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Four-dimensional uncertain multi-objective multi-item transportation problem

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Abstract

This paper considers a four-dimensional multi-objective multi-item transportation problem (4DMOMITP), where all the parameters are regarded as uncertain variables. In this paper, three mathematical models, namely expected value model (EVM), optimistic value model (OVM) and dependent optimistic-constrained model (DOCM), are discussed for the uncertain model of 4DMOMITP. These models are converted into their corresponding deterministic forms using different ranking criteria from uncertainty theory. These deterministic models are then solved by using the Lingo 18.0 software, utilizing three different classical approaches for obtaining a solution. A numerical example is given to illustrate the application of the model and the solution algorithm. A sensitivity analysis for the OVM and DOCM models has also been performed with respect to the confidence levels.

Keywords: four-dimensional, transportation problem, uncertain programming, optimistic value model, fuzzy technique

1. Introduction

In today's world, transportation plays a major role in the daily life of human beings. It is necessary for things to be moved around and as transportation systems have developed over time, the speed and efficiency of these systems have improved drastically. To model situations involving the transportation of goods, the transportation problem (TP) was introduced by Hitchock [13] in 1941. The objective is to transfer goods from available source centres to destination centres such that the total transportation cost is minimized. The transportation model takes into account restrictions in the form of natural constraints. These constraints are generally considered for handling the two main features of TP, namely: the availability of sources (source constraints) and demand from consumers (demand constraints). However, in a real system, we always deal with other constraints, e.g. regarding the type of the product or the mode of transportation. For handling such cases, the classical TP can be extended to the solid transportation problem (STP), which deals with an additional constraint based on transportation mode (conveyance),

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besides the two key constraints of the classical TP. STP was first presented by Schell [33] and a procedure for solving it was developed by Haley [12] in 1962. STP has gained an immense amount of interest among various researchers in the last few years.

Since most industrial problems require the consideration of multiple conflicting objectives simultaneously rather than only a single objective, multi-objective transportation problems (MOTPs) were introduced. These treat all the objectives simultaneously. Zimmermann [39] gave a fuzzy programming technique to solve MOTPs. In STP, it is assumed that a single homogeneous product is to be transported from various sources to destinations, but there are situations in which a company produces two or more products at a source. Due to this, the TP was extended to the multi-item transportation problem (MITP). When STP is studied along with multiple objectives and multiple items, it is known as a multi-objective multi-item solid transportation problem (MOMISTP). When dealing with practical situations, it is often difficult to evaluate the precise/actual values of the parameters involved, such as transportation cost, delivery time, the capacity of the suppliers and demand of the consumers and conveyance capacities. These parameters can fluctuate due to several factors, such as uncertainty in judgement, weather conditions or incomplete information. Therefore, to deal with an imprecise environment, different theories were developed , e.g. fuzzy set theory [36], probability theory [17], interval theory [24]. In 2007, Liu [20] introduced another way to represent imprecise data, known as uncertainty theory. This theory is based on human's degree of belief in various scenarios and is used to appropriately evaluate the personal degree of beliefs of experts in terms of an uncertainty measure. In recent years, there have been a wide number of real-world applications based on such models of imprecise environments. Erik Kropat et al. [18] studied the concept of fuzzy target environment networks with fuzzy-regression models by considering fuzzy data sets. Erik Kropat et al. [19] also went beyond traditional stochastic approaches and proposed semi-algebraic gene-environment networks that rely on data originating from different disciplines, such as chemistry and biology. Haripriya et al. [1] studied an economic production quantity model for deteriorating items with time-dependent demand and shortages including partially back-ordered demand with a cloudy fuzzy environment. Roy et al. [30] considered a probabilistic inventory model for deteriorating items with a price discount on back-orders. Lotfi et al. [23] considered interdependent demand in the two-period newsvendor problem with probabilistic demand.

In the field of transportation problems, plenty of research work has already been carried out with various models of imprecise environments, such as fuzzy theory, probability theory or interval theory. In 1963, Williams [35] constructed a model for TP in a stochastic environment, where the undetermined parameters were assumed to be random variables. Cerulli et al. [3] and D'Ambrosio et al. [9] have studied TP models with variables considered as interval numbers. Jimenez and Verdegay [14] studied STP with two kinds of indeterminacy in the data by considering interval STP and fuzzy STP. Ojha et al. [26] proposed an entropy-based STP under a fuzzy environment. Pratihar et al. [27] gave a modified version of Vogel's approximation method for a fuzzy transportation problem with interval type-2 fuzzy sets. Muthuperumal et al. [25] provided an algorithmic approach to solving unbalanced triangular fuzzy TP.

Probability theory is applicable when we can derive a probability distribution very close to the actual frequencies observed and to obtain such a distribution, we must have an adequate sample size. However, under certain circumstances, we might face problems in accessing historical data and have no samples available. In such situations, we need to employ experts in a related domain to estimate the degree of belief for a range of possible scenarios. However, a number of surveys have found that humans generally estimate a much wider scope of values for parameters than occur in reality. This means that degrees of belief deviate a lot from actual frequencies. Thus, degrees of belief and probability distributions cannot be treated equivalently, otherwise some absurd situation may arise. As a result, in 2007 Liu [20] presented a theory called uncertainty theory to handle the degree of belief using mathematical concepts. In 2009, Liu [21] introduced the theory of uncertain programming. A wide range of applications of uncertain programming has been introduced in various fields of operations research. In 2012, Sheng and Yao [34] examined TP in an uncertain environment treating all the variables as uncertain variables, instead of random variables or fuzzy variables. Furthermore, STP formulated as an expected constrained programming model in an uncertain environment was solved by Cui and Sheng [7]. Guo et al. [10] considered a TP that took into account costs as uncertain variables and supply capacity under a random environment. Chen et al. [5] proposed an entropy based STP in an uncertain environment. Recently, Zhao and Pan [38] generalized models of transportation under an uncertain environment by proposing a new transportation model with transfer costs in which all the variables, including transfer costs, are assumed to be uncertain variables.

Generally, the mathematical model of TP does not consider the possibility of choosing the paths or routes between the sources and destinations. In real life, different routes or paths are available for travel between sources and destinations. In transportation, such choices play a vital role, and if the possible routes are considered, as well as the source-destination-conveyance constraints, in STP, then it is known as the four-dimensional TP (4DTP). Halder et al. [11] solved a four dimensional fixed charge MITP in crisp and fuzzy environments. Bera et al. [2] formulated a 4D multi-item TP with a budget constraint under rough and fuzzy intervals. Recently, Samanta et al. [32] presented a novel multi-objective multi-item 4DTP in an intuitionistic fuzzy environment. Sahoo et al. [31] established a four-dimensional multi-objective multi-item TP using the GP technique. Revathi et al. [29] considered a four-dimensional multi-objective multi-item TP with vehicle costs under an uncertain environment. A literature survey of transportation problems under uncertainty is presented in Table 1. In fact, a core issue in such transportation problems is the ordering (ranking) of the uncertain variables. Therefore, to tackle the uncertain model, Liu [22] introduced four ranking criteria to fix this issue. These four criteria are: the expected value criterion, the optimistic value criterion, the pessimistic value criterion and the chance-criterion. To the best of our knowledge, in the available literature the transportation problem has been addressed using the expected value criterion and the chance criterion to transform the model under uncertainty into a deterministic form, but no one has yet used the optimistic value criterion to deal with such TPs. Hence, this paper aims to solve an uncertain four-dimensional MOMITP (4DUMOMITP) using the expected and optimistic value criteria along with the chance-criterion. Also, Charnes and Cooper's transformation [4] has been applied to convert the fractional objectives in a deterministic DOCM model into linear form.

Table 1. Review of existing literature and proposed work applying uncertainty theory to transportation problems

Author name	Dimension	Item	Objective	Ranking criteria used		
				EVC	OVC	CC
Sheng & Yao [34]	2	single	single	✓	×	×
Zhao et al. [38]	2	single	single	✓	×	×
Kakran & Dhodiya [16]	2	single	multi	✓	×	✓
Cui & Sheng [7]	3	single	single	✓	×	×
Zhang et al.[37]	3	single	single	✓	×	✓
Chen et al. [6]	3	single	multi	✓	×	✓
Dalman [8]	3	multi	multi	✓	×	×
Kakran & Dhodiya [15]	3	multi	multi	✓	×	×
Sahoo et al. [31]	4	multi	multi	✓	×	✓
Revathi et al. [29]	4	multi	multi	✓	×	✓
Proposed work	4	multi	multi	✓	✓	✓

The remaining sections of this paper are as follows. Section 2 focuses on some key concepts in uncertainty theory that are required for this paper's study. The mathematical description of 4DMOMITP is given in Section 3, and the model for 4DMOMITP under uncertainty with its respective EVM, OVM, and DOCM models, is given in Section 4. The deterministic models for EVM, OVM and DOCM obtained after the corresponding transformations are provided in Section 5 which employs uncertainty theory. The following Section 6 describes Charnes and Cooper's transformations for converting the DOCM into its linear form, and Section 7 introduces the solution methodologies for solving the 4DMOMITP. Section 8 contains a numerical illustration of the application of these models. The sensitivity of the models to confidence levels is examined in Section 9. Section 10 displays the obtained results, as well as a comparison to other methods. Finally, the last section presents a conclusion and overall summary of the paper.

2. Preliminaries

In this section, we introduce some essential definitions and notions about uncertainty theory.

Definition 1 (Liu [20])). A function $\mathcal{M}\colon \mathcal{L} \to [0,1]$ (where $\mathcal{L}$ is a σ-algebra over any non-empty set Ω), is known as an uncertain measure if it satisfies the stated three axioms

- Axiom 1: $\mathcal{M}\{\Omega\} = 1$.
- Axiom 2: $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$, for event $\Lambda \in \mathcal{L}$.
- Axiom 3: $\mathcal{M}\left\{\bigcup_{j=1}^{\infty} \Lambda_j\right\} \leq \sum_{j=1}^{\infty} \mathcal{M}\{\Lambda_j\}$ for any countable sequence of events $\{\Lambda_j\}$.

Here, $(\Omega, \mathcal{L}, \mathcal{M})$ is called an uncertainty space.

Definition 2 (Liu [20]). An uncertain variable is a measurable function ζ from $(\Omega, \mathcal{L}, \mathcal{M})$ to the set $\mathcal{R}$ of real numbers such that $\{\zeta \in \mathcal{B}\Big\}$ is an event for any Borel set $\mathcal{B}$ of real numbers.

Definition 3 (Liu [20]). For any uncertain variable ζ, the uncertainty distribution denoted by $\Psi : \mathcal{R} \to [0,1]$ is defined as $\Psi(y) = \mathcal{M}\{\zeta \leq y\},\ y \in \mathcal{R}$.

