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Neutrosophic compromise programming approach for multiobjective nonlinear transportation problem with supply and demand following the exponential distribution

Ahmad Yusuf Adhami*[1] Mohd Faizan[1] Anas M[1]

[1]*Department of Statistics and Operations Research, Aligarh Muslim Universty, Aligarh, India*
**Corresponding author:* *yadhami.st@amu.ac.in*

Abstract

Decision-making is a tedious and complex process. In the present competitive scenario, any incorrect decision may excessively harm an organization. Therefore, the parameters involved in the decision-making process should be looked into carefully as they may not always be of a deterministic nature. In the present study, a multiobjective nonlinear transportation problem is formulated, wherein the parameters involved in the objective functions are assumed to be fuzzy and both supply and demand are stochastic. Supply and demand are assumed to follow the exponential distribution. After converting the problem into an equivalent deterministic form, the multiobjective problem is solved using a neutrosophic compromise programming approach. A comparative study indicates that the proposed approach provides the best compromise solution, which is significantly better than the one obtained using the fuzzy programming approach.

Keywords: *fuzzy approach, multiobjective optimization, supply chain, transportation, uncertainty*

1. Introduction

Transportation is one of the primary components of supply chain management. The costs of service and time play crucial roles in firms being competitive. Thus, improving transportation provides a firm an edge over others. The transportation problem, as a mathematical problem, was first introduced by Hitchcock [18]. The work done by Koopsman [20] was instrumental in initial studies on the transportation problem. Later on, Shetty [29] formulated and solved a transportation problem with nonlinear costs. Thereafter, a floodgate was opened for research in this field. At present, any supply chain problem is incomplete without considering and addressing the transportation problem. In recent years, many novel ideas and mathematical approaches have been introduced to address the problem of transportation.

Rani and Gulati [27] introduced a new approach to solve an unbalanced transportation problem in an imprecise environment. Goczyła and Cielątkowski [15] investigated a routing problem in a public transportation network over a given timeframe. Bandopadhyaya and Puri [5] studied the impairment

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of flows in a multi-index transportation problem with axial constraints. Biswas et al. [8] investigated a multiobjective fixed charge transportation problem in crisp and interval environments. Gupta et al. [16] worked on a multiobjective capacitated transportation problem and solved it using an α-cut approach. Ahmad and Adhami [3] studied the transportation problem with varying supply and demand, and with a probabilistic cost function.

Multiobjective transportation problems form a special class, which usually involve multiple, conflicting, and incommensurate objective functions. Several exhaustive studies have been conducted on multiobjective linear transportation problems. Among them, Zimmermann [34] fuzzy programming is remarkable for obtaining the optimal compromise solution to a multiobjective transportation problem. In recent years, Nomani et al. [25] developed an approach for multiobjective transportation problems. Gupta et al. [17] carried out a case study on multiobjective capacitated transportation problems in an uncertain environment. Roy et al. [28] investigated a multiobjective multi-item fixed-charge solid transportation problem in a fuzzy-rough environment for a transportation system. As mentioned above, most transportation problems (TP) involve multiple, conflicting, and incommensurate objective functions. However, in real life applications, it is not always viable to determine the exact values of the different parameters involved in a problem. Insufficient information is generally available based on previous know-how and experiences, resulting in uncertainty. Therefore, parameters can take different forms of uncertainty, such as fuzzy numbers and random variables with known mean and variance. If the parameters are random variables following some probability distribution, then such problems may be solved using a stochastic programming approach. Fuzzy techniques can be used if the uncertainties are due to vagueness or ambiguity.

Only a few approaches have been described in the literature to solve transportation problems that involve both stochasticity and fuziness and even fewer for those also involving multiple objectives. Chanas and Kuchta [9] introduced a concept for the optimal solution of a transportation problem with fuzzy cost coefficients. Liu and Kao [21] solved a fuzzy transportation problem by employing the extension principle. Daneva et al. [11] compared various methods for solving probabilistic transportation problems. Gi and Ishii [14] studied transportation problems with different types of uncertainty. Najafi et al. [24] proposed a method for solving fuzzy linear programming problems. Also, Das et al. [13] developed an approach for fully fuzzy fractional programming problems. Ahmad and Adhami [2] developed a neutrosophic programming approach to the multiobjective nonlinear transportation problem with fuzzy parameters. The goal programming technique for solving fully interval-valued intuitionistic fuzzy multiple objective transportation problems was studied by Malik and Gupta [23]. Adhami and Ahmad [1] proposed an approach based on a Pythagorean-hesitant fuzzy decision set for multiobjective transportation problems. Recent contributions related to this area can be found in [6], [10], [19], [26], [32] and [33].

