each for a certain sort of product, can be installed in a DC.

Because a range of products are constantly released to better meet the diversified needs of customers, supply chain management must focus on minimising both cost and time to market. Retail outlets can only show product products in tiny amounts due to shelf space limitations, which requires frequent replenishment from distribution centres [10].

2.4. Linear and non-linear mathematical models

Warehouse management systems keep track of warehousing and picking processes, producing massive amounts of data in the millions to billions of entries. Without actively processing the collected data to monitor their business processes, ease warehouse flows, and support strategic choices, logistics operators pay enormous costs to maintain these IT systems [47].

There are a number of studies related to the design of warehouses and DCs as well as the integration of the different processes. Therefore, various models have been developed in the literature, including mathematical models.

Lee et al. [34] studied the practical problem of constructing a supply chain network, including the location of facilities, the location of facilities, and the routing decisions. The goal of this research is to find the most cost-effective placement, allocation, and routing for the supply chain network. To solve the model with routing, the study provides two mixed-integer programming models, one without routing and one with routing, as well as a heuristic approach based on LP-relaxation.

Non-rectilinear warehouse layouts with piecewise diagonal cross aisles and nonparallel picking aisles are presented by Gue and Meller [23] to reduce the travel distance in single-command warehouses, i.e., warehouses at which a single SKU is picked. For comparablely larger warehouses, one of their solutions reduces the projected distance of transport by more than 20%. Gajjar and Adil [22] used the piecewise linearisation approach to reformulate an existing non-linear model for the shelf space allocation issue with space elasticity and construct a local search strategy to solve it. They also provided two mathematical models: a non-linear model and a linear model for the problem.

The best storage system configuration (the number, length, and height of the storage aisles) is one of the most important factors in the design of a distribution system in a DC [41].

In a very basic type of DC, the SKUs come on pallets and leave on pallets in a "unit-load" area. The storage area is the same as the picking area in such DCs, and space-time (workforce) models are simple linear models. As a result, the time required to use each storage space and the time required to move each item through the DC could be predicted. This allows you to specify exactly where each pallet should be stored in order to reduce labour costs [4].

Muppani and Adil [38] created a non-linear integer programming algorithm for class-based storage arrangements that took into account space reduction, handling costs, and storage area costs. For resolving the created non-linear model, they used the branch-and-bound approach.

In most cases, the number of orders and goods to pick is too large to allow for order batching and picking sequence optimisation at the same time. As a result, batching must be determined while measuring the distance of the sequencing problem to be solved [8].

Chen et al. [9] studied the solution quality of a hybrid algorithm using a non-linear mixed-integer optimisation model for integrated order batching, sequencing and routing. Oncan [39] proposed formulations for batch ordering using mixed integer linear programming under three different policies: traversal, return, and midway.

Sung and Jang [44] solved the assort-packing and distribution problem in the fashion apparel industry. They suggested an optimisation model for solving the assort-packing and distribution problem quantitatively in terms of how many container types should be configured, how to configure the size distribution for each container, and how many of each form of the container should be supplied to stores. To obtain the exact approach, they implemented a linearised version of the basic optimisation model and solved it with a commercial solver, with its default setting being without any heuristic approach [44].

In some situations, the non-linear integer programming model could be transformed into a linear one. In this study, we explore how integer programming techniques could be adopted for facings and capping allocation methods. This approach is very useful in DCs and retail stores.

3. Methodology

3.1. Nomenclature

In this article, we use the variables and parameters listed below. Subscripts indicate variable's indexes; superscripts represent variable's description or mnemonics of the variable of the variable and should not be read as indexes.

Parameters and indices

S – the total number of shelves
P – the total number of products
i – shelf index, $i = 1, \ldots, S$
j – product index, $j = 1, \ldots, P$

$$r \text{ – orientation index, } r = \begin{cases} 1, & \text{for front orientation} \\ 2, & \text{for side orientation} \end{cases}$$

Parameters of the shelf i

s_i^w – width
s_i^d – depth
s_i^h – height

Parameters of the product j

p_j^w – width
p_j^d – depth
p_j^h – height
p_j^s – supply limit
p_j^u – unit profit
p_{jr}^o – orientation binary parameter, $p_{jr}^o = \begin{cases} 1, & \text{if orientation } r \text{ is available for product } j \\ 0, & \text{otherwise} \end{cases}$
$f_j^{\min}$ – the minimum number of facings
$f_j^{\max}$ – the maximum number of facings
$c_j^{\min}$ – the minimum number of cappings per facings group
$c_j^{\max}$ – the maximum number of cappings per facings group

Additional variables

ε – additional coefficient, $\varepsilon \approx 1, \varepsilon < 1$
g_{ijr} – the number of horizontal capping groups above facings of the product j on the shelf i on orientation r

Decision variables in SSAP-NL

x_{ij} – product placement binary variable, $x_{ij} = \begin{cases} 1, & \text{if product } j \text{ is placed on the shelf } i \\ 0, & \text{otherwise} \end{cases}$
f_{ij} – the number of facings of the product j on the shelf i
c_{ij} – the number of cappings of the product j on the shelf i
$y_{ijr}^o = \begin{cases} 1, & \text{if product } j \text{ is placed on the shelf } i \text{ in orientation } r \\ 0, & \text{otherwise} \end{cases}$

Decision variables in SSAP-L

f_{ijr} – the number of facings of the product j on the shelf i on orientation r
c_{ij} – the number of cappings of the product j on the shelf i

Solution evaluation parameters

Z' – profit ratio as the profit of SSAP-NL divided by the profit of SSAP-L;
Z^{opt} – profit ratio as the profit of SSAP-NL divided by the optimal value;
Z'' – profit ratio as the profit of SSAP-L (in 5 min) divided by the profit of SSAP-L obtained in extended time (10 min)
$V^{\text{SSAP-NLP}}$ – total profit of SSAP-NL
$V^{\text{SSAP-LP}}$ – total profit of SSAP-L

3.2. Problem definition

In this research, the Shelf Space Allocation Problem (SSAP) concerns a retail or DC rack of shelves of the same length. The goal is to determine the number of SKUs for each product that should be distributed on each shelf to maximise the total profit of the rack. The SKU could be placed on the shelf as facing or as capping. The facing is a visible SKU on the shelf. The capping is the SKU placed on the top of the facing in the other orientation (Fig. 1). Shelf length, height, and depth determine the space limit for all products. The shelf height considers the facings and capping row. Obviously, capping cannot be placed on the shelf if there is no facing of this product on the same shelf.

The product supply limit determines the maximum number of SKU if the product is placed on multiple shelves, as well as if the SKU is presented with facings or capping. The range of minimum and maximum number of facings for the product SKUs on the shelf is based on its movement.

The product can be placed on the shelf in front or side orientations. This parameter is defined by the package type and the label on the package visibility to the DC picker or store client. In the first case, the product width is taken as a linear parameter for the calculation of the occupied space of the product in front orientation. In the second case, the product depth is taken as a linear parameter for the calculation of the occupied space of the product in the side orientation. To make the product more visible, only one orientation is available for the product if it is placed on multiple shelves.

In this research, all products may be placed on all shelves; there are no specific shelf levels for the product. However, the range of the number of shelves on which the product could be placed is defined. Easy access to the products is required, allowing the picker to quickly place the product in the container of retail customers to make a quick purchase.