Definition 4 (Liu [20]). An uncertain variable ζ with $\Psi(y)$ defined as

$$\Psi(x) = \begin{cases} 0 & \text{if } y \leq p \\ \frac{y-p}{2(q-p)} & \text{if } p \leq y \leq q \\ \frac{y+r-2q}{2(r-q)} & \text{if } q \leq y \leq r \\ 1 & \text{if } y \geq r \end{cases}$$

is called a zigzag uncertain variable, denoted by $\mathcal{Z}(p,q,r),\ p,q,r \in \mathcal{R}$ and $p < q < r$.

Definition 5 (Liu [20]). The inverse uncertainty distribution function of $\mathcal{Z}(p,q,r)$, denoted by Ψ^{-1}, is given by

$$\Psi^{-1}(\beta) = \begin{cases} (1-2\beta)p + 2\beta q & \text{if } \beta < 0.5 \\ (2-2\beta)q + (2\beta-1)r & \text{if } \beta \geq 0.5 \end{cases}$$

Theorem 1 (Liu [20]). The expected value of an uncertain variable ζ, if it exists, is given by

$$E[\zeta] = \int_0^1 \Psi^{-1}(\beta)d\beta$$

For a zigzag uncertain variable $\mathcal{Z}(p,q,r)$, this value is given by $E[\zeta] = \dfrac{p+2q+r}{4}$.

Theorem 2. Consider non-negative decision variables $y_1, y_2, \ldots, y_n$ and $\zeta_1, \zeta_2, \ldots, \zeta_n$, representing independent zigzag uncertain variables $\mathcal{Z}(p_1, q_1, r_1), \mathcal{Z}(p_2, q_2, r_2), \ldots, \mathcal{Z}(p_n, q_n, r_n)$, respectively.

$$\text{When } \bar{f} \in \left[\sum_{j=1}^{n} p_j y_j, \sum_{j=1}^{n} q_j y_j\right], \text{then } \mathcal{M}\left\{\sum_{j=1}^{n} \zeta_j y_j \leq \bar{f}\right\} = \frac{\bar{f} - \sum_{j=1}^{n} p_j y_j}{2\sum_{j=1}^{n}(q_j - p_j)y_j}$$

$$\text{When } \bar{f} \in \left[\sum_{j=1}^{n} q_j y_j, \sum_{j=1}^{n} r_j y_j\right], \text{then } \mathcal{M}\left\{\sum_{j=1}^{n} \zeta_j y_j \leq \bar{f}\right\} = \frac{\bar{f} + \sum_{j=1}^{n}(r_j - 2q_j)y_j}{2\sum_{j=1}^{n}(r_j - q_j)y_j}$$

The uncertain measure is 0, if $\bar{f}$ lies to the left side of the interval $[\sum_{j=1}^{n} p_j y_j, \sum_{j=1}^{n} q_j y_j]$ and 1, if $\bar{f}$ is to the right side of the interval $[\sum_{j=1}^{n} q_j y_j, \sum_{j=1}^{n} r_j y_j]$.

Definition 6 (Liu [20]). The β-optimistic and β-pessimistic values of ζ are defined by

$$\zeta_{\sup}(\beta) = \sup\{t \mid \mathcal{M}\{\zeta \geq t\} \geq \beta\} = \Psi^{-1}(1-\beta), \beta \in (0,1]$$
$$\zeta_{\inf}(\beta) = \inf\{t \mid \mathcal{M}\{\zeta \leq t\} \geq \beta\} = \Psi^{-1}(\beta), \beta \in (0,1]$$

For example, if we have a zigzag uncertain variable $\mathcal{Z}(p, q, r)$, then

$$\zeta_{\sup}(\beta) = \begin{cases} 2\beta q + (1-2\beta)r & \text{if } \beta < 0.5 \\ (2\beta-1)p + (2-2\beta)q & \text{if } \beta \geq 0.5 \end{cases} \tag{1}$$

$$\zeta_{\inf}(\beta) = \begin{cases} (1-2\beta)p + 2\beta q & \text{if } \beta < 0.5 \\ (2-2\beta)q + (2\beta-1)r & \text{if } \beta \geq 0.5 \end{cases} \tag{2}$$

Theorem 3. Let ζ be an uncertain variable and $\beta \in (0,1]$. Then we have

(a) $\zeta_{\inf}(\beta)$ is an increasing and left-continuous function of β,

(b) $\zeta_{\sup}(\beta)$ is a decreasing and left-continuous function of β.

Theorem 4. If $f : \mathcal{R}^n \to \mathcal{R}$ is a continuous and strictly increasing function, then $\zeta = f(\zeta_1, \zeta_2, \ldots, \zeta_n)$ is an uncertain variable and

(a) $\zeta_{\sup}(\beta) = f\Big(\zeta_{1\sup}(\beta), \zeta_{2\sup}(\beta), \ldots, \zeta_{n\sup}(\beta)\Big)$,

(b) $\zeta_{\inf}(\beta) = f\Big(\zeta_{1\inf}(\beta), \zeta_{2\inf}(\beta), \ldots, \zeta_{n\inf}(\beta)\Big)$.

Theorem 5. Let ζ and η be uncertain variables and $\beta \in (0,1]$, then

$$(a\zeta)_{\sup}(\beta) = \begin{cases} a\zeta_{\sup}(\beta) & \text{if } a \geq 0 \\ a\zeta_{\inf}(\beta) & \text{if } a < 0 \end{cases} \quad \text{and} \quad (a\zeta)_{\inf}(\beta) = \begin{cases} a\zeta_{\inf}(\beta) & \text{if } a \geq 0 \\ a\zeta_{\sup}(\beta) & \text{if } a < 0 \end{cases}$$

One of the major issues in a model involving uncertainty is the ranking of the uncertain variables, since uncertain variables do not always follow a natural order in an uncertain environment. For any two uncertain variables ζ and η, Liu [21] presented four ranking criteria:

- Expected value criterion: we say $\zeta < \eta$ iff $E[\zeta] < E[\eta]$.
- Optimistic value criterion: we say $\zeta < \eta$ iff $\zeta_{\sup}(\beta) < \eta_{\sup}(\beta)$, for some $\beta \in (0, 1]$.
- Pessimistic value criterion: we say $\zeta < \eta$ iff $\zeta_{\inf}(\beta) < \eta_{\inf}(\beta)$, for some $\beta \in (0, 1]$.
- Chance-criterion: we say $\zeta < \eta$ iff $\mathcal{M}\{\zeta \geq \bar{t}\} < \mathcal{M}\{\eta \geq \bar{t}\}$ for some predefined level $\bar{t}$.

3. Description of the problem

A mathematical description of the four-dimensional multi-objective multi-item transportation problem is expressed by a description of the available sources, destinations, the vehicles used, items to be conveyed and the set of accessible routes. Let there be m sources, n destinations, K vehicles, P items and R routes in the four-dimensional multi-objective multi-item transportation problem. The 4DMOMITP model can be mathematically formulated as shown below.

$$\begin{array}{lll} \min & Z^t = \sum\limits_{i,j,k,r,p} (c^{tp}_{ijkr} x^p_{ijkr}) & \forall t \\ \text{subject to} & \sum\limits_{j,k,r} x^p_{ijkr} \leq a^p_i & \forall i, \forall p \\ & \sum\limits_{i,k,r} x^p_{ijkr} \geq b^p_j & \forall j, \forall p \\ & \sum\limits_{i,j,p} x^p_{ijkr} \leq e_{kr} & \forall k, \forall r \\ & x^p_{ijkr} \geq 0 & \end{array} \tag{3}$$

The notation used in this model are as follows: m defines the number of available sources, n defines the number of available destinations, K defines the number of available vehicles, R defines the number of available routes, P defines the number of items to be transported, S defines the number of objectives, a^p_i defines the quantity available of item p at source i, b^p_j defines the quantity of item p required at destination j and e_{kr} defines the maximum capacity of vehicle k during transportation along route r. c^{tp}_{ijkr} is the penalty cost (representing a coefficient matrix for t different objectives) for delivering one unit of item p from source i to destination j by vehicle k using route r for objective t and x^p_{ijkr} is the number of units of item p transported from source i to destination j by vehicle k along route r. We have also used the notation $\forall\, t$: $t = 1, 2, \ldots, S$, $\forall\, i$: $i = 1, 2, \ldots, m$, $\forall\, j$: $j = 1, 2, \ldots, n$, $\forall\, k$: $k = 1, 2, \ldots, K$, $\forall\, r$: $r = 1, 2, \ldots, R$, $\forall\, p$: $p = 1, 2, \ldots, P$, throughout this paper.

In a large majority of transportation systems, it is not the case that one can accurately estimate all the related parameters. In reality, decision-makers may encounter various uncertainties in the application of TP, such as the availability of raw materials at the sources, demand at the destinations, and unit transportation costs, due to a variety of uncontrollable factors. These factors may arise due to weather conditions, road conditions or the impossibility of accurately estimating total demand for a newly released product on the market. Considering the fact that there may be no historical reference available about a transportation plan, we take into account the uncertainty theory introduced by Liu [20]. So, keeping all these things in mind, we introduce a model for 4DMOMITP involving uncertainty which considers all the parameters, like $a^p_i, b^p_j, e_{kr}, c^{tp}_{ijkr}$ as uncertain variables $\tilde{a}^p_i$, $\tilde{b}^p_j$, $\tilde{e}_{kr}$ and ζ^{tp}_{ijkr}, respectively.

4. Mathematical model involving uncertainty

The model presented in (4) is a mathematical expression of the 4DMOMITP involving uncertain variables $\tilde{a}^p_i$, $\tilde{b}^p_j$, $\tilde{e}_{kr}$ and ζ^{tp}_{ijkr} in the place of crisp values as seen in model (3). Model (4) has more natural applications to real-world scenarios than the crisp model (3), because of the presence of uncertain parameters in the transportation plan.

$$\begin{aligned} \min \quad & Z_t(x;\zeta) = \sum_{i,j,k,r,p} \zeta_{ijkr}^{tp} x_{ijkr}^{p} && \forall t \\ \text{subject to} \quad & \sum_{j,k,r} x_{ijkr}^{p} \leq \tilde{a}_i^p && \forall i, \forall p \\ & \sum_{i,k,r} x_{ijkr}^{p} \geq \tilde{b}_j^p && \forall j, \forall p \\ & \sum_{i,j,p} x_{ijkr}^{p} \leq \tilde{e}_{kr} && \forall k, \forall r \\ & x_{ijkr}^{p} \geq 0 \end{aligned} \tag{4}$$

The model involving uncertainty (4) cannot be solved directly with any of the primary methods. One of the four ranking criteria proposed by Liu [22] must be applied first, as discussed in Section 2.

