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A heuristic approach to optimizing the loading of homogeneous marine cargo

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Abstract

In this article, the optimal loading of homogeneous marine cargo is considered. A mathematical formulation in terms of a mixed-integer linear program can be given. Still, the level of complexity turns out to be too high to perform full-scale computations. On the one hand, the reasons for this are the multitude of variables and constraints. On the other hand, feasible solutions to such problems may often be economically unacceptable or simply empty. Therefore, a heuristic is presented, according to which the relaxation of the limiting conditions influencing the solution's feasibility and its economic profitability was parametrized. Under this heuristic, shifting the deadlines of selected orders is allowed. Also, the assignment of orders to vessels is separated from the allocation of vessels to piers in loading and unloading ports. The solution presented can be easily generalized by adding additional restrictions or features like indirect vessels, founding cost, or differentiation between materials.

Keywords: large-scale optimization, cargo loading, heuristic, marine transportation, suboptimality

1. Introduction

Marine transportation drives more than 80% of global trade volume, moving 11.08 billion tons of goods each year [22]. From the deliverer perspective, including order handling, loading, marine transport, and unloading, require automation and optimization at every stage of the cargo flow process. Savings from such optimization are calculated in millions of dollars per year. Transport and logistics processes cover single-criterion optimization like determination of the optimal route of the ship [5, 7], minimization of collision risk [24], ship loading for the purpose of its stabilization [3, 23], logistics of cargo transportation between ports [2], optimization from the port perspective [6], etc. Most optimization problems related to marine operations are demanding for at least two reasons. The first one is the need to get results in near real-time. It significantly narrows the range of possible approaches to the problem. Therefore, heuristics [21] or genetic algorithms [13] are often used. The second aspect of the difficulty of the marine optimization problems, closely related to the first one, is their high computational complexity. The berth

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allocation could be given as an example of such a problem. There are many variants of this problem (see [10], [9]), but each of them is affected to a greater or lesser degree by the complexity of the class of NP-complete problems.

An overview of the problems and static and dynamic optimization methods in marine transport can be found, for example, in [16] or [8]. The extensive range of issues related to this subject translates into many possible objectives, from optimization of vessel arrival times, through the minimization of vessel total service times or vessel load maximization, to minimization of fuel consumption or maximization of berth productivity. It often results in the inability to choose one correct optimization criterion. This is why the marine logistics is an excellent field for multi-criteria optimization, e.g. minimization of transport cost with a minimum delivery time [25, 4], route selection [12], or cost with the service reliability and the shipping emission [20]. Various applications of multi-criteria decision making (MCDM) in maritime transportation can be found, for example, in [15] or [19].

In this article, optimization is limited to cargo loading, where the problems of vessel stabilization or load maximization are neglected. Therefore it is suitable for relatively homogeneous materials like wood pellets [1], gas [17] or oil [18]. This optimization formulation is also appropriate for the primary aluminum transportation problem. Real data related to this issue were used to verify the proposed solution. The complexity of the problem turns out to be so high that optimization in terms of the linear programming problem is not possible. The reason is not the difficulty of the mathematical formulation of the problem but the multitude of variables and constraints. Secondly, for real-world data, the set of feasible solutions is often economically unacceptable. The multi-criteria strictly defined optimization problem was transformed into a relaxation of the original problem, where selected constraints were relaxed. The goal of this article was to present a heuristic, which splits the optimization problem into highly parametrized, sequentially connected subproblems. Theoretically, an exact solution of each subproblem does not have to be equivalent to the global optimum of the original problem, but getting into account daily granulation of the problem and highly probable delays during the vessel transportation, the solution approximation returned by heuristic is close enough, which was confirmed by the real-data computations. The advantage of this approach over, for example, genetic algorithms (see [14]) manifests itself in shorter computation time, easy generalization by adding additional requirements, but most of all, more accurate and fully interpretable results thanks to the deterministic nature of the solution.

This article is constructed as follows. After the introduction, there is a section with the problem formulation and some comments concerning the complexity of the real-life data. The arguments and the examples of the situation, where a feasible solution does not exist are given. The following section describes the heuristic method of finding an approximation of the solution. The fourth section concentrates on the computer calculations on the prepared test examples. A sensitivity analysis was also performed concerning selected parameters. This article ends with a discussion about the possibilities of generalization of the proposed solution and its applicability in other transport and logistics processes.

2. Problem formulation

The problem discussed in this article should be considered from at least three points of view. From the end consumer perspective, the delivery should be simply on time. The role of the port management in the logistic transportation process is to ensure the availability of the pier with specific parameters at a given time and the organization of loading and unloading works. Finally, from the deliverer point of view, the goal is to minimize the costs of delivery, including the storage in the port and the rental of vessels, meeting the expectations of the end customer and the infrastructure capabilities of the ports. The formulation of the discussed problem comes down to the definition of constraints. For the sake of clarity of the description, we will limit ourselves to presenting the issue in a simplified version, disregarding the specific requirements related to the transport of aluminum. Information on possible generalizations will be provided in one of the sections.

The set of assumptions will be presented first in a descriptive form, and then the mathematical formulation of the problem will be given. We consider the following assumptions.

1. Each order must be delivered within deadline_from and deadline_to.
2. The time of loading and unloading depends on the pier and the size of the vessel.
3. Loading and unloading take place only on working days (varies by port).
4. Marine transport time depends on the distance between the source and destination port and the size of the vessel.
5. Piers are limited by vessel size, and their availability is determined by the loading and unloading schedule.
6. There can be only one vessel loading or unloading on the pier each day.
7. Orders cannot be split between vessels.
8. Cargo storage charges depend on the port, orders volume, and the number of storage days.
9. Vessels are shipped directly to a destination port (without intermediate ports).