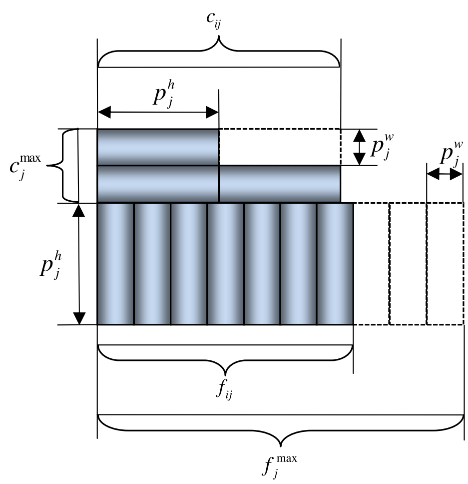

Figure 1. Capping allocation for front orientation

In the current model, we take into consideration one (top) facings row with or without cappings above it. To simplify the model, facings in vertical and depth dimensions are not considered. So, only the visible facings row is investigated. Shelf height s_i^h and shelf depth parameters s_i^d are also applied to one visible facings row. To solve the problem, the goal is to define the number of facings and cappings of a product on each shelf defining its orientation. The criteria function is to maximise the total retailer's profit subject to shelf, facings, capping, multi-shelves, and orientation constraints. In this investigation, only the basic set of SSAP constraints is used. Later the problem could be enriched with other constraints.

3.3. SSAP-NL problem formulation

The model can then be formulated as follows:

$$\max \sum_{j=1}^{P} \sum_{i=1}^{S} p_j^u \left(f_{ij} + c_{ij} \right) \tag{1}$$

subject to:

• Shelf constraints

– shelf length

$$\forall (i) \left[\sum_{j=1}^{P} f_{ij} (y_{ij1}^o p_j^w + y_{ij2}^o p_j^d) \leq s_i^l \right] \tag{2}$$

– shelf depth

$$\forall (i,j) [y_{ij1}^o p_j^w + y_{ij2}^o p_j^d \leq s_i^d] \tag{3}$$

– shelf height

$$\forall (i,j) \left[p_j^h x_{ij} + \left\lceil \frac{c_{ij} x_{ij}}{\max\left(\frac{f_{ij}(y_{ij1}^o p_j^w + y_{ij2}^o p_j^d)}{p_j^h}, 1 \right)} \right) (y_{ij1}^o p_j^w + y_{ij2}^o p_j^d) \leq s_i^h \right] \tag{4}$$

• Facing constraints

– supply limit

$$\forall (j) \left[\sum_{i=1}^{S} (f_{ij} + c_{ij}) \leq p_j^s \right] \tag{5}$$

– minimum and maximum number of facings

$$\forall (j) \left[f_j^{\min} \leq \sum_{i=1}^{S} f_{ij} \leq f_j^{\max} \right] \tag{6}$$

• Capping constraints

– minimum and maximum number of caps

$$\forall (i,j) \left[c_j^{\min} \leq \sum_{i=1}^{S} c_{ij} \leq c_j^{\max} \left\lfloor \frac{f_{ij}(y_{ij1}^o p_j^w + y_{ij2}^o p_j^d)}{p_j^h} \right\rfloor \right] \tag{7}$$

– no capping without facings

$$\forall (i,j) [\min(c_{ij}, 1) \leq \min(f_{ij}, 1)] \tag{8}$$

• Multi-shelves constraints
– minimum and maximum number of shelves

$$\forall(j)\left[s_j^{\min} \leq \sum_{i=1}^{S} x_{ij} \leq s_j^{\max}\right] \tag{9}$$

• Orientation constraints
– orientation is possible

$$\forall(i,j,r)[y_{ijr}^o \leq p_{jr}^o] \tag{10}$$

– only one orientation is available

$$\forall(i,j)\left[\sum_{r=1}^{R} y_{ijr}^o \leq 1\right] \tag{11}$$

• Relationships constraints
– facings relationships

$$\forall(i,j)\left[x_{ij}s_i^l\left(\frac{y_{ij1}^o}{p_j^w} + \frac{y_{ij2}^o}{p_j^d}\right) \geq f_{ij}\right] \tag{12}$$

– facings and orientation relationships

$$\forall(i,j)\left[x_{ij} = \sum_{r=1}^{R} \min(y_{ijr}^o f_{ij}, 1)\right] \tag{13}$$

– capping and orientation relationships

$$\forall(i,j)\ \left[c_{ij} \leq x_{ij}c_j^{\max}\left\lfloor\frac{f_{ij}(y_{ij1}^o p_j^w + y_{ij2}^o p_j^d)}{p_j^h}\right\rfloor\right] \tag{14}$$

• Decision variables
– product is placed on the shelf

$$x_{ij} \in \{0,1\} \tag{15}$$

– number of facings

$$f_{ij} = \{f_j^{\min} \ldots f_j^{\max}\} \tag{16}$$

– number of cappings

$$c_{ij} = \left\{c_j^{\min} \ldots c_j^{\max}\left\lfloor\frac{f_{ij}(y_{ij1}^o p_j^w + y_{ij2}^o p_j^d)}{p_j^h}\right\rfloor\right\} \tag{17}$$

– orientation

$$y_{ijr}^o \in \{0,1\} \tag{18}$$

3.4. Linearisation technique

In the defined constraints of SSAP-NL, some constraints are not linear; there are products of binary and continuous variables (constraints (2), (4), (7), (13), (14)) and products of two binaries (constraints (4), (12), (14)).

Let f_{ijr} be the number of facings of the product j on the shelf i on orientation r. This allows us to exclude the orientation decision variable y^o_{ijr}. Therefore, the product with this decision variable will be eliminated.

The allocation decision variable x_{ij} could be represented as follows:

$$x_{ij} = \max_{r=1,\ldots,R}(\min(f_{ijr}, 1))$$

Therefore, it could also be excluded from the model. In this case, the product will also be eliminated. The floor function is the mathematical function that takes as input a certain real number and gives as output the highest integer less than or equal to x, denoted $\lfloor x \rfloor$. The ceiling function is the mathematical function that takes as input a certain real number and gives as output the least integer less than or equal to x, denoted $\lceil x \rceil$.

Constraints (4), (7) are not linear, so we use the linearisation technique, which allows us to eliminate the floor and ceiling functions [1].

The value of the function $z(x)$ can be represented as $z(x) = t + \varepsilon$, where t is an integral part of $z(x)$ and $0 \le \varepsilon < 1$. Therefore, the floor function $\lfloor z(x) \rfloor$ can be linearised by replacing it with the integer variable $t(t\lfloor z(x) \rfloor)$ and adding the following constraints:

$$t \le z(x) < t + 1 \tag{19}$$

$$t \in Z \tag{20}$$

Equation (19) can also be represented as

$$t - \varepsilon \le z(x) \le t$$

where $\varepsilon = 0.999$.

A similar approach can be used to linearise the ceiling function $\lceil z(x) \rceil$ by replacing it with the integer variable $t(t\lceil z(x) \rceil)$ and adding the following constraints to the problem:

$$t - 1 \le z(x) < t \tag{21}$$

$$t \in Z \tag{22}$$

Equation (21) can also be represented as

$$t \le z(x) \le t + \varepsilon$$

where $\varepsilon = 0.999$. Equations (20), (22) are the integrality constraints.