With the development of fuzzy sets, neutrosophic sets (NSs) have recently emerged. Smarandache [31] introduced the concept of an NS. "Neutrosophic" literally means knowledge of neutral thoughts, Mahapatra [22]. Das [12] considered a transportation problem with pentagonal neutrosophic numbers where the supply, demand and transportation are uncertain. The neutrality/indeterminacy concept involved in the NS directed the premises for future research in this area. The neutrosophic compromise programming approach (NCPA), based on NS, was developed to obtain an optimum solution for multiobjective optimization problems. The NCPA considers three parameters, namely truth maximization (belongingness), indeterminacy (belongingness to some extent), and falsity minimization (non-belongingness). A perusal of the literature reveals that one missing development is an approach to solving transportation problems, when there are both fuzzy parameters and stochastic parameters. Such an approach would be especially benificial in the case of multiobjective nonlinear transportation problems. The present study aimed to fill this gap.

The paper is designed as follows. An introduction and literature review are included in Section 1, the definitions of a fuzzy set and neutrosophic set are presented in Section 2. Section 3 is dedicated to presenting the multiobjective nonlinear transportation problem. In Section 4, the exponential distribution and its involvement in the constraints of transportation problems is discussed. Also, it is explained how problems involving the exponential distribution can be converted into a deterministic form. In Section 5, a mathematical model of a multiobjective transportation problem, wherein the parameters involved in the objective functions are fuzzy in nature and the parameters involved in the constraints are stochastic, is presented. Section 6 describes the neutrosophic compromise programming approach, which is used to obtain a solution to such multiobjective transportation problems. An illustrative example to demonstrate the proposed approach is given in Section 8. In Section 10, some conclusions and the advantages of the approach used in the study are presented and future directions for research are given.

2. Definitions

2.1. Fuzzy set (FS)

Definition 1 ([7]). Let Y be a universal set and let $y \in Y$. A fuzzy set X in Y is a function $X : Y \to [0, 1]$.

Often $\mu_X(y)$ is used to denote the function X, and it is said that the fuzzy set X is characterized by the membership function $\mu_X(y) : Y \to [0, 1]$. The value $\mu_X(y)$ is interpreted as the degree to which y belongs to X.

Definition 2 ([7]). The triplets $\tilde{X}(p, q, r)$ denoting the lower, middle, and upper value of a membership function, is said to be a parabolic fuzzy number if its membership function is given by

$$\mu_{\tilde{X}}(y) = \begin{cases} \left(\frac{y-p}{q-p}\right)^2 & \text{if } p \leq y \leq q \\ 1 & \text{if } y = q \\ \left(\frac{r-y}{r-q}\right)^2 & \text{if } q \leq y \leq r \\ 0 & \text{otherwise} \end{cases}$$

The process of finding a crisp or deterministic value of a fuzzy number is called defuzzification [2]. The defuzzified value function d of the parabolic fuzzy number $\tilde{X}(p, q, r)$ is given by

$$d(\tilde{X}) = \frac{(p + 2q + r)}{4} \tag{1}$$

2.2. Intuitionistic fuzzy set (IFS)

Definition 3 ([4]). Let Y be a universal set. Then, an IFS W in Y, is given by the ordered triplet $W = \{y, \mu_W(y), \upsilon_W(y) | y \in Y\}$, where $\mu_W(y) : Y \to [0, 1]$, $\upsilon_W(y) : Y \to [0, 1]$, under the condition $0 \leq \mu_W(y) + \upsilon_W(y) \leq 1$, where $\mu_W(y)$ and $\upsilon_W(y)$ denote the membership and non-membership functions of the elements $y \in Y$ in the set W.

Definition 4 ([30]). An intuitionistic fuzzy number $\tilde{W}$ is said to be a trapeziodal intuitionistic fuzzy number (TrIFN) if the membership function $\mu_{\tilde{W}}(y)$ and non-membership function $\upsilon_{\tilde{W}}(y)$ are given by

$$\mu_{\tilde{W}}(y) = \begin{cases} \frac{y-a_1}{a_2-a_1} & \text{if } a_1 \leq y \leq a_2 \\ 1 & \text{if } a_2 \leq y \leq a_3 \\ \frac{a_4-y}{a_4-a_3} & \text{if } a_3 \leq y \leq a_4 \\ 0 & \text{otherwise} \end{cases}$$

and

$$\upsilon_{\tilde{W}}(y) = \begin{cases} \frac{a'_2-y}{a'_2-a'_1} & \text{if } a'_1 \leq y \leq a'_2 \\ 0 & \text{if } a'_2 \leq y \leq a'_3 \\ \frac{y-a'_3}{a'_4-a'_3} & \text{if } a'_3 \leq y \leq a'_4 \\ 1 & \text{otherwise} \end{cases}$$

where $a_1' \leq a_1 \leq a_2' \leq a_2 \leq a_3 \leq a_3' \leq a_4 \leq a_4'$.