4.1. Expected value model

The basic idea of the expected value model (EVM) is to find an optimal solution to the problem by considering the expected values of the uncertain variables in the objective function and constraints. Mathematically, the EVM model for 4DUMOMITP can be stated as follows:

$$\begin{aligned} \min \quad & E[Z_t] = E\left[\sum_{i,j,k,r,p} \zeta_{ijkr}^{tp} x_{ijkr}^{p}\right] && \forall t \\ \text{subject to} \quad & E\left[\sum_{j,k,r} x_{ijkr}^{p} - \tilde{a}_i^p\right] \leq 0 && \forall i, \forall p \\ & E\left[\sum_{i,k,r} x_{ijkr}^{p} - \tilde{b}_j^p\right] \geq 0 && \forall j, \forall p \\ & E\left[\sum_{i,j,p} x_{ijkr}^{p} - \tilde{e}_{kr}\right] \leq 0 && \forall k, \forall r \\ & x_{ijkr}^{p} \geq 0 \end{aligned} \tag{5}$$

In this model, throughout this paper we will use the notation Z_{tE} to represent the objective functions $E[Z_t]$ formed by taking the expected values of $Z_t(x, \zeta)$.

4.2. Optimistic value model

The optimistic value criterion defined for uncertain variables can also be used to develop an optimistic value model (OVM) to deal with uncertainty in models. For model (4), the OVM is as follows:

$$\begin{aligned} \min \quad & [Z_t]_{\sup(\alpha_t)} = \left[\sum_{i,j,k,r,p} \zeta_{ijkr}^{tp} x_{ijkr}^{p}\right]_{\sup} (\alpha_t) && \forall t \\ \text{subject to} \quad & \left[\sum_{j,k,r} x_{ijkr}^{p} - \tilde{a}_i^p\right]_{\sup} (\alpha_i^p) \leq 0 && \forall i, \forall p \\ & \left[\sum_{i,k,r} x_{ijkr}^{p} - \tilde{b}_j^p\right]_{\sup} (\beta_j^p) \geq 0 && \forall j, \forall p \\ & \left[\sum_{i,j,p} x_{ijkr}^{p} - \tilde{e}_{kr}\right]_{\sup} (\gamma_{kr}) \leq 0 && \forall k, \forall r \\ & x_{ijkr}^{p} \geq 0 \end{aligned} \tag{6}$$

Here $\alpha_t, \alpha_i^p, \beta_j^p$ and γ_{kr}, $\forall i, j, k, r, p, t$, are chosen confidence levels. In this model, throughout this paper we will use Z_{tS} to represent the objective functions $[Z_t]_{\sup(\alpha_t)}$ formed by taking the optimistic values of $Z_t(x, \zeta)$.

4.3. Dependent optimistic-constrained model

In the dependent-optimistic criterion model (DOCM), the basic criterion is to choose a decision which maximizes the chance of achieving a goal. Here, we denote by $\bar{f}_t$ the predetermined maximal cost available to optimize an uncertain measure, such that the total cost incurred does not exceed the predetermined value $\bar{f}_t$. The DOCM model (7) is formulated such that the chance of carrying out the required tasks in an uncertain environment is maximized.

$$\begin{array}{l} \max \; Z_{tM} = \mathcal{M}\left\{ \sum_{i,j,k,r,p} \zeta_{ijkr}^{tp} x_{ijkr}^p \leq \bar{f}_t \right\} \quad \forall t \\ \text{subject to the constraints of model (6)} \end{array} \tag{7}$$

Since the DOCM model considers constraints based on optimistic values, the constraints used in the model (7) are same as those in the OVM model (6). The notation Z_{tM} is used to denote the objective function $\mathcal{M}\{Z_t\}$ based on an uncertain measure.

5. Deterministic formulations

It is worth noting that the above models contain a large number of uncertain variables. To solve the proposed models, we must compute the expected value, optimistic value or uncertain measure. In general, it is natural for us to convert these models into deterministic forms whenever possible, for the sake of calculation.

5.1. Expected value model

If $\tilde{a}_i^p, \tilde{b}_j^p$ and $\tilde{e}_{kr}$ are independent uncertain variables, then model (5) is equivalent to the following model:

$$\begin{array}{lll} \min & Z_{tE} = \sum_{i,j,k,r,p} \left(x_{ijkr}^p \int_0^1 \Psi^{-1}_{\zeta_{ijkr}^{tp}}(\alpha_t) d\alpha_t \right) & \forall t \\ \text{subject to} & \sum_{j,k,r} x_{ijkr}^p - \int_0^1 \Psi^{-1}_{\tilde{a}_i^p}(\alpha_i^p) d\alpha_i^p \leq 0 & \forall i, \forall p \\ & \int_0^1 \Psi^{-1}_{\tilde{b}_j^p}(\beta_j^p) d\beta_j^p - \sum_{i,k,r} x_{ijkr}^p \leq 0 & \forall j, \forall p \\ & \sum_{i,j,p} x_{ijkr}^p - \int_0^1 \Psi^{-1}_{\tilde{e}_{kr}}(\gamma_{kr}) d\gamma_{kr} \leq 0 & \forall k, \forall r \\ & x_{ijkr}^p \geq 0 & \end{array} \tag{8}$$

Here $\alpha_t, \alpha_i^p, \beta_j^p$ and γ_{kr}, $\forall i, j, k, r, p, t$, are chosen confidence levels.

5.2. Optimistic value model

The corresponding formulation of the OVM model (6) is obtained by using the basic definitions and theorems defined in Section 2. This formulation is expressed in the following model (9):

$$\begin{array}{lll}
\min & Z_{tS} = \sum\limits_{i,j,k,r,p} x_{ijkr}^{p} \Psi_{\zeta_{ijkr}^{tp}}^{-1}(1-\alpha_t) & \forall t \\
\text{subject to} & \sum\limits_{j,k,r} x_{ijkr}^{p} - \Psi_{\tilde{a}_i}^{-1}(\alpha_i^p) \le 0 & \forall i, \forall p \\
& \Psi_{\tilde{b}_j}^{-1}(1-\beta_j^p) - \sum\limits_{i,k,r} x_{ijkr}^{p} \le 0 & \forall j, \forall p \\
& \sum\limits_{i,j,p} x_{ijkr}^{p} - \Psi_{\tilde{e}_k}^{-1}(\gamma_{kr}) \le 0 & \forall k, \forall r \\
& x_{ijkr}^{p} \ge 0 &
\end{array} \tag{9}$$

The terms α_t, α_i^p, β_j^p and γ_{kr}, $\forall i, j, k, r, p, t$, represent chosen confidence levels.

5.3. Dependent optimistic-constrained model

The DOCM model shown in model (7) corresponds to the model (10). Since the DOCM model considers constraints based on optimistic values, these are same as the constraints used in the OVM model (9).

$$\begin{array}{l}
\max\ Z_{tM} = \mathcal{M}\left\{ \sum\limits_{i,j,k,r,p} \zeta_{ijkr}^{tp} x_{ijkr}^{p} \le \bar{f}_t \right\} \quad \forall t \\
\text{subject to the constraints of model (9)}
\end{array} \tag{10}$$

6. Charnes and Cooper's transformation for the DOCM model

This section describes Charnes and Cooper's transformations [4] used for converting the linear fractional problem (LFP) described in model (11) to the linear programming problem described in model (12).

$$\begin{array}{ll}
\max & \dfrac{p^{'}x+\alpha}{q^{'}x+\beta} \\
\text{subject to} & Ax \le b \\
& x \ge 0
\end{array} \tag{11}$$

Let $\mathcal{S} = \{x : Ax \le b, x \ge 0\}$ be a compact set and $q^{'}x + \beta > 0$ for each $x \in \mathcal{S}$. Setting $t = \frac{1}{q^{'}x+\beta}$ and $y = tx$, the LFP in model (11) converts into the following model:

$$\begin{array}{ll}
\max & p^{'}y + \alpha t \\
\text{subject to} & q^{'}y + \beta t = 1 \\
& Ay - bt \le 0 \\
& y \ge 0, t \ge 0
\end{array} \tag{12}$$

Note that, if (y, t) is a feasible solution to the model (12), then $t > 0$ and if $(\bar{y}, \bar{t})$ is an optimal solution to the above linear program, then $\bar{x} = \frac{\bar{y}}{\bar{t}}$ is an optimal solution to the fractional program.

Suppose that $A\bar{x} \le b$ and $\bar{x} \ge 0$, so that $\bar{x}$ is a feasible solution to LFP. To prove the optimality of $\bar{x}$, take x such that $Ax \le b$ and $x \ge 0$. Note that $q^{'}x + \beta > 0$ by assumption and the vector (y, t) is a feasible solution to LPP in model (12), where $y = \frac{x}{q^{'}x+\beta}$ and $t = \frac{1}{q^{'}x+\beta}$. Also, $\bar{y}, \bar{t}$ is an optimal solution to the LPP which gives $p^{'}\bar{y} + \alpha\bar{t} \le p^{'}y + \alpha t$. Substituting in the values for $\bar{y}, y$ and t this inequality leads to $\bar{t}(p^{'}\bar{x} + \alpha) \le \frac{(p^{'}x+\alpha)}{q^{'}x+\beta}$. This proves the result immediately when we divide the left hand side by $1 = q^{'}\bar{y} + \beta\bar{t}$. Now, if $q^{'}x + \beta < 0$ for all $x \in \mathcal{S}$, then letting $-t = \frac{1}{q^{'}x+\beta}$ and $y = tx$ gives the following LPP:

$$\begin{array}{ll} \min & -p'y - \alpha t \\ \text{subject to} & Ay - bt \leq 0 \\ & -q'y - \beta t = 1 \\ & y \geq 0,\ t > 0 \end{array} \tag{13}$$

The form of the LPP used depends on whether $q'x + \beta > 0$ for all $x \in \mathcal{S}$ or $q'x + \beta < 0$ for all $x \in \mathcal{S}$.

7. Solution methodologies

This section describes the three classical approaches: weighted sum method, minimizing distance method, and fuzzy programming technique, used to obtain a compromise solution for multi-objective optimization problems. These three methods will be used to find a compromise solution for the EVM, OVM and DOCM crisp models.

7.1. Weighted sum method

The weighted sum method (WSM) converts multi-objective problems into a single objective problem by pre-multiplying the vector of objective functions by a specified weight vector representing the relative importance of each objective function to the decision-maker. This method helps achieve a Pareto-optimal solution of a multi-objective problem by optimizing the weighted sum of objective functions. For instance, the single objective model using WSM is displayed below:

$$\begin{array}{l} \min / \max \displaystyle\sum_{t=1}^{S} w_t Z_t \\ \text{subject to the constraints of model (8) or (9) or (10)} \end{array}$$

Here, Z_t is a generalized notation for the objective functions in the EVM, OVM and DOCM models. The objective function in WSM should be minimized or maximized according to the type of individual objective functions in any model. Also, the weights $w_1, w_2, \dots w_S$ are non-negative numbers such that $w_1 + w_2 + \cdots + w_S = 1$.