Let g_{ijr} be the additional variable that corresponds to the number of horizontal capping groups above the facings of the product j on the shelf i on orientation r

$$g_{ij1} = \left\lfloor \frac{f_{ij1} p_j^w}{p_j^h} \right\rfloor, \quad g_{ij2} = \left\lfloor \frac{f_{ij2} p_j^d}{p_j^h} \right\rfloor$$

The shelf height constraint (4) could be rewritten as follows:

$$\forall (i,j) \left[c_{ij} \leq g_{ij1} \left\lfloor \frac{s_i^h - p_j^h}{p_j^w} \right\rfloor + g_{ij2} \left\lfloor \frac{s_i^h - p_j^h}{p_j^d} \right\rfloor \right]$$

which represents the number of capping above facings depending on the shelf height. The floor function $\lfloor g_{ijr} \rfloor$ can be linearised as follows:

$$\forall (i,j) \left[\frac{f_{ij1} p_j^w}{p_j^h} - \varepsilon \leq g_{ij1} \leq \frac{f_{ij1} p_j^w}{p_j^h} \right]$$

where g_{ij1} represents the number of horizontal capping groups above facings in front orientation.

$$\forall (i,j) \left[\frac{f_{ij2} p_j^d}{p_j^h} - \varepsilon \leq g_{ij2} \leq \frac{f_{ij2} p_j^d}{p_j^h} \right]$$

where g_{ij2} represents the number of horizontal capping groups above facings in side orientation.

In constraint (7), the number of caps should be rewritten in order to consider not only the available shelf height but also the maximum number of caps. Because we excluded two decision variables (y_{ijr}^o and x_{ij}), relationship constraints (12), (14) are no longer needed. The following section presents a complete linearised equivalent model.

3.5. SSAP-L problem formulation

The model can then be formulated as follows:

$$\max \sum_{j=1}^{P} \sum_{i=1}^{S} \left(\sum_{r=1}^{R} f_{ijr} + c_{ij} \right) p_j^u \tag{23}$$

subject to:

• Shelf constraints

– shelf length

$$\forall (i) \left[\sum_{j=1}^{P} (f_{ij1} p_j^w + f_{ij2} p_j^d) \leq s_i^l \right] \tag{24}$$

– shelf depth

$$\forall (i,j) \left[\min(f_{ij1}, 1) p_j^d + \min(f_{ij2}, 1) p_j^w \leq s_i^d \right] \tag{25}$$

– capping depending on the shelf height

$$\forall(i,j)\left[c_{ij} \le g_{ij1}\left\lfloor \frac{s_i^h - p_j^h}{p_j^w} \right\rfloor + g_{ij2}\left\lfloor \frac{s_i^h - p_j^h}{p_j^d} \right\rfloor\right] \tag{26}$$

– number of horizontal capping groups above facings in front orientation

$$\forall(i,j)\left[\frac{f_{ij1}p_j^w}{p_j^h} - \varepsilon \le g_{ij1} \le \frac{f_{ij1}p_j^w}{p_j^h}\right] \tag{27}$$

– number of horizontal capping groups above facings in side orientation

$$\forall(i,j)\left[\frac{f_{ij2}p_j^d}{p_j^h} - \varepsilon \le g_{ij2} \le \frac{f_{ij2}p_j^d}{p_j^h}\right] \tag{28}$$

• Facing constraints

– supply limit

$$\forall(j)\left[\sum_{i=1}^{S}\left(\sum_{r=1}^{R} f_{ijr} + c_{ij}\right) \le p_j^s\right] \tag{29}$$

– minimum and maximum number of facings

$$\forall(j)\left[f_j^{\min} \le \sum_{i=1}^{S}\sum_{r=1}^{R} f_{ijr} \le f_j^{\max}\right] \tag{30}$$

• Capping constraints

– minimum and maximum number of caps

$$\forall(i,j)\left[c_j^{\min} \le \sum_{i=1}^{S} c_{ij} \le c_j^{\max}(g_{ij1} + g_{ij2})\right] \tag{31}$$

– no capping without facings

$$\forall(i,j)\left[\min(c_{ij},1) \le \max_{r=1,\ldots,R}(\min(f_{ij},1))\right] \tag{32}$$

• Multi-shelves constraints

– minimum and maximum number of shelves

$$\forall(j)\left[s_j^{\min} \le \sum_{i=1}^{S}\max_{r=1,\ldots,R}(\min(f_{ijr},1)) \le s_j^{\max}\right] \tag{33}$$

• Orientation constraints

– orientation is possible

$$\forall(i,j,r)\left[\min(f_{ijr},1) \le p_{jr}^o\right] \tag{34}$$

– only one orientation is available

$$\forall(i,j)\left[\sum_{r=1}^{R}\min(f_{ijr},1)\leq 1\right] \tag{35}$$

• Decision variables
– number of facings

$$f_{ijr}=\{f_j^{\min}\dots f_j^{\max}\} \tag{36}$$

– number of cappings

$$c_{ij}=\left\{c_j^{\min}\dots c_j^{\max}\left\lfloor\frac{f_{ij1}p_j^w+f_{ij2}p_j^d}{p_j^h}\right\rfloor\right\} \tag{37}$$

4. Computational experiment

The computational experiments compare the solutions of linear and non-linear SSAPs. Due to linearisation techniques, an optimal solution for SSAP-L could be found. This is extremely important in large instances. The experimental data were simulated based on real-store data. According to Yang [52], Lim et al. [35], and Bai and Kendall [3], there were 23 sets of products randomly generated with a normal distribution with different parameters.

The 23 sets of products given contained 10, 15,. . . , 60, 70,. . . , 100, 125,. . . , 300 products. Each set of products included 5, 10 or 25 products more than the previous one. The planograms to allocate this set of products were modelled in five widths: 250, 375, 500, 625, and 750 cm. Each planogram had 3, 4 or 5 shelves. The same planogram widths for all shelves were selected because the most common case in a store is to have all shelves of the same widths on the whole planogram. Optimal and feasible solutions to both problems have been found using the IBM ILOG CPLEX Optimisation Studio version 2 commercial solver.

Let us define some additional variables dedicated to evaluating the solution quality in both mathematical formulations. This evaluation is done by linear programming. The basic solution time was set to 5 min. Next, we tried to solve the instances for which this time was too short to solve in 10 min.

$$V^{\mathrm{SSAP-NLP}}=\sum_{j=1}^{P}\sum_{i=1}^{S}(f_{ij}+c_{ij})p_j^u,\quad V^{\mathrm{SSAP-LP}}=\sum_{j=1}^{P}\sum_{i=1}^{S}\left(\sum_{r=1}^{R}f_{ij}+c_{ij}\right)p_j^u$$

$$V^{\mathrm{opt}}=V^{\mathrm{SSAP-LP}},\quad Z'=\frac{V^{\mathrm{SSAP-NLP}}(t_1)}{V^{\mathrm{SSAP-LP}}(t_1)},\quad \text{where } t_1=5\text{ min}$$

$$Z^{\mathrm{opt}}=\frac{V^{\mathrm{SSAP-NLP}}(t_1)}{V^{\mathrm{opt}}(t_1)},\ \text{where } t_1=5\text{ min}$$

$$Z''=\frac{V^{\mathrm{SSAP-LP}}(t_1)}{V^{\mathrm{SSAP-LP}}(t_2)},\ \text{where } t_1=5\text{ min}, t_2=10\text{ min}$$

Table 1. SSAP-L and SSAP-NL solutions for 10–55 products [%]