2.3. Neutrosophic set (NS)

Definition 5 ([31]). Let Y be a universal set, such that $y \in Y$, then a neutrosophic set A in Y is defined by three membership functions, viz., truth $T_A(y)$, indeterminancy $I_A(y)$, and falsity $F_A(y)$ and is denoted by the following form:

$$A = \{\langle y, T_A(y), I_A(y), F_A(y)\rangle | y \in Y\}$$

where $T_A(y), I_A(y)$, and $F_A(y)$ are real standard or non-standard subsets belonging to $]0^-, 1^+[$ also given as $T_A(y) : Y \rightarrow]0^-, 1^+[$, $I_A(y) : Y \rightarrow]0^-, 1^+[$, and $F_A(y) : Y \rightarrow]0^-, 1^+[$. Also, there is no restriction on the sum of $T_A(y), I_A(y)$, and $F_A(y)$, so we have

$$0^- \leq \sup T_A(y) + I_A(y) + \sup F_A(y) \leq 3^+$$

Definition 6 ([31]). A single valued neutrosophic set (SVNS) A, over a universal set Y, is defined as

$$A = \{\langle y, T_A(y), I_A(y), F_A(y)\rangle | y \in Y\}$$

where $T_A(y), I_A(y)$, and $F_A(y) \in [0, 1]$ and $0 \leq T_A(y) + I_A(y) + F_A(y) \leq 3$, for each $y \in Y$.

Definition 7 ([31]). Let A and B be two SVNSs. Then the union of A and B is also a single valued neutrosophic set C, that is $C = (A \cup B)$, whose truth $T_C(y)$, indeterminacy $I_C(y)$ and falsity $F_C(y)$ membership functions are given by

$$T_C(y) = \max(T_A(y),_B(y))$$
$$I_C(y) = \max(I_A(y), I_B(y))$$
$$F_C(y) = \min(F_A(y), F_B(y))$$

for each $y \in Y$.

Definition 8 ([31]). Let A and B be two SVNSs. Then the intersection of A and B is also a single valued neutrosophic set C, that is, $C = (A \cap B)$, whose truth $T_C(y)$, indeterminacy $I_C(y)$ and falsity $F_C(y)$ membership functions are given by

$$T_C(y) = \min(T_A(y), T_B(y))$$
$$I_C(y) = \min(I_A(y), I_B(y))$$
$$F_C(y) = \max(F_A(y), F_B(y))$$

for each $y \in Y$.

3. Multiobjective nonlinear transportation problems

The classical transportation problem can be described as a special case of the linear programming problem and its model is applied for determining the number of units of some commodity to be transported from each origin to various destinations, satisfying the supply and demand constraints while achieving the prescribed objective(s). In the present study, a multiobjective transportation problem with K objectives is considered. There are m origins with availability of a_i, $i = 1, \ldots, m)$ units and n destinations with demand of b_j, $j = 1, \ldots, n$ units. The cost associated with the kth objective is represented by c_{ij}^k. The decision variables are the number of units to be transported from the ith origin to the jth destination,

$i = 1, \dots, m$, $j = 1, \dots, n$, and denoted by x_{ij}. Thus, the model for the multiobjective transportation problem is given as follows:

$$\begin{aligned}
\min \quad & Z_k = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^k x_{ij} & \\
\text{s.t.} \quad & \sum_{j=1}^{n} x_{ij} \le a_i & i = 1, \dots, m \\
& \sum_{i=1}^{m} x_{ij} \ge b_j & j = 1, \dots, n \\
& \sum_{i=1}^{m} a_i \ge \sum_{j=1}^{n} b_j & \\
& x_{ij} \ge 0 & i = 1, \dots, m, \ j = 1, \dots, n
\end{aligned}$$

Maity and Roy [28] argued that due to the instability of various players in the market, there may exist cases where the cost parameter per unit of commodity changes according to the number of goods delivered to a destination from the point of origin, along with the source capacity of the supply. Because of this, an extra cost is incurred, which can be given as follows:

$$\text{Extra cost} = \frac{a_i - \text{goods transported from } i\text{th origin to } j\text{th destination}}{\text{availability of goods at } i\text{th origin}}$$

$$c_{ij}^k = \frac{a_i - x_{ij}}{a_i} c_{ij}^k$$

After incorporating this change, the multiobjective nonlinear transportation problem becomes

$$\begin{aligned}
\min \quad & Z_k = 2 \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^k x_{ij} - \sum_{i=1}^{m} \frac{1}{a_i} \sum_{j=1}^{n} c_{ij}^k x_{ij}^2 & \\
\text{s.t.} \quad & \sum_{j=1}^{n} x_{ij} \le a_i & i = 1, \dots, m \\
& \sum_{i=1}^{m} x_{ij} \ge b_j & j = 1, \dots, n \\
& x_{ij} \ge 0 & i = 1, \dots, m, \ j = 1, \dots, n
\end{aligned}$$

In the above problem, the cost function is nonlinear if at least one $\sum_{j=1}^{n} x_{ij} \le a_i$, $i = 1, \dots, m$ is satisfied, otherwise the cost function is linear.