7.2. Minimizing distance method

This method transforms a multi-objective problem into single objective by minimizing the sum of deviation of the ideal vector from the corresponding objective functions. Here, we use the L_2 norm to convert the crisp multi-objective models EVM, OVM and DOCM into their corresponding compromise models. The single objective model formed using minimizing distance method (MDM) is described below:

$$\begin{array}{l} \min \sqrt{\displaystyle\sum_{t=1}^{S} (Z_t - Z_t^o)^2} \\ \text{subject to the constraints of model (8) or (9) or (10)} \end{array}$$

Here, Z_t^o represents the ideal values of the individual objective functions Z_t in the EVM, OVM and DOCM models.

7.3. Fuzzy programming technique

The fuzzy programming technique (FPT), introduced by Zimmerman [39], is used to obtain a solution to multi-objective problems and its sequential steps for 4DMOMITP are described below:

Step 1. Solve each of optimization problems derived from the deterministic models (8), (9) and (10) by considering a single objective at a time and ignoring all of the other objectives w.r.t the given constraints.

Step 2. Obtain the minimum value (say L_t) and maximum value (say U_t) for each of the S objective functions.

Step 3. Construct the exponential membership function $\mu_t(Z_t)$ for the tth objective function defined by:

$$\mu_t(Z_t) = \begin{cases} 1 & \text{if } Z_t \leq L_t \\ \dfrac{e^{-s_t\psi_t(x)} - e^{-s_t}}{1 - e^{-s_t}} & \text{if } L_t < Z_t < U_t \\ 0 & \text{if } Z_t \geq U_t \ \ \forall t \end{cases}$$

Here, $\psi_t(x) = \frac{Z_t - L_t}{U_t - L_t}$ if an objective function is of the minimization type and $\psi_t(x) = \frac{U_t - Z_t}{U_t - L_t}$ if an objective function is of the maximization type. Also, s_t is a non-zero shape parameter given by the decision-maker.

Step 4. Now, we formulate the fuzzy linear programming problem using the max-min operator for the exponential membership function (mf) as:

$$\begin{aligned} &\max\ \lambda \\ &\text{subject to } \mu_t(Z_t) \geq \lambda \ \ \forall t \\ &\text{and the constraints of model (8), (9) or (10)with } \lambda \geq 0 \text{ and } \lambda = \min \mu_t(Z_t) \end{aligned} \tag{14}$$

Here, Z_t is generalized notation for each of the three objectives $Z_{tE},\ Z_{tS},\ Z_{tM}$. One can form such a single objective model for each of the three models EVM, OVM and DOCM by considering the exponential mf.

Step 5. Now this converted single-objective problem is solved using the LINGO 18.0 software to obtain a pareto-optimal solution of the 4DMOMITP problem.

8. Numerical illustration

In this section, the application of the proposed models is demonstrated with the help of the following example of a 4DMOMITP. In this illustration, we assume two origins, three destinations, two vehicles, two items and two routes between origins and destinations. In this problem, all the parameters, such as transportation cost/damage cost, capacity of suppliers, demand at destinations, capacity of suppliers, are considered as independent zigzag uncertain variables. The primary objective of the 4DMOMITP problem is to determine the number of goods to be delivered from the sources to the destinations via the appropriate routes in such a way that transportation costs and item damage costs are minimized. The data for this 4DMOMITP are listed in Tables 2–5 and the rest of the uncertain variables used are defined as follows: $\tilde{a}_1^1 \sim \mathcal{Z}(56,58,60)$, $\tilde{a}_2^1 \sim \mathcal{Z}(53,55,57)$, $\tilde{a}_1^2 \sim \mathcal{Z}(56,58,61)$, $\tilde{a}_2^2 \sim \mathcal{Z}(52,54,56)$, $\tilde{b}_1^1 \sim \mathcal{Z}(32,34,36)$, $\tilde{b}_2^1 \sim \mathcal{Z}(34,36,38)$, $\tilde{b}_3^1 \sim \mathcal{Z}(31,34,37)$, $\tilde{b}_1^2 \sim \mathcal{Z}(32,34,36)$, $\tilde{b}_2^2 \sim \mathcal{Z}(30,32,34)$, $\tilde{b}_3^2 \sim \mathcal{Z}(29,32,35)$, $\tilde{e}_{11} \sim \mathcal{Z}(100,150,170)$, $\tilde{e}_{21} \sim \mathcal{Z}(130,150,180)$ $\tilde{e}_{12} \sim \mathcal{Z}(120,150,160)$, $\tilde{e}_{22} \sim \mathcal{Z}(150,200,250)$.

Solution. The mathematical model of the 4DMOMITP under uncertainty can be framed using the parameters displayed in Tables 2–5 and solved by utilizing any of the deterministic models mentioned in Section 5. We discuss here the results from the application of the three described solution methodologies to each model.

Table 2. The shipping costs for the first objective and each of two possible routes when using rail transport

	ζ^{11}_{ij1r}	1	2	3	ζ^{12}_{ij1r}	1	2	3
$r=1$	1	(7,8,11)	(4,6,8)	(2,4,7)	1	(5,7,9)	(4,6,8)	(2,4,6)
	2	(4,6,8)	(7,9,11)	(5,7,9)	2	(3,6,9)	(4,7,10)	(3,6,9)
$r=2$	1	(5,7,9)	(2,5,8)	(1,4,7)	1	(5,7,9)	(5,9,13)	(4,8,12)
	2	(4,8,12)	(8,11,14)	(6,9,12)	2	(4,7,10)	(5,7,9)	(3,5,7)

Table 3. The shipping costs for the first objective and each of two possible routes when using cargoship transport

	ζ^{11}_{ij2r}	1	2	3	ζ^{12}_{ij2r}	1	2	3
$r=1$	1	(4,7,10)	(7,9,11)	(5,7,9)	1	(3,6,9)	(5,8,11)	(4,7,10)
	2	(4,7,10)	(7,8,9)	(5,8,11)	2	(3,5,7)	(5,7,9)	(4,8,12)
$r=2$	1	(4,6,8)	(4,7,10)	(4,6,8)	1	(4,8,12)	(5,7,9)	(3,6,9)
	2	(6,8,10)	(8,11,14)	(4,7,10)	2	(5,8,11)	(4,7,10)	(4,6,8)

Table 4. The damage costs for the second objective and each of two possible routes when using rail transport

	ζ^{21}_{ij1r}	1	2	3	ζ^{22}_{ij1r}	1	2	3
$r=1$	1	(2,5,8)	(6,9,12)	(7,10,13)	1	(4,7,10)	(4,7,10)	(3,8,13)
	2	(1,5,9)	(7,11,15)	(7,12,17)	2	(3,6,9)	(3,6,9)	(3,5,7)
$r=2$	1	(1,4,7)	(7,9,11)	(8,11,14)	1	(3,5,7)	(4,7,10)	(1,3,5)
	2	(4,7,10)	(10,12,14)	(7,10,13)	2	(4,8,12)	(5,9,13)	(5,8,11)

Table 5. The damage costs for the second objective and each of two possible routes when using cargoship transport

	ζ^{21}_{ij2r}	1	2	3	ζ^{22}_{ij2r}	1	2	3
$r=1$	1	(2,4,8)	(6,8,10)	(6,8,11)	1	(5,7,10)	(4,7,10)	(4,6,9)
	2	(5,7,9)	(6,9,12)	(7,10,13)	2	(3,6,9)	(4,6,8)	(6,8,11)
$r=2$	1	(8,10,13)	(10,12,16)	(8,10,12)	1	(5,8,12)	(4,6,10)	(4,7,11)
	2	(7,10,13)	(7,10,13)	(7,11,15)	2	(6,8,11)	(7,11,16)	(5,7,9)

8.1. Expected value model

The EVM model can be formulated by calculating the expected values of the uncertain data for supplier capacities, demand requirements, conveyance capacities and all of the uncertain variables given in Tables 2–5. Substituting these expected values into model (8), we obtain a deterministic multi-objective EVM model, which can be solved using any of the three solution methodologies considered. We present here the results obtained for the EVM model using the three solution methodologies described in Section 7.

a) Weighted sum method.
The solutions obtained for the EVM model using the weighted sum method are displayed in Table 6. In this table, Z^*_{1E} and Z^*_{2E} denote the Pareto-optimal values obtained after substituting the solution vector into the objective functions Z_{1E} and Z_{2E}, respectively.

b) Minimizing distance method.
The results obtained using the single objective model obtained via MDM as described in Section 7.2 for the EVM model are as follows: $x^1_{1311} = 34$, $x^1_{2111} = 34$, $x^2_{1211} = 17.13$, $x^2_{2211} = 14.45$, $x^2_{2311} = 4.68$, $x^1_{1212} = 24$, $x^2_{1312} = 27.31$, $x^1_{2221} = 12$, $x^2_{2121} = 34$, $x^2_{2221} = 0.42$ with the ideal values taken to be $Z^o_{1E} = 1051.750$ and $Z^o_{2E} = 1216.250$. The compromise values of the objective functions for the EVM model obtained using the MDM method are $Z^*_{1E} = 1188.0$ and $Z^*_{2E} = 1352.50$.

c) Fuzzy programming technique.
Here, the results are obtained using FPT as described in Section 7.3. In order to apply FPT, the U_t and L_t values are taken to be: $L_1 = 1051.750,\ L_2 = 1216.250,\ U_1 = 1986.250,\ U_2 = 2372.50$.

Table 6. Solutions for the EVM model according to the weighted sum method using various weight vectors

w_1	w_2	Z_{1E}^*	Z_{2E}^*	Solution
1	0	1051.75	1558.25	$x_{2111}^1 = 34, x_{2311}^1 = 12, x_{1211}^2 = 26.25, x_{1311}^2 = 32, x_{2211}^2 = 5.75, x_{1212}^1 = 36, x_{1312}^1 = 22, x_{2121}^2 = 34$
0.8	0.2	1057.25	1524.25	$x_{1311}^1 = 22, x_{2111}^1 = 34, x_{1211}^2 = 26.25, x_{1311}^2 = 32, x_{2211}^2 = 5.75, x_{1212}^1 = 36, x_{2121}^2 = 34, x_{2322}^1 = 12$
0.6	0.4	1066.00	1500.75	$x_{1311}^1 = 34, x_{2111}^1 = 34, x_{1211}^2 = 32, x_{1311}^2 = 26.25, x_{2311}^2 = 5.75, x_{1212}^1 = 24, x_{2221}^1 = 12, x_{2121}^2 = 34$
0.5	0.5	1142.50	1398.00	$x_{1311}^1 = 34, x_{2111}^1 = 34, x_{1211}^2 = 32, x_{2311}^2 = 20, x_{1212}^1 = 24, x_{1312}^2 = 12, x_{2221}^1 = 12, x_{2121}^2 = 34$
0.4	0.6	1202.50	1338.00	$x_{1311}^1 = 34, x_{2111}^1 = 34, x_{1211}^2 = 12, x_{2211}^2 = 20, x_{1212}^1 = 24, x_{1312}^2 = 32, x_{2221}^1 = 12, x_{2121}^2 = 34$
0.2	0.8	1360.50	1240.25	$x_{2111}^1 = 34, x_{2211}^2 = 32, x_{1212}^1 = 24, x_{1112}^2 = 26.25, x_{1312}^2 = 32, x_{1321}^1 = 34, x_{2221}^1 = 12, x_{2121}^2 = 7.75$
0	1	1464.25	1216.25	$x_{2111}^1 = 34, x_{2111}^2 = 7.75, x_{2211}^2 = 32, x_{1112}^2 = 26.25, x_{1312}^2 = 32, x_{1221}^1 = 24, x_{1321}^1 = 34, x_{2221}^1 = 12$

Solution for the EVM model using FPT were obtained by considering two different sets of shape parameters in the exponential mf ($s_1 = 2, s_2 = 3$) and ($s_1 = -2, s_2 = -2$), where the s_1 shape parameter corresponds to the Z_{1E} objective and the s_2 shape parameter corresponds to the Z_{2E} objective. The results for these two cases are shown in Table 7.