Products	Shelf width	No. of shelves eq. (3)			No. of shelves eq. (4)			No. of shelves eq. (5)		
		Z'	Z^{opt}	Z''	Z'	Z^{opt}	Z''	Z'	Z^{opt}	Z''
	250	100	100		100	100		100	100	
	375	100	100		100	100		100	100	
10	500	100	100		100	100		100	100	
	625	100	100		100	100		100	100	
	750	100	100		100	100		100	100	
	250	100	100		100	100		100	100	100
	375	100	100		100		100	99.99		100
15	500	100	100		100	100		99.96		100
	625	99.89	99.89		99.88	99.88		99.71	99.71	100
	750	99.95	99.95		99.90	99.90		99.84	99.84	100
	250	100	100		99.68	99.68		99.43		100
	375	100	100		100	100		99.93		100
20	500	100	100		99.82	99.82		99.51		99.99
	625	99.95	99.95		99.98		100	99.70	99.70	
	750	100	100		99.99		100	99.69		100
	250	100	100		99.78	99.78		99.88	99.88	
	375	100	100		99.48		100	99.71		100
25	500	100		100	99.76		100	99.77		100
	625	100		100	99.77		100	99.61		100
	750	99.98		100	99.81		100	99.74		100
	250	100	100		100		100	99.99		100
	375	100		100	99.81	99.81		99.35	99.35	100
30	500	100	100		99.84	99.84		99.49	99.49	
	625	99.81	99.81		99.69	99.69		99.69		100
	750	100	100		99.68	99.68	100	99.51		100
	250	100	100		99.45	99.45		99.00		100
	375	99.84	99.84		98.80		100	99.82		100
35	500	99.96		100	99.47		100	99.42		100
	625	100	100	100	99.68		100	99.50	99.50	
	750	99.65	99.65		99.67	99.67		99.77	99.77	
	250	100	100		99.73	99.73		98.97		100
	375	99.89		100	98.98		100	99.10		100
40	500	100		100	99.59		100	99.29	99.29	
	625	100		100	99.71	99.71		99.45		100
	750	100	100		99.76		100	99.29		100
	250	100		100	99.86	99.86		99.13	99.13	
	375	99.88	99.88		99.31		100	99.55	99.55	
45	500	99.81	99.81		99.45	99.45		99.46		100
	625	99.88	99.88		99.84		100	99.29	99.29	
	750	99.75	99.75		99.57		100	99.29		100
	250				100	100		99.88		100
	375	100	100		99.44	99.44		98.80		100
50	500	99.84	99.84		99.29		100	99.71		100
	625	99.67	99.67		99.15	99.15		99.48	99.48	
	750	99.74	99.74		99.42	99.42	100	99.11		100
	500				100	100		97.97	97.97	
	625				98.61	98.61		98.45		100
	750	99.06	99.06		99.07		100	98.53		100

Tables 1 and 2 present the quality of SSAP-L and SSAP-NL solutions. The tables report the settings for which solution exists, i.e. there are no rows for which solution could not be found because even the lowest numbers of SKUs exceed the shelf limits. So, there is no solution in 56 cases for 3 shelves, for 42 cases for 4 shelves, and for 34 cases for 5 shelves. This shows that when we add the number of shelves, additional space appears; therefore, more test instances than previous could be solved.

Table 2. SSAP-L and SSAP-NL solutions' quality for 60–300 products [%]

Products	Shelf width	No. of shelves eq. (3)			No. of shelves eq. (4)			No. of shelves eq. (5)		
		Z'	Z^{opt}	Z''	Z'	Z^{opt}	Z''	Z'	Z^{opt}	Z''
60	250	100	100		99.53	99.53		97.47		100
	375	100	100		98.79	98.79		97.51		100
	500	99.44	99.44		99.03		100	98.91	98.91	
	625	99.45	99.45		99.36	99.36		98.08		100
	750	99.56	99.56		99.53		100	99.28	99.28	
70	375				99.46	99.46		98.19		100
	500	100	100		97.62		100	97.55		100
	625	99.14	99.14		95.22		100	96.15	96.15	
	750	99.75	99.75		99.14		100	97.87		100
80	500				100	100		98.30		100
	625				99.37		100	93.86		100
	750	98.62		100	99.64		100	98.85		100
90	375							97.26	97.26	
	500				99.82		100	97.12		100
	625	99.81	99.81		98.98		100	97.06		100
	750	98.96	98.96		95.60		100	96.03		100
100	375							98.30	98.30	
	500				99.26		100	96.13		100
	625	99.52	99.52		96.73		100	95.43	95.43	
	750	99.62		100	98.56	98.56		98.72		100
125	500							98.63		100
	625				97.59		100	96.84		100
	750	99.34	99.34		95.24		100	94.87		100
150	625							93.38	93.38	
	750				95.59	95.59		90.19	90.19	
175	500							99.17	99.17	
	625				99.70	99.70		87.13	87.13	
	750				95.98	95.98		92.25		100
200	625							95.73		100
	750				96.95	96.95		88.64	88.64	
225	750							92.26	92.26	
275	625							93.65	93.65	100
	750				98.25	98.25		89.02		100
300	250							100	100	
	375							100	100	
	500							100	100	
	625							100	100	
	750							100	100	

Unfortunately, the solver could not find optimal solutions for all instances. This was indicated by the CPLEX solution status. So, it reports that in the time limit of 5 min, it could not prove that the solution it finds is optimal. Z'' shows for which test instances the solution was not proved to be optimal in 5 min. So we tried to solve them in 10 min. There are 12 such cases for 3 shelves, 35 cases for 4 shelves, and 53 cases for 5 shelves. These results show that the problem becomes more complex as the number of shelves increases. The empty cells in the column Z'' indicate that the solution was optimal in 5 min.

Z^{opt} indicates for which test instances in 5 or 10 min an optimal solution was found. So, an optimal solution was found for 48 of 59 tests for which a solution exists on 3 shelves, for 40 of 73 tests on 4 shelves, for 33 tests of 81 on 5 shelves, which is quite good. Z' compares the total profit of SSAP-L and SSAP-NL in 5 min. 100% indicates that the same solution was achieved. It also shows that the solution of the SSAP-L model was better than SSAP-NL in 28 cases for 3 shelves, 60 cases for 4 shelves, and 75 cases for 5 shelves. The average Z' among the tests for which solutions exist, not including values with 100% equals 99.63% for 3 shelves, 98.99% for 4 shelves, and 97.82% for 5 shelves. This shows how the solution quality of SSAP-NL decreases with increasing number of shelves.

Column Z^{opt} compares the total profit of SSAP-L and SSAP-NL for which an optimal solution was obtained. Some cells with Z^{opt} are empty even if the values in Z' exist because the solution of SSAP-L was not proven to be optimal. The number of tests for which optimal solutions exist and therefore Z^{opt} was calculated equal to 23 for 3 shelves, 29 for 4 shelves and 27 for 5 shelves. The average Z^{opt} among the tests for which this value was calculated without including the values with 100% is 99.64% for 3 shelves, 99.13% for 4 shelves, and 97.10% for 5 shelves.

Z'' compares the total profit of SSAP-L in 10 min, for which the solution was not shown to be optimal in 5 min. 100% indicates that the solution was the same in 5 and 10 min. Only in 1 test for 20 products on 500 cm shelves on 5 shelves did the SSAP-L solution improve in 10 min. In the rest of the 99 tests, the solution was the same. Therefore, we did not conduct the test with SSAP-NL in 10 min.

However, the SSAP-L solution was the same at 5, and 10 min. In some cases, CPLEX could prove that it is an optimal solution. So, the solution is proved to be optimal in 8 cases: in the test with 35 products on 625 cm on 3 shelves, in the test with 30 and 50 products on 750 cm on 4 shelves, in the test with 15 products on 250, 625, 750 cm on 5 shelves, in the test with 30 products on 375 cm on 5 shelves, in the test with 275 products on 625 cm on 5 shelves. In the rest 92 tests (in 11, 33, 48 tests for 3, 4, and 5 shelves, respectively), the solution was not proved to be optimal, and CPLEX stopped due to a time limit.

Generally, the experiment shows that the SSAP-L model could obtain better results. This proves the rationality of using linearisation techniques. For small instances, the results are the same as in the SSAP-NL model.