3.1. Multiobjective nonlinear transportation problem with fuzzy parameters

As discussed in Section 1, uncertainty plays a pivotal role in formulating a model for decision making, and uncertainty can be probabilistic or fuzzy. Fuzziness in the parameters is due to the vagueness caused by the non-exact estimates of the parameters provided by the decision maker. In such circumstances, this uncertainty should also be considered while solving the problem. In this study, it is assumed that the cost parameter involved in the objective function is of the fuzzy type. This fuzziness is transformed into a crisp or deterministic number by using ranking function techniques based on fuzzy set theory. The fuzzified transportation problem under consideration can be given as follows:

$$\begin{aligned}
\min \quad & Z_k = 2\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}^{k}x_{ij} - \sum_{i=1}^{m}\frac{1}{a_i}\sum_{j=1}^{n}\tilde{c}_{ij}^{k}x_{ij}^{2} & \\
\text{s.t.} \quad & \sum_{j=1}^{n}x_{ij} \le a_i & i=1,\dots,m \\
& \sum_{i=1}^{m}x_{ij} \ge b_j & j=1,\dots,n \\
& x_{ij} \ge 0, & i=1,\dots,m,\ j=1,\dots,n
\end{aligned}$$

The cost parameter involved in the objective function of the above problem is assumed to be a parabolic fuzzy number and can be defuzzified using equation (1).

$$\begin{aligned}
\min \quad & Z_k = 2\sum_{i=1}^{m}\sum_{j=1}^{n}d(\tilde{c}_{ij}^{k})x_{ij} - \sum_{i=1}^{m}\frac{1}{a_i}\sum_{j=1}^{n}d(\tilde{c}_{ij}^{k})x_{ij}^{2} & \\
\text{s.t.} \quad & \sum_{j=1}^{n}x_{ij} \le a_i & i=1,\dots,m \\
& \sum_{i=1}^{m}x_{ij} \ge b_j & j=1,\dots,n \\
& x_{ij} \ge 0 & i=1,\dots,m,\ j=1,\dots,n
\end{aligned}$$

where $d(\tilde{c}_{ij}^{k})$ is the defuzzified value of $\tilde{c}_{ij}^{k}$.

4. Multiobjective nonlinear transportation problem with exponentially distributed constraint parameters

In this section, we consider a stochastic model for multiobjective nonlinear transportation problems where supply and demand follow exponential distributions.

$$\begin{aligned}
\min \quad & Z_k = 2\sum_{i=1}^{m}\sum_{j=1}^{n}d(\tilde{c}_{ij}^{k})x_{ij} - \sum_{i=1}^{m}\frac{1}{a_i}\sum_{j=1}^{n}d(\tilde{c}_{ij}^{k})x_{ij}^{2} & \\
\text{s.t.} \quad & P\left(\sum_{j=1}^{n}x_{ij} \le a_i\right) \ge 1-\alpha_i & i=1,\dots,m \qquad (2) \\
& P\left(\sum_{i=1}^{m}x_{ij} \ge b_j\right) \ge 1-\beta_j & j=1,\dots,n \qquad (3) \\
& x_{ij} \ge 0 & i=1,\dots,m,\ j=1,\dots,n
\end{aligned}$$

where $0 < a_i < 1$ and $0 < \beta_i < 1$, $i = 1,\dots,m$, $j = 1,\dots,n$.

Let a_i, $i = 1,\dots,m$, be independent exponential random variables with known means θ_i. The probability density function (pdf) and distribution function (df) of a_i are given by

$$f(a_i) = \frac{1}{\theta_i}\mathrm{e}^{-\frac{1}{\theta_i}a_i}, a_i \ge 0 \text{ and } \theta_i > 0$$

and

$$F(a_i) = 1 - \mathrm{e}^{\frac{1}{\theta_i}a_i}, a_i \ge 0 \text{ and } \theta_i > 0 \qquad (4)$$

Inequality (2) can be expressed as

$$P\left(a_i \leq \sum_{j=1}^{n} x_{ij}\right) \leq \alpha_i$$

Using (4), we get

$$1 - e^{\frac{1}{\theta_i}\sum_{j=1}^{n} x_{ij}} \leq \alpha_i \tag{5}$$

After simplification of (5), we obtain equivalent deterministic constraints, as given by

$$\sum_{j=1}^{n} x_{ij} \leq \theta_i \ln(1-\alpha_i) \quad i = 1, \ldots, m$$

Similarly, let b_j, $j = 1, \ldots, n$, be independent exponential random variables with known means μ_j, the pdf and df of b_j are given by

$$f(b_j) = \frac{1}{\mu_j}\mathrm{e}^{-\frac{1}{\mu_j}b_j} \quad , b_j \geq 0 \text{ and } \mu_j > 0$$

and

$$F(b_j) = 1 - \mathrm{e}^{-\frac{1}{\mu_j}b_j} \quad , b_j \geq 0 \text{ and } \mu_j > 0 \tag{6}$$