Table 7. Solutions for the EVM model using fuzzy programming with shape parameters (s_1, s_2) in the exponential mf

(s_1, s_2)	Z_{1E}^*	Z_{2E}^*	λ	Solution
(2,3)	1193.536	1346.964	0.6973	$x_{1311}^1 = 34, x_{2111}^1 = 34, x_{1211}^2 = 14.99, x_{2211}^2 = 10.36, x_{2311}^2 = 2.99, x_{1212}^1 = 24, x_{1312}^2 = 29.01, x_{2221}^1 = 12, x_{2121}^2 = 34, x_{2221}^2 = 6.65$
(-2,-2)	1173.549	1366.951	0.9534	$x_{1311}^1 = 34, x_{2111}^1 = 34, x_{1211}^2 = 26, x_{2211}^2 = 2.998, x_{2311}^2 = 7.47, x_{1212}^1 = 24, x_{1312}^2 = 24.53, x_{2221}^1 = 12, x_{2121}^2 = 34, x_{2221}^2 = 2.998$

8.2. Optimistic value model

As the OVM involves predetermined confidence levels $\alpha_t, \alpha_i^p, \beta_j^p, \gamma_{kr} \in (0, 1]$ to determine the corresponding deterministic model (9), the OVM for the problem under uncertainty may be formulated by considering two classes of confidence levels.

Class 1 (*cl*$_1$). In this class, we consider all the cases of confidence levels in the interval $[0.5, 1]$, i.e. $\alpha_t, \alpha_i^p, \beta_j^p, \gamma_{kr} \in [0.5, 1]$. To solve the numerical problem presented, let us assume that all the confidence levels are equal to 0.9. The three solution methodologies listed in Section 7 can be used to solve the OVM model for class cl_1. The results obtained for each of these three methods are displayed below.

a) Weighted sum method.
The solutions obtained for the OVM model (cl_1) using WSM are given in Table 8.

b) Minimizing distance method.
The solution obtained for the OVM model (cl_1) using MDM is: $x_{1311}^1 = 25.2, x_{2111}^1 = 32.4, x_{1211}^2 = 7.2, x_{1311}^2 = 0.58, x_{2211}^2 = 23.2, x_{1212}^1 = 34.4, x_{1312}^2 = 29.02, x_{2121}^2 = 32.4, x_{2322}^1 = 6.4$. The compromise values obtained for the objective functions are $Z_{1S}^* = 711.1706$ and $Z_{2S}^* = 830.545$.

c) Fuzzy programming technique.
To apply the fuzzy programming technique with exponential mf, the U_t and L_t values are taken to be: $L_1 = 616.720, L_2 = 743.360, U_1 = 1494.840,\ U_2 = 1825.84$. Solving the OVM model (cl_1) for the two

Table 8. Solutions for the OVM using the weighted sum method with various weight vectors

w_1	w_2	Z^*_{1S}	Z^*_{2S}	Solution
1	0	616.72	954.40	$x^1_{2111} = 32.4, x^2_{1211} = 30.4, x^2_{1311} = 29.6, x^1_{1212} = 34.4, x^1_{1312} = 25.2, x^2_{2121} = 32.4, x^1_{2322} = 6.4$
0.8	0.2	621.36	931.20	$x^1_{2111} = 32.4, x^2_{1211} = 7.2, x^2_{1311} = 29.6, x^2_{2211} = 23.2, x^1_{1212} = 34.4, x^1_{1312} = 25.2, x^2_{2121} = 32.4, x^1_{2322} = 6.4$
0.6	0.4	621.36	931.20	$x^1_{2111} = 32.4, x^2_{1211} = 7.2, x^2_{1311} = 29.6, x^2_{2211} = 23.2, x^1_{1212} = 34.4, x^1_{1312} = 25.2, x^2_{2121} = 32.4, x^1_{2322} = 6.4$
0.5	0.5	712.56	829.04	$x^1_{1311} = 25.2, x^1_{2111} = 32.4, x^2_{1211} = 7.2, x^2_{2211} = 23.2, x^1_{1212} = 34.4, x^2_{1312} = 29.6, x^2_{2121} = 32.4, x^1_{2322} = 6.4$
0.4	0.6	712.56	829.04	$x^1_{1311} = 25.2, x^1_{2111} = 32.4, x^2_{1211} = 7.2, x^2_{2211} = 23.2, x^1_{1212} = 34.4, x^2_{1312} = 29.6, x^2_{2121} = 32.4, x^1_{2322} = 6.4$
0.2	0.8	888.96	753.68	$x^1_{1211} = 28, x^1_{2111} = 32.4, x^2_{2211} = 30.4, x^2_{1112} = 7.2, x^2_{1312} = 29.6, x^1_{1321} = 31.6, x^1_{2221} = 6.4, x^2_{2121} = 25.2$
0	1	1020.48	743.36	$x^1_{2111} = 32.4, x^2_{2111} = 1.6, x^2_{2211} = 30.4, x^2_{1112} = 30.8, x^2_{1312} = 29.6, x^1_{1221} = 28, x^1_{1321} = 31.6, x^1_{2221} = 6.4$

different sets of shape parameters $(s_1 = 2, s_2 = 3)$ and $(s_1 = -2, s_2 = -2)$, we obtain the solutions shown in Table 9.

Table 9. Solutions for the OVM model using fuzzy programming with shape parameters (s_1, s_2) in the exponential mf (cl_1)

(s_1, s_2)	Z^*_{1S}	Z^*_{2S}	λ	Solution
(2,3)	711.615	830.064	0.7752	$x^1_{1311} = 25.2, x^1_{2111} = 32.4, x^2_{1211} = 7.2, x^2_{1311} = 0.394, x^2_{2211} = 23.2, x^1_{1212} = 34.4, x^2_{1312} = 29.21, x^2_{2121} = 32.4, x^1_{2322} = 6.4$
(-2,-2)	698.543	844.225	0.9679	$x^1_{1311} = 25.2, x^1_{2111} = 32.4, x^2_{1211} = 7.2, x^2_{1311} = 5.84, x^2_{2211} = 23.2, x^1_{1212} = 34.4, x^2_{1312} = 23.76, x^2_{2121} = 32.4, x^1_{2322} = 6.4$

Class 2 (cl_2). In this case, $\alpha_t, \alpha^p_i, \beta^p_j, \gamma_{kr} \in [0, 0.5)$. Let us assume that all of the confidence levels are equal to 0.1. The results for the class cl_2 of confidence levels can be obtained in a similar way to the solutions obtained for the OVM model with confidence levels in class cl_1. The results obtained for this class cl_2 are displayed in Table 17 of Section 10.

8.3. Dependent optimistic-constrained model

The deterministic model for the DOCM, model (10), is obtained using Theorem 2 and depends on the chosen values of $\bar{f}_1$ and $\bar{f}_2$. The values of $\bar{f}_1$ and $\bar{f}_2$ considered in this model are 1400 and 1600, respectively. A deterministic model (10) can be obtained by setting the confidence levels to 0.9 and transforming the fractional objectives (represented by Z_{tM}) into their linear form (represented by Z'_{tM}) using the transformation given by Charnes and Cooper [4]. The results obtained for the linear DOCM model using the three solution methodologies considered are presented below.

a) Weighted sum method.
The solutions obtained for the linear DOCM model using WSM are given in Table 10. The solutions presented in this table are retransformed back into terms of 'X' decision variables using $x = y/t$. The retransformed solutions are shown in Table 11.

It may happen that the Pareto optimal solution found in terms of an uncertain measure is not suitable for the decision-maker. So, a solution vector given in Table 10 can be substituted into the original model involving uncertainty by considering lower, middle and upper values of the uncertain variables to give a range of uncertainty to the decision-maker. For instance, consider the solution vector obtained using the weights $(w_1 = 0.5, w_2 = 0.5)$ given in Table 10. We substitute this solution vector into the original model with uncertainty to obtain the uncertain values $[690.4, 1143.2, 1622.4]$ and $[792, 1271.2, 1750.4]$ for the objectives $\tilde{Z}_1$ and $\tilde{Z}_2$, respectively. Such uncertain values can be obtained for each of the various cases of weight vectors considered in a similar way.

Table 10. Solutions for the linear DOCM model using WSM with various weight vectors

w_1	w_2	Z'^*_{1M}	Z'^*_{2M}	Solution
1	0	0.8455414	0.6385350	$t = 0.00099522, y^2_{1211} = 0.03025478, y^2_{1311} = 0.00636943, y^2_{2311} = 0.02229299, y^1_{1212} = 0.03423567, y^1_{1312} = 0.02507962, y^2_{2312} = 0.00079618, y^1_{2121} = 0.03224522, y^2_{2121} = 0.03224522, y^1_{2322} = 0.00636942$
0.8	0.2	0.8345128	0.7170659	$t = 0.00102069, y^1_{1311} = 0.03225367, y^1_{2111} = 0.00707676, y^2_{1211} = 0.03102885, y^2_{2311} = 0.03021230, y^1_{1212} = 0.02857921, y^2_{1112} = 0.00653239, y^1_{2121} = 0.02599347, y^1_{2221} = 0.00653239, y^2_{2121} = 0.02653783$
0.6	0.4	0.7719883	0.8370154	$t = 0.00104210, y^1_{1311} = 0.02326094, y^1_{2111} = 0.03376407, y^2_{1211} = 0.00187578, y^2_{2311} = 0.02813672, y^1_{1212} = 0.03584827, y^2_{1112} = 0.03376407, y^2_{1312} = 0.00270946, y^1_{2321} = 0.00666945, y^2_{2221} = 0.02980409$
0.5	0.5	0.7679466	0.8430718	$t = 0.00104341, y^1_{1311} = 0.02754591, y^1_{2111} = 0.03380634, y^2_{2311} = 0.02629382, y^1_{1212} = 0.03464107, y^2_{1112} = 0.03380634, y^2_{1312} = 0.00459098, y^1_{2221} = 0.00125209, y^1_{2321} = 0.00542571, y^2_{2221} = 0.03171953$
0.4	0.6	0.6377220	0.9466040	$t = 0.00104493, y^1_{2111} = 0.03385580, y^2_{2311} = 0.00167189, y^1_{1212} = 0.03542320, y^2_{1112} = 0.03385580, y^2_{1312} = 0.02925810, y^1_{1321} = 0.02685475, y^1_{2221} = 0.000522470, y^1_{2321} = 0.00616510, y^2_{2221} = 0.03176594$
0.2	0.8	0.6192231	0.9519422	$t = 0.00106323, y^1_{2111} = 0.02608449, y^1_{1212} = 0.02977034, y^2_{1112} = 0.03274738, y^2_{1312} = 0.03147151, y^1_{1321} = 0.03359796, y^1_{2121} = 0.00836405, y^1_{2221} = 0.00680465, y^2_{2121} = 0.00170116, y^2_{2221} = 0.03232209$
0	1	0.4599311	0.9814735	$t = 0.00107712, y^1_{2111} = 0.01831107, y^2_{2111} = 0.00172339, y^1_{1112} = 0.01658768, y^2_{1112} = 0.03317536, y^2_{1312} = 0.03188281, y^1_{1221} = 0.03705299, y^1_{1321} = 0.01055579, y^1_{2321} = 0.02348126, y^2_{2221} = 0.03274451$