Figures 2, 4, and 6 demonstrate the profit ratio expressed by Z' found in 5 min on 3, 4, 5 shelves, respectively. Figures 3, 5, and 7 demonstrate the profit ratio expressed by Z^{opt} found in 5 or 10 min on 3 shelves. Comparing Z' and Z^{opt} on 3 shelves (Figs. 2 and 3), a slight difference could be observed. For 4 shelves, this difference is more visible.

For 5 shelves, this difference is definitely visible. The least number of optimal solutions was received for 5 shelves. The quality of SSAP-NL expressed by Z' is high enough; the profit ratio Z' is higher than 98.5% (Fig. 2), 95% (Fig. 4), 89% (Fig. 6) for 3, 4, 5 shelves, respectively. The quality of SSAP-NL expressed by Z^{opt} is also high enough, but slightly lower than Z'. So Z^{opt} is higher than 98% (Fig. 3), 95% (Fig. 5), 87% (Fig. 7) for 3, 4, 5 shelves, respectively. Table 3 presents the numbers of integer variables in the SSAP-L and SSAP-NL problems. These numbers are the same in both problems despite the fact that we substituted some variables while linearising. This shows the next advantage of the linearisation technique in SSAP-L, which does not increase the number of variables. Table 3 also shows that even the largest 7500 number of variables could be solved sufficiently fast by the CPLEX solver.

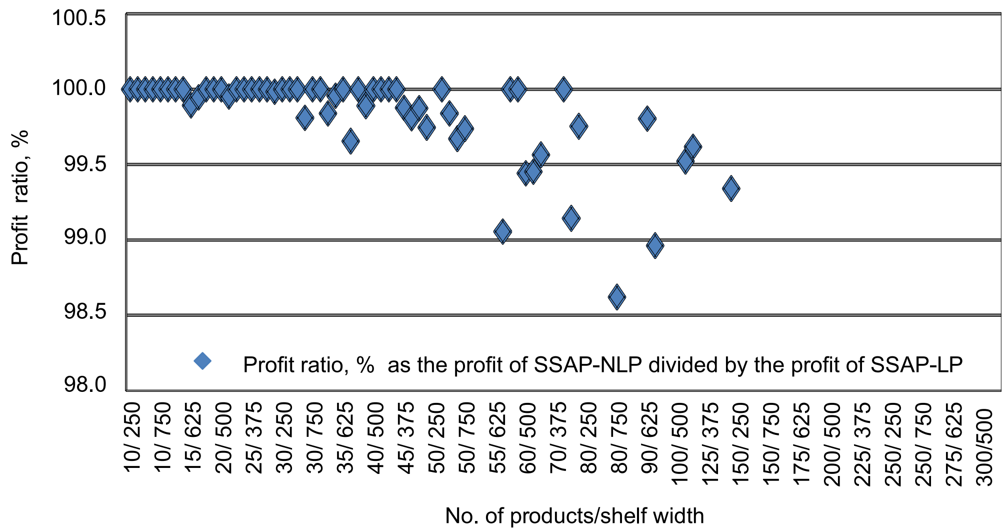

Figure 2. Profit ratio expressed by Z' found in 5 min on 3 shelves

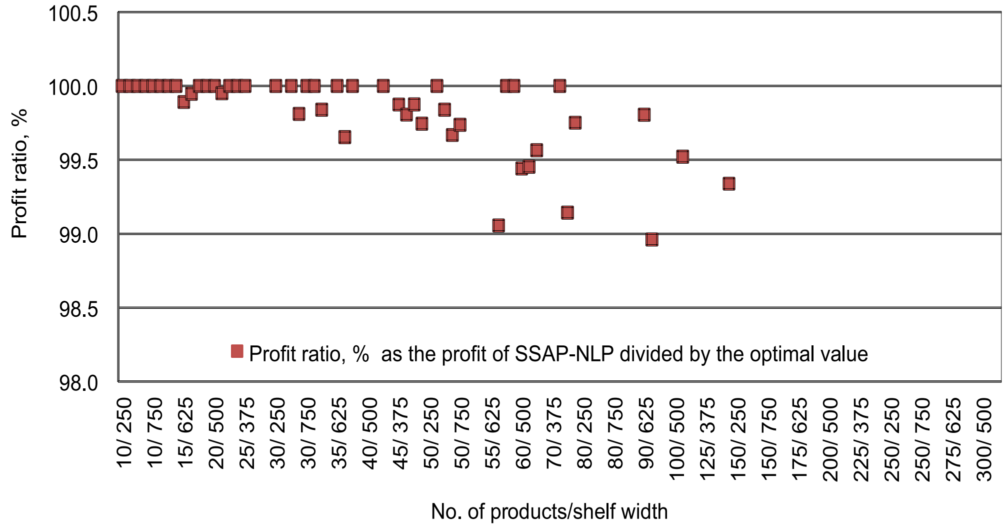

Figure 3. Profit ratio expressed by Z' found in 5 min on 3 shelves

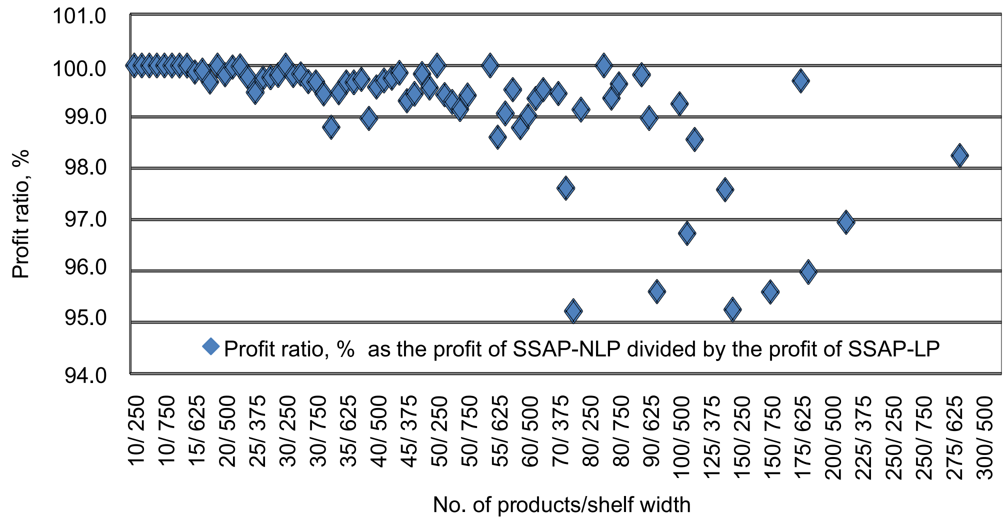

Figure 4. Profit ratio expressed by Z' found in 5 min on 4 shelves

5. Conclusion

Order picking (sometimes known as picking) is the gathering and merging of non-unitary loads in order to put together a customer order. This can happen in practically any kind of DC, and it always happens