Using (6), inequality (3) can be transformed into deterministic constraints given by

$$\sum_{i=1}^{m} x_{ij} \geq -\mu_j(\ln \beta_j) \quad j = 1, \ldots, n$$

5. Multiobjective nonlinear transportation problem with fuzzy and stochastic parameters

Before coming to the final model, the case when all the parameters are fuzzy numbers is discussed. The multiobjective nonlinear transportation problem with fuzzy parameters can be mathematically presented as follows:

$$\begin{aligned}
\min \quad & Z_k = 2\sum_{i=1}^{m}\sum_{j=1}^{n} d(\tilde{c}_{ij}^k)x_{ij} - \sum_{i=1}^{m}\frac{1}{a_i}\sum_{j=1}^{n} d(\tilde{c}_{ij}^k)x_{ij}^2 & \\
\text{s.t.} \quad & \sum_{j=1}^{n} x_{ij} \leq \tilde{a}_i & i = 1, \ldots, m \\
& \sum_{i=1}^{m} x_{ij} \geq \tilde{b}_j & j = 1, \ldots, n \\
& x_{ij} \geq 0 & i = 1, \ldots, m, \; j = 1, \ldots, n
\end{aligned}$$

The cost, supply and demand parameters involved in the objective function of the above problem are assumed to be parabolic fuzzy numbers and can be defuzzified using equation (1). Now it is assumed that both types of parameters, supply and demand, follow an exponential distribution. It is also assumed that the mean and variance of a_i and b_j are known and defined as previously. Considering the properties of the exponential distribution discussed in Section 4, the model for the multiobjective nonlinear transportation problem with fuzzy and stochastic parameters will be given as follows:

$$\begin{aligned}
\min \quad & Z_k = 2\sum_{i=1}^{m}\sum_{j=1}^{n} d(\tilde{c}_{ij}^k)x_{ij} - \sum_{i=1}^{m}\frac{1}{a_i}\sum_{j=1}^{n} d(\tilde{c}_{ij}^k)x_{ij}^2 & \\
\text{s.t.} \quad & P\left(\sum_{j=1}^{n} x_{ij} \le a_i\right) \ge 1-\alpha_i & i=1,\ldots,m \\
& P\left(\sum_{i=1}^{m} x_{ij} \ge b_j\right) \ge 1-\beta_j & j=1,\ldots,n \\
& x_{ij} \ge 0 & i=1,\ldots,m,\ j=1,\ldots,n
\end{aligned}$$

The equivalent deterministic form, as discussed in Section 4 of the above problem, is given as follows:

$$\begin{aligned}
\min \quad & Z_k = 2\sum_{i=1}^{m}\sum_{j=1}^{n} d(\tilde{c}_{ij}^k)x_{ij} - \sum_{i=1}^{m}\frac{1}{a_i}\sum_{j=1}^{n} d(\tilde{c}_{ij}^k)x_{ij}^2 & \\
\text{s.t.} \quad & \sum_{j=1}^{n} x_{ij} \le \theta_i \ln(1-\alpha_i) & i=1,\ldots,m \\
& \sum_{i=1}^{m} x_{ij} \ge -\mu_j(\ln\beta_j) & j=1,\ldots,n \\
& x_{ij} \ge 0 & i=1,\ldots,m,\ j=1,\ldots,n
\end{aligned}$$

6. Neutrosophic compromise programming approach (NCPA)

An approach has been proposed to solve multiobjective nonlinear transportation problems with fuzzy and stochastic parameters. In this approach, three membership functions are considered, namely maximization of the degree of truth, maximization of indeterminacy, and minimization of the falsity membership function. If a fuzzy decision is denoted by F, fuzzy goal by D, and fuzzy constraints by C, then the neutrosophic decision set, denoted by F_N, can be defined as

$$F_N = (\cap_{k=1}^{K} D_k)(\cap_{l=1}^{L} C_l) = (x, T_F(x), I_F(x), D_F(x)) \tag{7}$$

where

$$T_F(x) = \max\left\{ \begin{array}{c} T_{D_1}(x), T_{D_2}(x), T_{D_3}(x) \\ T_{C_1}(x), T_{C_2}(x) \end{array} \right\} \forall x \in X \tag{8}$$

$$I_F(x) = \max\left\{ \begin{array}{c} I_{D_1}(x), I_{D_2}(x), I_{D_3}(x) \\ I_{C_1}(x), I_{C_2}(x) \end{array} \right\} \forall x \in X \tag{9}$$

$$D_F(x) = \min\left\{ \begin{array}{c} F_{D_1}(x), F_{D_2}(x), F_{D_3}(x) \\ F_{C_1}(x), F_{C_2}(x) \end{array} \right\} \forall x \in X \tag{10}$$

where $T_F(x), I_F(x)$, and $D_F(x)$ are the truth membership function, indeterminacy membership function, and falsity membership function of the neutrosophic decision set F_N.The lower and upper bounds for each objective function at different levels can be obtained as follows. Firstly, under given constraints, the objective function at a given level is derived. A set of K solutions is obtained by solving the problem for a given set of constraints with an objective at each level. Let these solution sets be denoted by $X^1, X^2, \cdots, X^K$. These solutions provide lower and upper bounds for each objective as follows:

$$U_k = \max[Z_k(X^k)] \text{ and } L_k = \min[Z_k(X^k)], k = 1,\ldots,K \tag{11}$$