Table 11. Solutions (after retransformation) for the DOCM model using WSM with various weight vectors

w_1	w_2	Z^*_{1M}	Z^*_{2M}	Solution
1	0	0.8455414	0.6385350	$x^2_{1211} = 30.4, x^2_{1311} = 6.4, x^2_{2311} = 22.4, x^1_{1212} = 34.4, x^1_{1312} = 25.2, x^2_{2312} = 0.8, x^1_{2121} = 32.4, x^2_{2121} = 32.4, x^1_{2322} = 6.4$
0.8	0.2	0.8345128	0.7170659	$x^1_{1311} = 31.6, x^1_{2111} = 6.93, x^2_{1211} = 30.4, x^2_{2311} = 29.6, x^1_{1212} = 28.0, x^2_{1112} = 6.4, x^1_{2121} = 25.5, x^1_{2221} = 6.4, x^2_{2121} = 26$
0.6	0.4	0.7719883	0.8370154	$x^1_{1311} = 25.2, x^1_{2111} = 32.4, x^2_{1211} = 1.8, x^2_{2311} = 27, x^1_{1212} = 34.4, x^2_{1112} = 32.4, x^2_{1312} = 2.6, x^1_{2321} = 6.4, x^2_{2221} = 28.6$
0.5	0.5	0.7679466	0.8430718	$x^1_{1311} = 26.4, x^1_{2111} = 32.4, x^2_{2311} = 25.2, x^1_{1212} = 33.2, x^2_{1112} = 32.4, x^2_{1312} = 4.4, x^1_{2221} = 1.2, x^1_{2321} = 5.2, x^2_{2221} = 30.4$
0.4	0.6	0.6377220	0.9466040	$x^1_{2111} = 32.4, x^2_{2311} = 1.6, y^1_{1212} = 33.9, x^2_{1112} = 32.4, x^2_{1312} = 28, x^1_{1321} = 25.7, x^1_{2221} = 0.5, x^1_{2321} = 5.9, x^2_{2221} = 30.4$
0.2	0.8	0.6192231	0.9519422	$x^1_{2111} = 24.53, x^1_{1212} = 28, x^2_{1112} = 30.8, x^2_{1312} = 29.6, x^1_{1321} = 31.6, x^1_{2121} = 7.87, x^1_{2221} = 6.4, x^2_{2121} = 1.6, x^2_{2221} = 30.4$
0	1	0.4599311	0.9814735	$x^1_{2111} = 17, x^2_{2111} = 1.6, x^1_{1112} = 15.4, x^2_{1112} = 30.8, x^2_{1312} = 29.6, x^1_{1221} = 34.4, x^1_{1321} = 9.8, x^1_{2321} = 21.8, x^2_{2221} = 30.4$

b) Minimizing distance method.

The solution obtained for the linear DOCM model using MDM is: $t = 0.001042174$, $y^1_{1311} = 0.02633308$, $y^1_{2111} = 0.0337664$, $y^2_{1211} = 0.001770472$, $y^2_{2311} = 0.028003326$, $y^1_{1212} = 0.0357805$, $y^2_{1112} = 0.03376644$, $y^2_{1312} = 0.00281509$, $y^1_{2221} = 0.000070294$, $y^1_{2321} = 0.00659962$, $y^2_{2221} = 0.02991162$ and the compromise solutions obtained for the objective functions in the linear DOCM model is $Z'^*_{1M} = 0.7717614$ and $Z'^*_{2M} = 0.8373554$. After retransformation, the solution obtained for the DOCM using MDM is: $x^1_{1311} = 25.267, x^1_{2111} = 32.4, x^2_{1211} = 1.699, x^2_{2311} = 26.9, x^1_{1212} = 34.33, x^2_{1112} = 32.4, x^2_{1312} = 2.70, x^1_{2221} = 0.0674, x^1_{2321} = 6.33, x^2_{2221} = 28.701$.

This solution can be substituted into the original model with uncertainty to obtain a range of uncertainty for the objective functions i.e. $\tilde{Z}_1 \in [684.74, 1139.24, 1619]$ and $\tilde{Z}_2 \in [796.53, 1276.3, 1756.06]$.

c) Fuzzy programming technique.

To apply the FPT, the U_t and L_t values are taken to be $L_1 = 0.07144754,\ L_2 = 0.03139867,\ U_1 =$

0.9889604, $U_2 = 0.9823045$. Now, solving the linear DOCM model for two different sets of shape parameters $(s_1, s_2) = (2, 3)$ and $(s_1, s_2) = (-2, -2)$ in the exponential mf, we obtain the solutions shown in Table 12.

Table 12. Solutions for the DOCM model using fuzzy programming with shape parameters (s_1, s_2) in the exponential mf

(s_1, s_2)	Z'^*_{1M}	Z'^*_{2M}	λ	Solution
(2,3)	0.78157	0.82056	0.57939	$t = 0.001042029, y^1_{1311} = 0.02625913, y^1_{2111} = 0.03101790, y^2_{1211} = 0.005991410, y^2_{2311} = 0.03084405, y^1_{1212} = 0.03584579, y^2_{1112} = 0.03235525, y^1_{2121} = 0.002743839, y^1_{2321} = 0.006668985, y^2_{2121} = 0.001406483, y^2_{2221} = 0.02568627$
(-2,-2)	0.79956	0.78601	0.92000	$t = 0.001029264, y^1_{1311} = 0.02593745, y^1_{2111} = 0.02451297, y^2_{1211} = 0.01510546, y^2_{2311} = 0.03046622, y^1_{1212} = 0.03540668, y^2_{1112} = 0.02277146, y^1_{2121} = 0.008835188, y^1_{2321} = 0.006587290, y^2_{2121} = 0.01057670, y^2_{2221} = 0.01618417$

The solutions given in Table 12 are retransformed into the original decision variables using $x = y/t$ as shown in Table 13.

Table 13. Solutions (after retransformation) for the DOCM model using fuzzy programming with shape parameters (s_1, s_2) in the exponential mf

(s_1, s_2)	Z^*_{1M}	Z^*_{2M}	λ	Solution
(2,3)	[675.15, 1129.78, 1609.62]	[812.53, 1292.37, 1772.2]	0.57939	$x^1_{1311} = 25.2, x^1_{2111} = 29.767, x^2_{1211} = 5.750, x^2_{2311} = 29.6, x^1_{1212} = 34.4, x^2_{1112} = 31.05, x^1_{2121} = 2.63, x^1_{2321} = 6.4, x^2_{2121} = 1.35, x^2_{2221} = 24.65$
(-2,-2)	[648.37, 1108.96, 1594.74]	[836.34, 1322.12, 1807.90]	0.9200	$x^1_{1311} = 25.2, x^1_{2111} = 23.816, x^2_{1211} = 14.676, x^2_{2311} = 29.6, x^1_{1212} = 34.4, x^2_{1112} = 22.124, x^1_{2121} = 8.584, x^1_{2321} = 6.4, x^2_{2121} = 10.276, x^2_{2221} = 15.724$

9. Analysis of the sensitivity to confidence levels

This section investigates the sensitivity of the objective functions with respect to confidence levels. Of the three models discussed, only the OVM and DOCM models involve confidence levels in their objective functions and constraints. Therefore, a sensitivity analysis is performed for both of these models to investigate how variation in the confidence levels affects the objective values attained for these models. The sensitivity analysis is performed by varying the confidence levels α_i^p, β_j^p and γ_{kr} present in the constraints of the OVM and DOCM models. The confidence level α_t associated with the objective function can be varied over the interval $[0, 1]$. In this sensitivity analysis, two cases of the confidence levels α_t $(\alpha_t = 0.1, 0.9)$ have been considered for the objective functions. The sensitivity analysis is carried out by changing one confidence level (say α_i^p) in the range $[0.1, 0.9]$ and assuming that the other two confidence levels (β_j^P, γ_{kr}) are fixed to be 0.9. Once the variation of α_i^p is completed, one can vary β_j^p between $[0.1, 0.9]$ and the other two confidence levels $(\alpha_i^p, \gamma_{kr})$ are fixed to be 0.9. This step is repeated for γ_{kr} by fixing α_i^p and β_j^p to 0.9. This procedure of varying each of the three confidence levels is performed for two cases of α_t, i.e. $\alpha_t = 0.1$ and $\alpha_t = 0.9$. In this section, results of the sensitivity analysis are only presented for FPT. However, a similar analysis can be done for the other two solution methodologies, WSM and MDM.

Table 14 and Table 15 display the objective values obtained for the OVM model during the sensitivity analysis. The column "Variation in α_i^p" shows that only α_i^p is varied in the range $[0.1, 0.9]$ and the other two confidence levels β_j^p and γ_{kr} are fixed to be 0.9. "CL" is the notation used for variation of the confidence levels α_i^p, β_j^p and γ_{kr}. A graphical representation of the sensitivity analysis performed with respect to all the confidence levels α_i^p, β_j^p and γ_{kr} is shown in Figure 1 and Figure 2. It is observed from

Figure 1 and Figure 2 that the values of the objective values attained are decreasing with respect to the tested confidence levels α_i^p, β_j^p and are constant with respect to γ_{kr}.