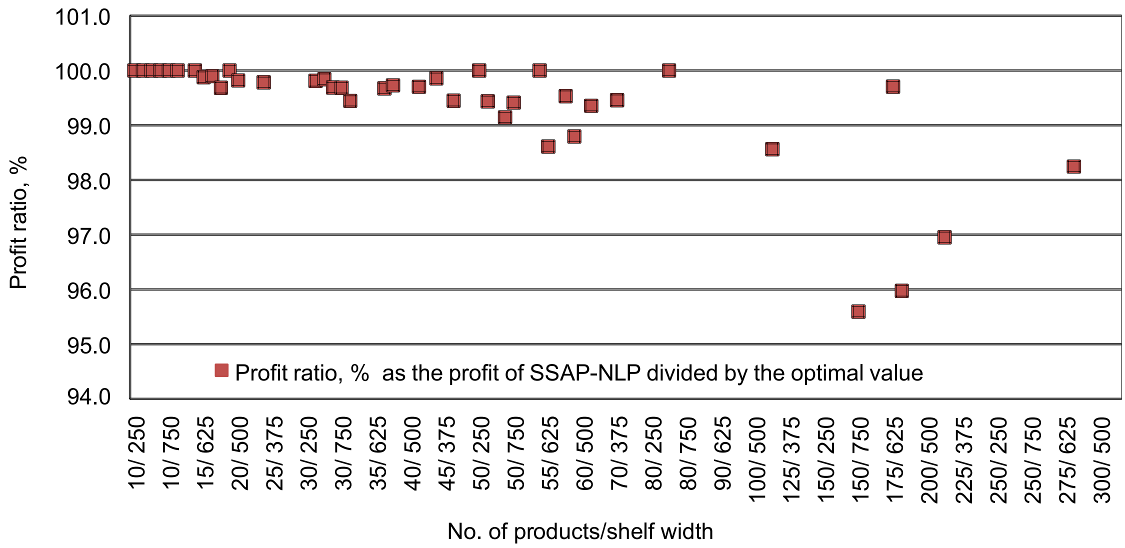

Figure 5. Profit ratio expressed by Z^{opt} found in 5 or 10 min on 4 shelves

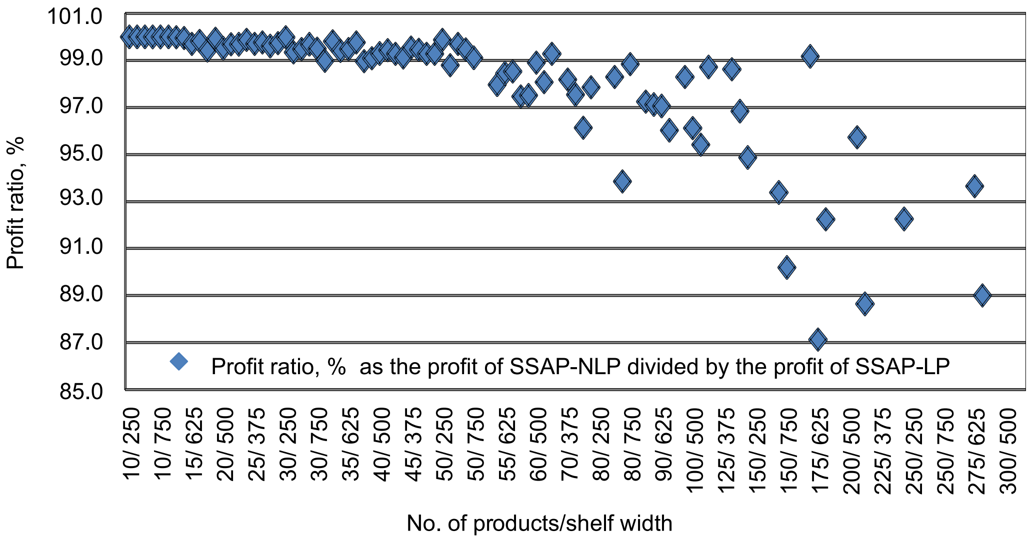

Figure 6. Profit ratio expressed by Z' found in 5 min on 5 shelves

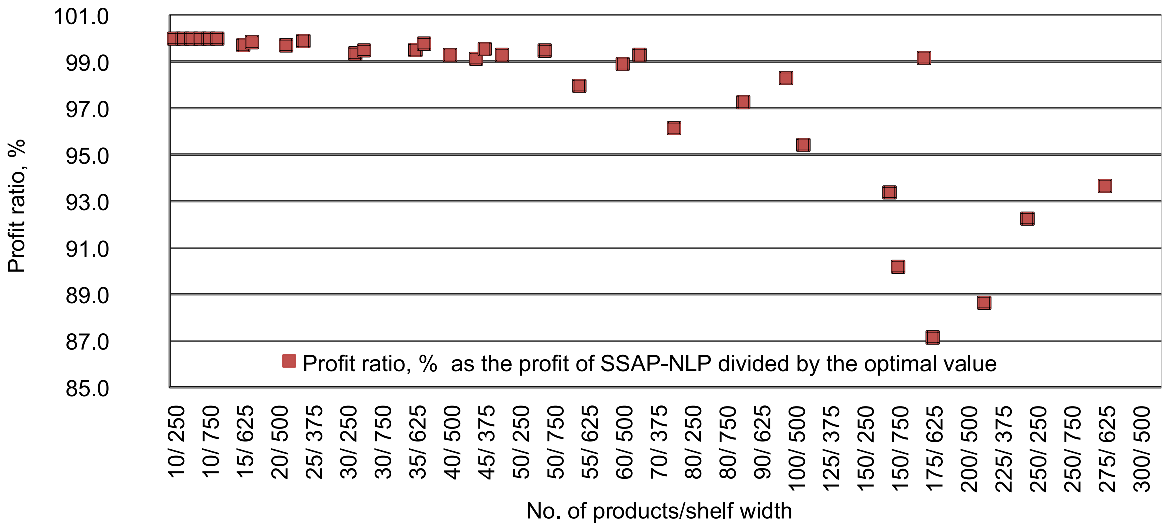

Figure 7. Profit ratio expressed by Z^{opt} found in 5 or 10 min on 5 shelves

Table 3. Numbers of integer variables in both problems

Shelves	Products	Variables	Shelves	Products	Variables	Shelves	Products	Variables
3	10	150	4	10	200	5	10	250
3	15	225	4	15	300	5	15	375
3	20	300	4	20	400	5	20	500
3	25	375	4	25	500	5	25	625
3	30	450	4	30	600	5	30	750
3	35	525	4	35	700	5	35	875
3	40	600	4	40	800	5	40	1000
3	45	675	4	45	900	5	45	1125
3	50	750	4	50	1000	5	50	1250
3	55	825	4	55	1100	5	55	1375
3	60	900	4	60	1200	5	60	1500
3	70	1050	4	70	1400	5	70	1750
3	80	1200	4	80	1600	5	80	2000
3	90	1350	4	90	1800	5	90	2250
3	100	1500	4	100	2000	5	100	2500
3	125	1875	4	125	2500	5	125	3125
3	150	2250	4	150	3000	5	150	3750
3	175	2625	4	175	3500	5	175	4375
3	200	3000	4	200	4000	5	200	5000
3	225	3375	4	225	4500	5	225	5625
3	250	3750	4	250	5000	5	250	6250
3	275	4125	4	275	5500	5	275	6875
3	300	4500	4	300	6000	5	300	7500

when shipments, parts, items, or supplies need to be integrated and then transferred. Before products, order fulfilment, and manufactured goods are shipped to wholesale or retail clients; distribution facilities are an essential part of the distribution chain. They act as a link between suppliers and clients.

The experiment shows how the solution quality of the non-linear formulation decreases with an increasing number of shelves. The average profit ratio, which compares linear and non-linear problem models with the results obtained in 5 min among the tests for which solutions exist, not including the values with equal results obtained by both models, equals 99.63% for 3 shelves, 98.99% for 4 shelves, and 97.82% for 5 shelves. The average profit ratio that compares the formulation of non-linear problems with the optimal solution obtained in 5 or 10 min among the tests for which this value was calculated, not including the tests for which the non-linear problem obtained an optimal solution, equals 99.64% for 3 shelves, 99.13% for 4 shelves, and 97.10% for 5 shelves.

The picking process has the largest impact on the product cost in DC. Order picking is likely the most significant function in fulfilment centres; therefore, having a good system in place is essential. Warehouses use a variety of order picking methods to increase productivity and accuracy.