The lower and upper bounds can now be obtained as follows [2]. For truth membership

$$U_k^T = U_k, L_k^T = L_k \tag{12}$$

For indeterminacy membership

$$U_k^I = L_k^T + s_k, L_k^I = L_k^T \tag{13}$$

For falsity membership

$$U_k^D = U_k^T, L_k^D = L_k^T + t_k, \tag{14}$$

where s_k and $t_k \in (0, 1)$ are predetermined real numbers assigned by the decision makers. Using these lower and upper bounds, under a neutrosophic environment the linear membership functions are defined as follows:

$$T_k(Z_k(x)) = \begin{cases} 1 & \text{if } Z_k(x) < L_k^T \\ 1 - \frac{U_k^T - Z_k(x)}{U_k^T - L_k^T} & \text{if } L_k^T \leq Z_k(x) \leq U_k^T \\ 0 & \text{if } Z_k(x) > U_k^T \end{cases} \tag{15}$$

$$I_k(Z_k(x)) = \begin{cases} 1 & \text{if } Z_k(x) < L_k^I \\ \frac{U_k^I - Z_k(x)}{U_k^I - L_k^I} & \text{if } L_k^I \leq Z_k(x) \leq U_k^I \\ 0 & \text{if } Z_k(x) > U_k^I \end{cases} \tag{16}$$

$$D_k(Z_k(x)) = \begin{cases} 1 & \text{if } Z_k(x) < L_k^D \\ 1 - \frac{z_k(x) - L_k^D(x)}{U_k^D - L_k^D} & \text{if } L_k^D \leq Z_k(x) \leq U_k^D \\ 0 & \text{if } Z_k(x) < L_k^D \end{cases} \tag{17}$$

For all the objective functions, $L_k^{(.)} \neq U_k^{(.)}$. If $L_k^{(.)} = U_k^{(.)}$, then the value of the corresponding membership function will be equal to 1. Using the approach described in Bellman and Zadeh [7], the problem can be presented as follows:

$$\begin{aligned} & \max \min_{k=1,\ldots,K} T_k(Z_k(x)) \\ & \max \min_{k=1,\ldots,K} I_k(Z_k(x)) \\ & \min \max_{k=1,\ldots,K} D_k(Z_k(x)) \\ \text{s.t.} & \sum_{j=1}^{n} x_{ij} \leq \theta_i \ln(1 - \alpha_i) && i = 1, \ldots, m && (18) \\ & \sum_{i=1}^{m} x_{ij} \geq -\mu_j (\ln \beta_j) && j = 1, \ldots, n && (19) \\ & x_{ij} \geq 0 && i = 1, \ldots, m, j = 1, \ldots, n \end{aligned}$$

The above problem can be transformed into the following form, using auxiliary parameters.

$$
\begin{aligned}
\max\ & \alpha && (20)\\
\max\ & \beta && (21)\\
\min\ & \gamma && (22)\\
\text{s.t. } & T_K(Z_k(x)) \geq \alpha && (23)\\
& I_K(Z_k(x)) \geq \beta && (24)\\
& D_K(Z_k(x)) \leq \gamma && (25)\\
& \sum_{j=1}^{n} x_{ij} \leq -\theta_i \ln(1-\alpha_i) \qquad i=1,\dots,m && (26)\\
& \sum_{i=1}^{m} x_{ij} \geq -\mu_j (\ln \beta_j) \qquad j=1,\dots,n && (27)\\
& x_{ij} \geq 0 \qquad i=1,\dots,m, j=1,\dots,n
\end{aligned}
$$

With the help of a linear membership function, the above problem can further be written as follows:

$$
\begin{aligned}
\max\ & \alpha + \beta - \gamma && (28)\\
\text{s.t. } & \sum_{j=1}^{n} x_{ij} \leq -\theta_i \ln(1-\alpha_i) \qquad i=1,\dots,m && (29)\\
& \sum_{i=1}^{m} x_{ij} \geq -\mu_j (\ln \beta_j) \qquad j=1,\dots,n && (30)\\
& x_{ij} \geq 0 \qquad i=1,\dots,m, j=1,\dots,n\\
& Z_k(x) + (U_k^T - L_k^T)\alpha \leq U_k^T && (31)\\
& Z_k(x) + (U_k^I - L_k^I)\beta \leq U_k^I && (32)\\
& Z_k(x) - (U_k^D - L_k^D)\gamma \leq L_k^D && (33)\\
& \alpha \geq \beta, \alpha \geq \gamma, \alpha + \beta + \gamma \leq 3 && (34)\\
& \alpha, \beta, \gamma \in (0,1) && (35)
\end{aligned}
$$

Thus, the above model gives a compromise solution to the multiobjective nonlinear transportation problem with fuzzy and stochastic parameters.