Table 14. Objective values obtained during the sensitivity analysis for the OVM models with exponential mf using shape parameters (2,3)

CL	Obj fun	Variation in α_i^p		Variation in β_j^p		Variation in γ_{kr}	
		$\alpha_t = 0.1$	$\alpha_t = 0.9$	$\alpha_t = 0.1$	$\alpha_t = 0.9$	$\alpha_t = 0.1$	$\alpha_t = 0.9$
0.1	Z_{1S}^*	1493.107	721.1515	1671.539	814.4289	1490.923	711.6150
	Z_{2S}^*	1689.152	831.6846	1895.513	931.0897	1681.916	830.0638
0.2	Z_{1S}^*	1492.775	719.9103	1648.945	801.5637	1490.923	711.6150
	Z_{2S}^*	1688.319	831.4453	1868.834	918.4052	1681.916	830.0638
0.3	Z_{1S}^*	1492.441	718.6667	1626.350	788.6978	1490.923	711.6150
	Z_{2S}^*	1687.487	831.2072	1842.157	905.7267	1681.916	830.0638
0.4	Z_{1S}^*	1492.108	717.4208	1603.753	775.8010	1490.923	711.6150
	Z_{2S}^*	1686.654	830.9705	1815.480	893.0540	1681.916	830.0638
0.5	Z_{1S}^*	1491.776	716.1725	1581.156	762.9044	1490.923	711.6150
	Z_{2S}^*	1685.821	830.7350	1788.805	880.3867	1681.916	830.0638
0.6	Z_{1S}^*	1491.523	714.9449	1558.559	749.9978	1490.923	711.6150
	Z_{2S}^*	1684.892	830.4964	1762.129	867.7249	1681.916	830.0638
0.7	Z_{1S}^*	1491.271	713.8256	1535.963	737.1348	1490.923	711.6150
	Z_{2S}^*	1683.962	830.3623	1735.453	855.1173	1681.916	830.0638
0.8	Z_{1S}^*	1491.019	712.7046	1513.364	724.3610	1490.923	711.6150
	Z_{2S}^*	1683.033	830.2300	1708.779	842.6056	1681.916	830.0638
0.9	Z_{1S}^*	1490.923	711.6150	1490.923	711.6150	1490.923	711.6150
	Z_{2S}^*	1681.916	830.0638	1681.916	830.0638	1681.916	830.0638

Table 15. Objective values obtained during the sensitivity analysis for the OVM models with exponential mf using shape parameters (-2,-2)

CL	Obj Fun	Variation in α_i^p		Variation in β_j^p		Variation in γ_{kr}	
		$\alpha_t = 0.1$	$\alpha_t = 0.9$	$\alpha_t = 0.1$	$\alpha_t = 0.9$	$\alpha_t = 0.1$	$\alpha_t = 0.9$
0.1	Z_{1S}^*	1471.550	707.6865	1647.927	799.2245	1468.456	698.5429
	Z_{2S}^*	1715.020	845.0929	1923.847	945.9535	1708.877	844.2251
0.2	Z_{1S}^*	1471.117	706.5402	1625.464	786.6545	1468.456	698.5429
	Z_{2S}^*	1714.307	844.9882	1897.011	933.2209	1708.877	844.2251
0.3	Z_{1S}^*	1470.685	705.3925	1603.003	774.0783	1468.456	698.5429
	Z_{2S}^*	1713.594	844.8848	1870.173	920.4952	1708.877	844.2251
0.4	Z_{1S}^*	1470.253	704.2435	1580.544	761.4961	1468.456	698.5429
	Z_{2S}^*	1712.881	844.7828	1843.332	907.7759	1708.877	844.2251
0.5	Z_{1S}^*	1469.821	703.0934	1558.086	748.9081	1468.456	698.5429
	Z_{2S}^*	1712.167	844.6822	1816.489	895.0628	1708.877	844.2251
0.6	Z_{1S}^*	1469.433	701.9482	1535.630	736.3148	1468.456	698.5429
	Z_{2S}^*	1711.400	844.5761	1789.644	882.3557	1708.877	844.2251
0.7	Z_{1S}^*	1469.046	700.8016	1513.176	723.7162	1468.456	698.5429
	Z_{2S}^*	1710.633	844.4715	1762.797	869.6541	1708.877	844.2251
0.8	Z_{1S}^*	1468.658	699.6537	1490.723	711.1126	1468.456	698.5429
	Z_{2S}^*	1709.866	844.3685	1735.949	856.9580	1708.877	844.2251
0.9	Z_{1S}^*	1468.456	698.5429	1468.456	698.5429	1468.456	698.5429
	Z_{2S}^*	1708.877	844.2251	1708.877	844.2251	1708.877	844.2251

The sensitivity of the linear DOCM model is examined likewise w.r.t. the confidence levels α_i^p, β_j^p, γ_{kr}. Table 16 shows the uncertain measures of the objective functions obtained during the sensitivity analysis for the linear DOCM model. Figure 3 and Figure 4 illustrate the sensitivity analysis for the DOCM model and it is observed that the uncertain measures of the objective functions are non-decreasing w.r.t. the confidence levels. This means that with the increase in the value of confidence levels, the value of the uncertain measure also increases eventually.

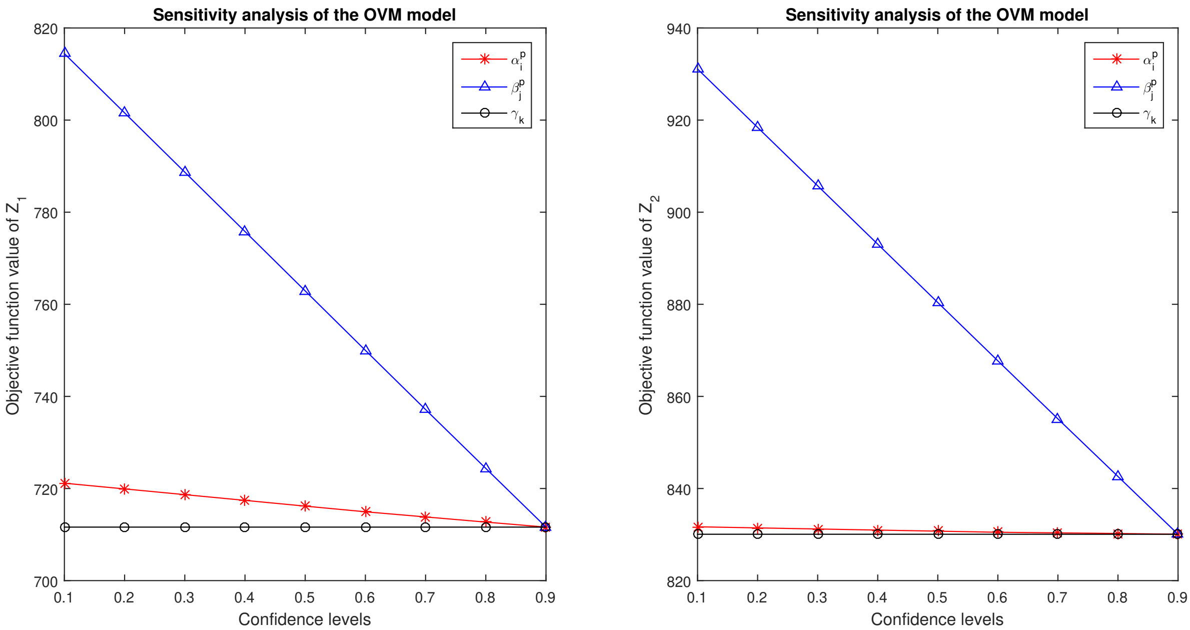

Figure 1. Sensitivity analysis for the OVM model w.r.t α_i^p, β_j^p and γ_{kr} using exponential mf with shape parameters $(2, 3)$ for class cl_1

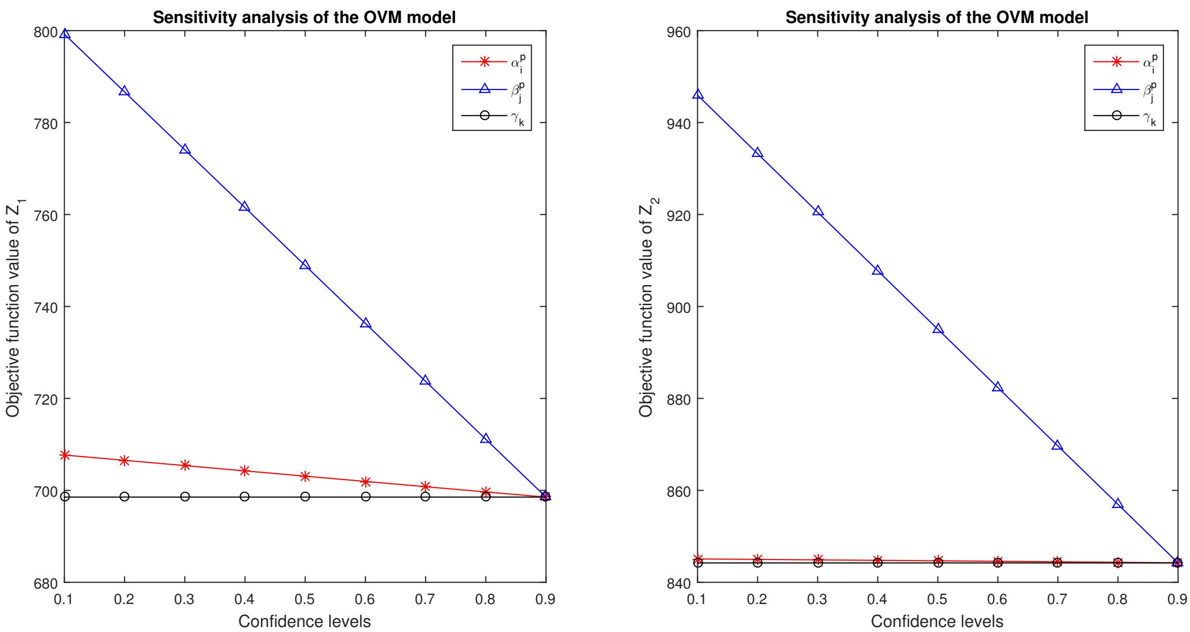

Figure 2. Sensitivity analysis for OVM model w.r.t α_i^p, β_j^p and γ_{kr} using exponential mf with shape parameters $(-2, -2)$ for class cl_1

10. Results and comparison

In this section, we present the results obtained for the 4DMOMITP under uncertainty using the EVM, OVM and DOCM models. The solution methodologies chosen for solving these three models are: the weighted sum method, minimizing distance method and fuzzy programming technique. Table 17 compares the results obtained for the EVM, OVM and DOCM models using each of the three solution methodologies. We can observe from Table 17 that no method dominates any other method, since the

Table 16. Values of the objective function obtained during the sensitivity analysis of the linear DOCM model