This research could give new insights into the methods of product allocation of DC's rack using multi-oriented capping as well as the methods to create linear problems when capping is included. The knowledge gained from this study is expected to be useful to practitioners working on allocations products on the racks in DCs.

6. Future research

In this investigation, the model without complicated constraints was investigated because the main goal was to explain the capping method and its linearisation technique. Next, the model can be expanded with the vertical and horizontal categories, product groupings, shelf segments, levels, and other constraints. Moreover, only two capping orientations were analysed. But facings and capping could be in any of six possible orientations (Fig. 8).

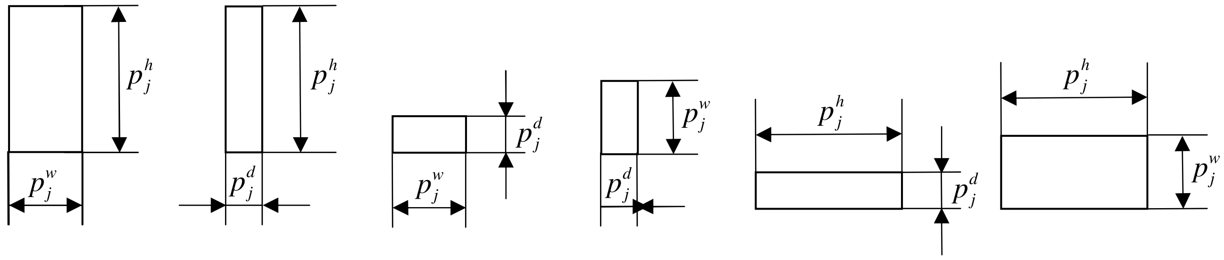

Figure 8. Orientation possibilities for facing and capping

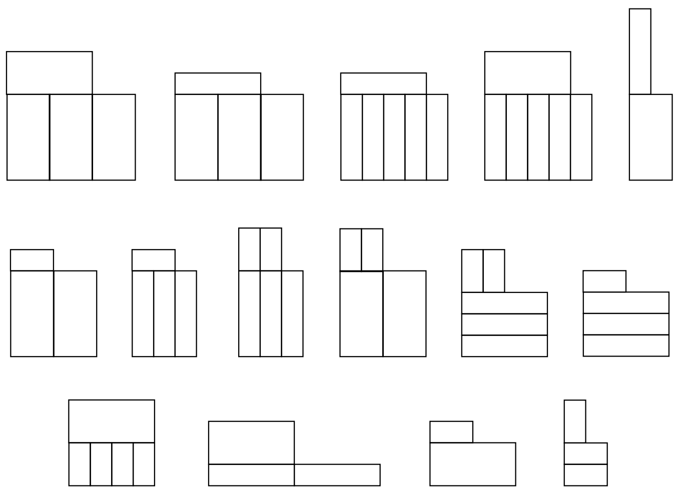

Figure 9. An example set of pairs of facing and capping orientations

Generally, all permutations of facing and capping are possible. Having 6 single orientations and selecting from them by 2 orientations gives us 15 permutations without repetitions.

$$C_n^m = \frac{n!}{m!(n-m)!} = \frac{6!}{2!(6-2)!} = \frac{4! \times 5 \times 6}{2! \times 4!} = 15$$

where C_n^m is the number of combinations, n – the total number of orientations, m – the number of orientations chosen from the set.

We assume that facings and capping cannot be in the same orientation because, in this case, capping and facing parameters are equal. Also, there is no need to check all 30 permutations with repetitions because the facings and capping rows could be swapped. This complicates the model with additional constraints but gives the same numerical result. Therefore, only valuable allocations of facing and capping should be analysed. The set of examples of facing and capping permutations is shown in Figure 9. Last but not least, the presented 2D model could be expanded as a 3D.

Acknowledgement

The authors are grateful to two anonymous reviewers for their valuable comments and suggestions made on the previous draft of this manuscript.

Funding

The project is financed by the Ministry of Science and Higher Education in Poland under the program *Regional Initiative of Excellence 2019–2022*, project No. 015/RID/2018/19, total funding amount 10 721 040,00 PLN.

References

[1] Asghari, M., Fathollahi-Fard, A. M., Mirzapour Al-e-hashem, S. M. J., and Dulebenets, M. A. Transformation and linearization techniques in optimization: A state-of-the-art survey. *Mathematics 10*, (2022), 283.

[2] Bahrami, B., Aghezzaf, E.-H., and Limère, V. Enhancing the order picking process through a new storage assignment strategy in forward-reserve area. *International Journal of Production Research 57*, 21 (2019), 6593–6614.

[3] Bai, R., and Kendall, G. An investigation of automated planograms using a simulated annealing based hyper-heuristic. In *Metaheuristics: Progress as real problem solvers. Operations Research/ Computer Science Interfaces Series, vol. 32*, T. Ibaraki, K. Nonobe, M. Yagiura, Eds., Springer, Boston MA, 2005, pp. 87–108.

[4] Bartholdi III, J. J., and Hackman, S. T. *Warehouse & distribution science: release 0.92*. The Supply Chain and Logistics Institute, Georgia Institute of Technology, Atlanta, GA, 2011.

[5] Bianchi-Aguiar, T., Hübner, A., Carravilla, M. A., and Oliveira, J. F. *Retail shelf space planning problems: A comprehensive review and classification framework 289*, 1 (2021), 1–16.

[6] Buttle, F. Retail space allocation. *International Journal of Physical Distribution & Logistics Management 14*, 4 (1984), 3–23.

[7] Cergibozan, Ç., and Tasan, A. S. Order batching operations: an overview of classification, solution techniques, and future research. *Journal of Intelligent Manufacturing 30*, (2019), 335-349.

[8] Chabot, T., Coelho, L. C., Renaud, J., and Côté, J.-F. Mathematical model, heuristics and exact method for order picking in narrow aisles. *Journal of the Operational Research Society 69*, 8 (2018), 1242–1253.

[9] Chen, T.-L., Cheng, C.-Y., Chen, Y.-Y., and Chan, L.-K. An efficient hybrid algorithm for integrated order batching, sequencing and routing problem. *International Journal of Production Economics 159*, (2015), 158–167.

[10] Chiang, D. M.-H., Lin, C.-P., and Chen, M.-C. Data mining based storage assignment heuristics for travel distance reduction. *Expert Systems 31*, 1 (2014), 81–90.

[11] Czerniachowska, K. A genetic algorithm for the retail shelf space allocation problem with virtual segments. *OPSEARCH 59*, 1 (2022), 364–412.

[12] Czerniachowska, K., and Hernes, M. A genetic algorithm for the shelf-space allocation problem with vertical position effects. *Mathematics 8*, 11 (2020), 1881.

[13] Czerniachowska, K., and Hernes, M. Optimisation models for the shelf space allocation problem with vertical position effects. In *Proceedings of the 36th International Business Information Management Association Conference (IBIMA), 4-5 November 2020, Granada, Spain. Sustainable Economic Development and Advancing Education Excellence in the era of Global Pandemic*, K. S. Soliman, Ed., IBIMA, Granada, 2020, pp. 1719–1729.

[14] Czerniachowska, K., and Hernes, M. A heuristic approach to shelf space allocation decision support including facings, capping, and nesting. *Symmetry 13*, 2 (2021), 314.

[15] Czerniachowska, K., and Hernes, M. Simulated annealing hyper-heuristic for a shelf space allocation on symmetrical planograms problem. *Symmetry 13*, 7 (2021), 1182.

[16] Czerniachowska, K., Sachpazidu-Wójcicka, K., Sulikowski, P., Hernes, M., and Rot, A. Genetic algorithm for the retailers' shelf space allocation profit maximization problem. *Applied Sciences 11*, 14 (2021), 6401.