7. Stepwise algorithm for NCPA

A summary of the steps involved in the proposed approach can be presented as follows:

Step 1. Formulate the multiobjective nonlinear transportation problem with fuzzy and stochastic parameters as discussed in Section 5.

Step 2. Using the defuzzification method, as given in equation (1), convert the problem into a crisp and deterministic form, as discussed in Section 5.

Step 3. Construct a payoff matrix by deriving the objective function for the set of constraints at each level individually.

Step 4. For each level, determine the upper and lower bounds for each objective function.

Step 5. Define the upper and lower bounds, as in equations (12)–(14), for the truth, indeterminacy, and falsity membership functions.

Step 6. Under a neutrosophic environment, define the linear membership function as in equations (15)–(17).

Step 7. Formulate the neutrosophic problem defined in equations (18)–(27) and transform it into a neutrosophic compromise programming problem as desribed in equations (28)–(35).
Step 8. Solve the transformed multiobjective nonlinear transportation problem with fuzzy and stochastic parameters using an optimization software package.

8. Illustrative example

To demonstrate the proposed approach, an illustrative example is considered, using the method described in Ahmad and Adhami [2], with some modifications. A new product is launched from four outlets a_1, a_2, a_3, a_4, to retailers at four sites b_1, b_2, b_3, b_4. Since the product is being launched for the first time, the decision makers are in a dilemma over the costs incurred in transporting the product. For the same reason, supply and demand are non-deterministic in nature. Due to the lack of appropriate information, the decision makers assume that the cost parameters are fuzzy and the supply and demand parameters are stochastic. The proposed NCPA approach is applied to solve a multiobjective nonlinear transportation problem with fuzzy and stochastic parameters with three different objectives. The transportation cost per unit from different sources to various destinations $(\tilde{c}_{ij})$, is given in Table 1. The means of the exponential random variables, together with specified probability levels of supplies and demands, the supply and demand data, is given in Table 2 and Table 3, respectively.

Table 1. Transportation cost per unit $\tilde{c}_{ij}$ in Thousand

	b_1	b_2	b_3	b_4
a_1	$(2.04, 2.08, 2.12)$	$(1.6, 1.9, 2.2)$	$(2.4, 2.5, 2.6)$	$(2.5, 3.5, 4.5)$
a_2	$(1.2, 2.2, 3.2)$	$(1.4, 1.8, 2.2)$	$(2.8, 3.8, 4.8)$	$(0, 2, 4)$
a_3	$(0, 2, 4)$	$(2, 2.5, 3)$	$(1.2, 1.7, 2.2)$	$(1.4, 1.6, 1.8)$
a_4	$(2, 2.2, 2.4)$	$(2.5, 2.9, 3.3)$	$(2.2, 2.8, 3.4)$	$(2.2, 2.4, 2.6)$

Assuming that the means of the exponential random variables, together with specified probability levels of supplies and demands, the supply and demand data, is given in Table 2 and Table 3, respectively.

Table 2. Mean and probability level of supply

Mean	Specified probability level
$E(a_1) = \theta_1 = 950$	$\alpha_1 = 0.03$
$E(a_2) = \theta_2 = 637$	$\alpha_2 = 0.04$
$E(a_3) = \theta_3 = 623$	$\alpha_3 = 0.05$
$E(a_4) = \theta_4 = 452$	$\alpha_4 = 0.06$

Table 3. Mean and probability level of demand

Mean	Specified probability level
$E(b_1) = \mu_1 = 950$	$\beta_1 = 0.06$
$E(b_2) = \mu_2 = 637$	$\beta_2 = 0.07$
$E(b_3) = \mu_3 = 623$	$\beta_3 = 0.08$
$E(b_4) = \mu_4 = 452$	$\beta_4 = 0.09$

Table 4. Pay-off matrix

Decision variables	Objective function values		
	Z_1	Z_2	Z_3
$(0, 0, 29, 0, 0, 22, 0, 0, 0, 32, 22, 0, 0, 0)$	235.856	2909.68	117.973
$(0, 0, 29, 0, 22, 0, 0, 2, 0, 0, 0, 32, 0, 22, 0, 0)$	264.71	2898.98	112.497
$(22, 0, 0, 7, 0, 0, 0, 26, 0, 0, 29, 1, 0, 22, 0, 0)$	286.435	3166.87	76.6186