CL	Z^*_{tM}	Variation in α^p_i		Variation in β^p_j		Variation in γ_{kr}	
		(2,3)	(-2,-2)	(2,3)	(-2,-2)	(2,3)	(-2,-2)
0.1	Z^*_{1M}	0.773720	0.791585	0.610116	0.627381	0.781573	0.799561
	Z^*_{2M}	0.806964	0.772996	0.638970	0.608550	0.820564	0.786012
0.2	Z^*_{1M}	0.774765	0.792639	0.628963	0.646606	0.781573	0.799561
	Z^*_{2M}	0.808528	0.774499	0.658306	0.627474	0.820564	0.786012
0.3	Z^*_{1M}	0.775808	0.793690	0.648272	0.666301	0.781573	0.799561
	Z^*_{2M}	0.810100	0.776013	0.678316	0.647077	0.820564	0.786012
0.4	Z^*_{1M}	0.776849	0.794738	0.668059	0.686480	0.781573	0.799561
	Z^*_{2M}	0.811680	0.777536	0.699038	0.667393	0.820564	0.786012
0.5	Z^*_{1M}	0.777811	0.795690	0.689388	0.707847	0.781573	0.799561
	Z^*_{2M}	0.813415	0.779248	0.721358	0.689171	0.820563	0.786012
0.6	Z^*_{1M}	0.778840	0.796766	0.711578	0.729950	0.781573	0.799561
	Z^*_{2M}	0.815026	0.780726	0.744750	0.711972	0.820564	0.786012
0.7	Z^*_{1M}	0.779789	0.797744	0.734339	0.752609	0.781573	0.799561
	Z^*_{2M}	0.816794	0.782394	0.769011	0.735634	0.820564	0.786012
0.8	Z^*_{1M}	0.780735	0.798718	0.757697	0.775846	0.781573	0.799561
	Z^*_{2M}	0.818573	0.784074	0.7941954	0.760212	0.820564	0.786012
0.9	Z^*_{1M}	0.781573	0.799561	0.781573	0.799561	0.781573	0.799561
	Z^*_{2M}	0.820564	0.786012	0.820564	0.786012	0.820564	0.786012

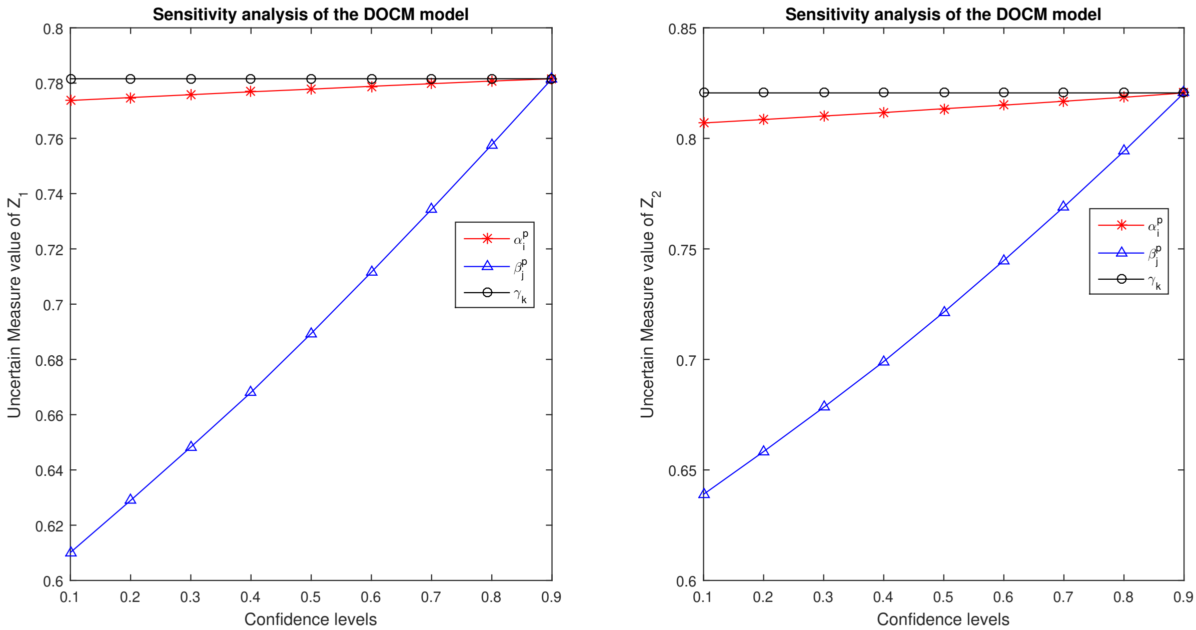

Figure 3. Sensitivity analysis for the DOCM model w.r.t α^p_i, β^p_j and γ_{kr} using exponential mf with shape parameters $(2, 3)$

solutions obtained are Pareto-optimal. From Table 17, it is clearly seen that if one method gives a better solution for one objective than a second method, then the second method gives a better solution for the second objective. The results for the DOCM model obtained using each of the three methods are non-dominating with respect to each other.

From the results obtained using the three given solution methodologies, we can say that the minimizing distance method will always lead to a single optimal solution, whereas the weighted sum method and fuzzy programming technique (with exponential mf) give various solutions according to the choice of weights and shape parameters, respectively. Also, we would like to add that the fuzzy technique with exponential mf is more efficient than the other two methodologies, because a number of alternatives

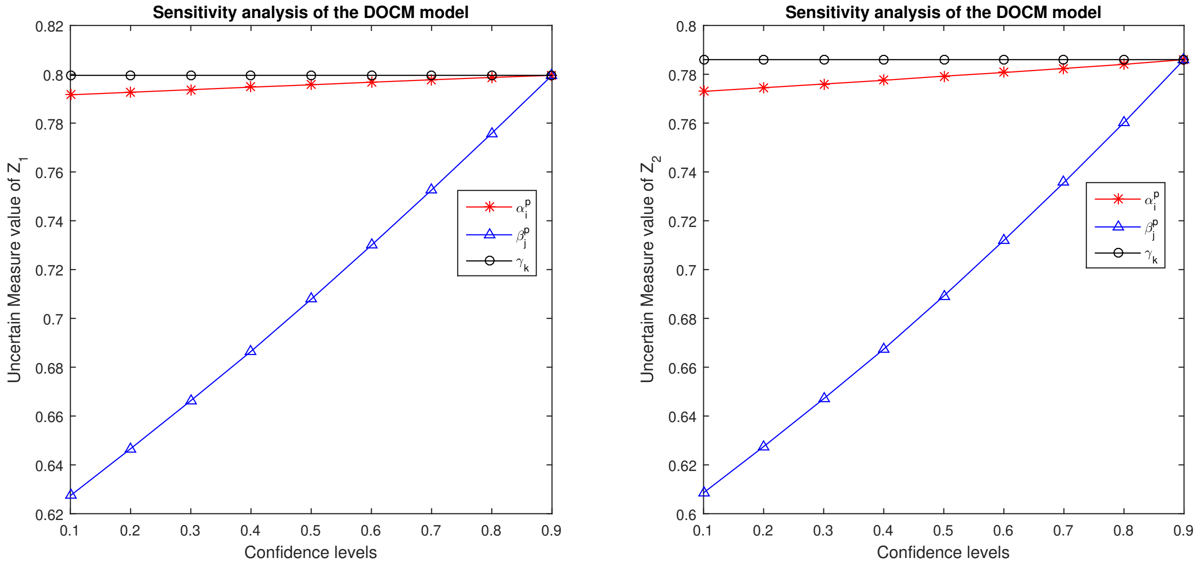

Figure 4. Sensitivity analysis for the DOCM model w.r.t α_i^p, β_j^p and γ_{kr} using exponential mf with shape parameters $(-2,-2)$

Table 17. Comparison of the results obtained using various methodologies

Model	Obj values	WSM with equal weights	MDM	FPT with exp mf	
				(2,3)	(-2,-2)
EVM	Z_{1E}^*	1142.50	1188.0	1193.536	1173.549
	Z_{2E}^*	1398.0	1352.50	1346.964	1366.951
OVM (Cl_1)	Z_{1S}^*	712.560	711.1706	711.6150	698.5429
	Z_{2S}^*	829.040	830.5452	830.0638	844.2251
OVM (Cl_2)	Z_{1S}^*	1734.160	1697.680	1678.140	1657.182
	Z_{2S}^*	1842.080	1878.560	1906.785	1937.294
DOCM	Z_{1M}^*	0.7679466	0.7717614	0.7815730	0.995611
	Z_{2M}^*	0.8430718	0.8373554	0.8205637	0.7860120

can be obtained with the help of shape parameters, which can be changed without any restriction unlike choosing a weight vector whose components sum to one, thus providing the decision-maker with a wide range of solutions.

Moreover, the four-dimensional problem under an uncertain environment is a relatively new area of research interest. Sahoo et al. [31] were the first to present the 4DMOMITP and obtained solutions using the goal programming technique. In their work, they considered an uncertain environment with normal uncertain variables. Recently, Revathi et al. [29] studied a four-dimensional MOMITP that took into account vehicle speed by considering linear uncertain variables. Later, in 2021, Revathi et al. [28] extended their work by studying four-dimensional TP with fractional objectives under uncertainty. It should be noted that the four-dimensional MOMITP presented by Sahoo et al. [31] deals with normal uncertain variables, whereas the 4DMOMITP discussed by Revathi et al. [29, 28] deals with two different problems. This article has considered zigzag uncertain variables for uncertain 4DMOMITP which differs from the previously discussed research articles and thus the results for such problems cannot be directly compared with the results from other articles. Also, it has been observed that solutions to four-dimensional problems or any variant of TP under a uncertain environment presented in the literature most commonly have utilized the expected value criterion or chance-criterion and no research article has considered the optimistic value criterion to convert a model involving uncertainty into a deterministic model. Hence, this article considered the optimistic value criterion along with two other ranking criteria to obtain deterministic models.

11. Conclusion

This paper has discussed a 4DMOMITP in an uncertain environment with zigzag uncertain variables. We solved the MOMITP model under uncertainty by transforming to one of three deterministic models: EVM, OVM and DOCM. Further, these deterministic models were solved using each of the three following approaches: WSM, MDM and FPT with exponential membership function. The EVM model gives a solution in terms of the expected values of the objective functions. The EVM model can give one or various solutions to the decision-maker depending on the solution methodology used. But, the OVM and DOCM models can always give various solutions to the decision-maker, independently of the solution method used, according to the confidence levels selected. Hence, these two models can provide the decision-maker with a number of alternative solutions by varying the confidence levels. Thus the OVM and DOCM models provide a wider range of solutions than the EVM model.

This paper has focused on four-dimensional multi-objective multi-item transportation problems in an uncertain environment with zigzag uncertain variables. In the future, this work can be extended by considering multi-objective transportation problems and their variants under two-fold uncertainty. Also, most of the variants of transportation problems have been solved using classical approaches and very little research has been carried out in the direction of evolutionary algorithms. Hence, there are a large number of applications of uncertainty theory (which is applicable when no historical references are available) to real-world complex problems that can be solved using various evolutionary algorithms, such as genetic algorithms, Jaya algorithms, Rao algorithms etc.

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