[17] De Koster, R. Performance approximation of pick-to-belt orderpicking systems. *European Journal of Operational Research 72*, 3 (1994), 558–573.

[18] De Koster, R. B., Le-Duc, T., and Zaerpour, N. Determining the number of zones in a pick-and-sort order picking system. *International Journal of Production Research 50*, 3 (2012), 757–771.

[19] De Vries, J., de Koster, R., and Stam, D. Aligning order picking methods, incentive systems, and regulatory focus to increase performance. *Production and Operations Management 25*, 8 (2016), 1363–1376.

[20] Frazelle, E. H. *World-class warehousing and material handling*. 2nd edition, McGraw Hill Education, New York, 2016.

[21] Frontoni, E., Marinelli, F., Rosetti, R., and Zingaretti, P. Shelf space re-allocation for out of stock reduction. *Computers and Industrial Engineering 106* (2017), 32–40.

[22] Gajjar, H. K., and Adil, G. K. A piecewise linearization for retail shelf space allocation problem and a local search heuristic. *Annals of Operations Research 179*, 1 (2010), 149–167.

[23] Gue, K. R., and Meller, R. D. Aisle configurations for unit-load warehouses. *IIE Transactions 41*, 3 (2009), 171–182.

[24] Ho, Y.-C., and Lin, J.-W. Improving order-picking performance by converting a sequential zone-picking line into a zone-picking network. *Computers and Industrial Engineering 113* (2017), 241–255.

[25] Ho, Y.-C., and Tseng, Y.-Y. A study on order-batching methods of order-picking in a distribution centre with two cross-aisles. *International Journal of Production Research 44*, 17 (2006), 3391–3417.

[26] Hong, S., Johnson, A. L., and Peters, B. A. Large-scale order batching in parallel-aisle picking systems. *IIE Transactions 44*, 2 (2012), 88–106.

[27] Huang, H., Yao, L., Chang, J.-S., Tsai, C.-Y., and Kuo, R. J. Using product network analysis to optimize product-to-shelf assignment problems. *Applied Sciences 9*, 8 (2019), 1581.

[28] Hübner, A. A decision support system for retail assortment planning. *International Journal of Retail and Distribution Management* 45, 7/8 (2017), 808–825.

[29] Hübner, A. H., and Kuhn, H. Retail category management: State-of-the-art review of quantitative research and software applications in assortment and shelf space management. *Omega* 40, 2 (2012), 199–209.

[30] Kök, A. G., Fisher, M. L., and Vaidyanathan, R. Assortment planning: Review of literature and industry practice. In *Retail Supply Chain Management*, N. Agrawal, and Smith S., Eds., vol. 223 of *International Series in Operations Research and Management Science*. Springer, Boston, MA, 2015, pp. 175–236.

[31] Kostrzewski, M. Analysis of chosen issues in high–bay warehouse designing. *Logistyka 4*, (2015), 481–490, CD 1 (in Polish).

[32] Kostrzewski, M. Application of simulation method in analysis of order-picking processes in a high-rack warehouse. *Research in Logistics and Production 6*, 4 (2016), 309–319.

[33] Lee, I. G., Chung, S. H., and Yoon, S. W. Two-stage storage assignment to minimize travel time and congestion for warehouse order picking operations. *Computers and Industrial Engineering 139*, (2020), 106129.

[34] Lee, J. H., Moon, I. K., and Park, J. H. Multi-level supply chain network design with routing. *International Journal of Production Research 48*, 13 (2010), 3957–3976.

[35] Lim, A., Rodrigues, B., and Zhang, X. Metaheuristics with Local Search Techniques for Retail Shelf-Space Optimization. *Management Science 50*, 1 (2004), 117–131.

[36] Masae, M., Glock, C. H., and Grosse, E. H. Order picker routing in warehouses: A systematic literature review, *International Journal of Production Economics 224*, (2020), 107564.

[37] Mason, J. M., and Mayer, M. L. *Modern retailing: Theory and practice*. BPI Irwin, Homewood, IL, 1990.

[38] Muppani (Muppant), V. R., and Adil, G. K. A branch and bound algorithm for class based storage location assignment. *European Journal of Operational Research 189*, 2 (2008), 492–507.

[39] Öncan, T. MILP formulations and an iterated local search algorithm with tabu thresholding for the order batching problem. *European Journal of Operational Research 243*, 1 (2015), 142–155.

[40] Parikh, P. J., and Meller, R. D. Selecting between batch and zone order picking strategies in a distribution center. *Transportation Research Part E: Logistics and Transportation Review 44*, 5 (2008), 696–719.

[41] Parikh, P. J., and Meller, R. D. A travel-time model for a person-onboard order picking system. *European Journal of Operational Research 200*, 2 (2010), 385–394.

[42] Revillot-Narváez, D., Pérez-Galarce, F., and Álvarez-Miranda, E. Optimising the storage assignment and order-picking for the compact drive-in storage system. *International Journal of Production Research 58*, 22 (2020), 6949–6969.

[43] Scholz, A., Henn, S., Stuhlmann, M., and Wäscher, G. A new mathematical programming formulation for the Single-Picker Routing Problem. *European Journal of Operational Research 253*, 1 (2016), 68–84.

[44] Sung, S. W., and Jang, Y. J. Heuristic for the assort-packing and distribution problem in the fashion apparel industry. *International Journal of Production Research 56*, 9 (2018), 3116–3133.

[45] Tanaka, K., Ihara, A., and Zhang, J. Introducing parallel zone picking to warehouse batch picking systems. In *Proceedings of the 2019 International Conference on Modeling, Simulation and Big Data Analysis (MSBDA 2019)*, D. N. Hoang Thanh, P. P. Ray, and A. Mathews, Eds., Vol. 91 of Advances in Computer Science Research Series, Atlantis Press, 2019, pp. 440–444.

[46] Tompkins, J. A, White, J. A., Bozer, Y. A., and Tanchoco, J. M. A. *Facilities Planning*, John Wiley & Sons, Inc., USA, 2010.

[47] Tufano, A., Accorsi, R., and Manzini, R. A machine learning approach for predictive warehouse design. *International Journal of Advanced Manufacturing Technology 119*, 3-4 (2022), 2369–2392.

[48] Van Gils, T., Ramaekers, K., Braekers, K., Depaire, B., and Caris, A. Increasing order picking efficiency by integrating storage, batching, zone picking, and routing policy decisions. *International Journal of Production Economics 197*, (2018), 243–261.

[49] Venkatadri, U., Vardarajan, V., and Das, B. Product placement within a fast-picking tunnel of a distribution centre. *International Journal of Advanced Manufacturing Technology 76*, 9-12 (2015), 1681–1690.

[50] Visser, L. R., and Visagie, S. E. Smoothing the outflow of stock from picking lines in a distribution centre. In *Computational Logistics. Proceedings of the 9th Conference, Vietri sul Mare, Italy, ICCL 2018*, R. Cerulli, A. Raiconi, and S. Voß, Eds., vol. 11184 of *Lecture Notes in Computer Science*, Springer, Cham 2018, pp. 431–445.

[51] Wang, M., Zhang, R.-Q., and Fan, K. Improving order-picking operation through efficient storage location assignment: A new approach. *Computers and Industrial Engineering 139*, (2020), 106186.

[52] Yang, M.-H. An efficient algorithm to allocate shelf space. *European Journal of Operational Research 131*, 1 (2001), 107–118.

[53] Yang, M.-H., and Chen, W.-C. A study on shelf space allocation and management. *International Journal of Production Economics 60-61*, (1999), 309–317.