As discussed in Section 6, the problem can be formulated as follows:

$$\begin{aligned}
\min \ \ Z_1 &= 2\sum_{i=1}^{4}\sum_{j=1}^{4} d(\tilde{c}_{ij}^{k})x_{ij} - \sum_{i=1}^{4}\frac{1}{a_i}\sum_{j=1}^{4} d(\tilde{c}_{ij}^{1})x_{ij}^2 \\
\min \ \ Z_2 &= 2\sum_{i=1}^{4}\sum_{j=1}^{4} d(\tilde{c}_{ij}^{2})x_{ij} - \sum_{i=1}^{4}\frac{1}{a_i}\sum_{j=1}^{4} d(\tilde{c}_{ij}^{2})x_{ij}^2 \\
\min \ \ Z_3 &= 2\sum_{i=1}^{4}\sum_{j=1}^{4} d(\tilde{c}_{ij}^{3})x_{ij} - \sum_{i=1}^{4}\frac{1}{a_i}\sum_{j=1}^{4} d(\tilde{c}_{ij}^{3})x_{ij}^2 \\
\text{s.t.} \ \ & \sum_{j=1}^{4} x_{ij} \le \theta_1 \ln(1-\alpha_1) \\
& \sum_{j=1}^{4} x_{ij} \le \theta_2 \ln(1-\alpha_1) \\
& \sum_{j=1}^{4} x_{ij} \le \theta_3 \ln(1-\alpha_1) \\
& \sum_{j=1}^{4} x_{ij} \le \theta_4 \ln(1-\alpha_1) \\
& \sum_{i=1}^{4} x_{ij} \ge -\mu_1 \ln(1-\beta_1) \\
& \sum_{i=1}^{4} x_{ij} \ge -\mu_2 \ln(1-\beta_2) \\
& \sum_{i=1}^{4} x_{ij} \ge -\mu_3 \ln(1-\beta_3) \\
& \sum_{i=1}^{4} x_{ij} \ge -\mu_4 \ln(1-\beta_4) \\
& x_{ij} \ge 0 \quad i = 1,\dots,m, j = 1,\dots,n
\end{aligned}$$

After deriving the payoff according to each individual objective, the crisp model of the multiobjective nonlinear transportation problem with fuzzy and stochastic parameters gives the pay-off matrix shown in Table 4

As discussed in Section 6, the upper and lower bounds for truth, indeterminacy, and falsity functions are obtained and the membership function for the three objectives based on neutrosophic sets are constructed. The simplified neutrosophic model for the multiobjective nonlinear transportation problem with fuzzy and stochastic parameters is then formulated and solved using LINGO 16.0, on Intel(R) core i5-4210U CPU @1.7 GHz, and 8 GB of RAM. The results are shown in Table 5.

Table 5. Optimal solution

Decision variables																Objective values		
x_{11}	x_{12}	x_{13}	x_{14}	x_{21}	x_{22}	x_{23}	x_{24}	x_{31}	x_{32}	x_{33}	x_{34}	x_{41}	x_{42}	x_{43}	x_{44}	Z_1	Z_2	Z_3
5	22	0	2	17	0	0	9	0	0	29	3	1	0	0	1	236.856	2899.98	77.6186

9. Comparative study

The solution of the neutrosophic model for the multiobjective nonlinear transportation problem with fuzzy and stochastic parameters is obtained using NCPA. Three membership functions are considered, viz., truth, indeterminacy, and falsity. This provides more elasticity in decision making and is ultimately able to improve the solution. To demonstrate the efficiency of the NCPA approach more rationally, the problem discussed in Section 8 is compared with the fuzzy programming approach (FPA) developed by Ahmad and Adhami [2]. These results are compared in Table 6.

Table 6. Comparison of the solutions derived by NCPA and FPA. Decision variables and objectives

Decision variables and objectives	Solution using NCPA	Solution using FPA
x_{11}	5	0
x_{12}	22	22
x_{13}	0	1
x_{14}	2	0
x_{21}	17	22
x_{22}	0	0
x_{23}	0	0
x_{24}	9	2
x_{31}	0	0
x_{32}	0	0
x_{33}	29	0
x_{34}	3	32
x_{41}	1	0
x_{42}	0	0
x_{43}	0	28
x_{44}	1	0
Z_1	236.856	249.942
Z_2	2899.98	2912.66
Z_3	77.6186	89.3642

10. Conclusion

The transportation problem plays an important role in the overall profitability of any manufacturing firm. Insufficient consideration of the variability involved in the transportation problem can result in chaos, due to which the firm may well lose both money and the goodwill of its clients. In the present study, uncertainties regarding the parameters in a mathematical model of a multiobjective transportation problem are considered. These uncertainties are of both fuzzy and stochastic types and are shown to be efficiently resolved. Thus, NCPA is proposed to obtain the best compromise solution, which is significantly better than the one obtained using the FPA as shown in Table 6. The main contribution of this paper is summarized below:

- Both fuzzy and stochastic parameters are considered in the model, which may be suitable for many real-life problems.
- To the best of the authors' knowledge, neutrality/indeterminacy is an area that is not well-explored.
- A comparative study of the proposed approach (NCPA) with an existing approach (FPA) indicates that the NCPA is significantly better than FPA.
- The proposed approach can be very efficiently applied to the problem of selecting suppliers, advertizing problem, and portfolio selection problem.

This study was limited to consideration of supply and demand that follow the exponential distribution. In addition, fuzzy parameters may be replaced by intuitionistic parameters. As future directions for research, the proposed approach should be applied in conjunction with different distributions and intuitionistic sets.

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