It is worth elaborating on the last two assumptions. Cargo storage charges may be charged on the loading or unloading port side. We can therefore decide on one of the two solutions. Either the order is shipped to the destination port before deadline_from, and there are storage costs in the destination port. Either we wait with the shipment to get the order within specified deadlines, and the storage costs are paid at the loading port. The decision should be made based on the different amounts of storage costs in ports, the availability of vessels and piers, and the possibility of combining with other orders.

The ninth assumption is the restriction in the number of destination ports for one vessel. A sequence of intermediate vessel destination ports is not obvious. On the one hand, shipping to the nearest destination port first would be cheaper, but on the other hand, the deadlines for some orders may not be fulfilled choosing this route.

To define the objective function, two types of costs must be considered:

(A) storage costs depending on the port, orders volume, and storage time,

(B) vessel costs depending on the vessel capacity and the number of rental days (including loading and unloading time).

Minimizing the sum of these two types of costs is the basis of the optimization problem that needs to be solved. Both objective function and set of constraints may be defined in terms of the mixed-integer linear programming problem. The linear optimization model is the mathematical formulation of the described constraints and costs is presented in Appendix A. In the remaining part of this section, we will introduce the notation and discuss the complexity of the problem.

Let $\mathcal{O}$ be the set of orders, $\mathcal{V}$ set of vessels, $\mathcal{P}$ set of ports, $\mathcal{R}_P$ set of piers in port P, and T set of days covering deadlines of all orders and dates of orders availability in the loading port. The presented in the appendix mathematical formulation can be transformed into the mixed-integer linear optimization problem by adding extra binary variables and exploring the so-called big M method [11]. The problem, therefore, lies not in difficulty in a formulation as a linear model but the high complexity and possible lack of feasible solutions. First, let us look closer at the matter of complexity. Ignoring the issues of many additional variables needed to formulate the problem in linear form, let us consider the case where there are 500 orders, 50 vessels, 20 ports with only two piers in each of them, and the time horizon of 90 days. The set of variables has a cardinality of 1.26 billion for these numbers. Detailed calculation is given in Table 1. With additional variables required to apply the big M approach, the size of the optimization problem could be even more if we take the number of variables as the measure of complexity. The number of constraints needed for the formulation is even greater. Therefore, finding the solution in a

Table 1. Estimation of the number of variables in the linear optimization for an example assumptions

Variable	Indexes	Count
$x^d_{o,v,k}$	$\lvert\mathcal{T}\rvert \times \lvert\mathcal{O}\rvert \times \lvert\mathcal{V}\rvert \times (2 \times \lvert\mathcal{P}\rvert) = 90 \times 500 \times 50 \times 40$	90 000 000
$\alpha^d_{o,v,k}$	$\lvert\mathcal{T}\rvert \times \lvert\mathcal{O}\rvert \times \lvert\mathcal{V}\rvert \times (2 \times \lvert\mathcal{P}\rvert) = 90 \times 500 \times 50 \times 40$	90 000 000
$y^d_{o,v,k}$	$\lvert\mathcal{T}\rvert \times \lvert\mathcal{O}\rvert \times \lvert\mathcal{V}\rvert \times (2 \times \lvert\mathcal{P}\rvert) = 90 \times 500 \times 50 \times 40$	90 000 000
$\beta^d_{o,v,k}$	$\lvert\mathcal{T}\rvert \times \lvert\mathcal{O}\rvert \times \lvert\mathcal{V}\rvert \times (2 \times \lvert\mathcal{P}\rvert) = 90 \times 500 \times 50 \times 40$	90 000 000
$c^d_{o,P}$	$\lvert\mathcal{T}\rvert \times \lvert\mathcal{O}\rvert \times \lvert\mathcal{P}\rvert = 90 \times 500 \times 20$	900 000
$\delta^d_{v,k}$	$\lvert\mathcal{T}\rvert \times \lvert\mathcal{V}\rvert \times (2 \times \lvert\mathcal{P}\rvert) = 90 \times 50 \times 40$	180 000
$\varepsilon^d_{v,k}$	$\lvert\mathcal{T}\rvert \times \lvert\mathcal{V}\rvert \times (2 \times \lvert\mathcal{P}\rvert) = 90 \times 50 \times 40$	180 000
$\gamma^d_{o,v,P1,P2}$	$\lvert\mathcal{T}\rvert \times \lvert\mathcal{O}\rvert \times \lvert\mathcal{V}\rvert \times \lvert\mathcal{P}\rvert \times \lvert\mathcal{P}\rvert = 90 \times 500 \times 50 \times 20 \times 20$	900 000 000
	sum	1 261 260 000

reasonable time is not possible, if we consider the solution defined as a result of the mixed-integer linear optimization problem.

Besides the time complexity, which could be potentially solved by, for example, parallelization of the computations or increasing the computer power, we have to deal with the more serious problem of lack of feasible solutions. In the real-world input data, the combination of deadlines preferred by the client, availability of aluminum in the loading ports, and availability of vessels, can sometimes result in an empty set of solutions. However, let us suppose that some feasible solution exists. The feasibility is understood in mathematical terms. Meanwhile, business feasibility is a completely different matter. Theoretically, it may be possible to ship each order on a separate vessel. It, however, would be extremely unprofitable. What is the remedy for these problems? One way is to reformulate the problem as a multiobjective optimization problem, where the balance between cost and orders delays is considered. Of course, the size of the optimization problem and its complexity will still be an issue. An alternative is to relax some of the assumptions by allowing for exceeding deadlines or simply accepting the unavailability of the possibility of shipping. This approach became the basis of heuristics, which allows finding the solution's approximation within a reasonable time. The heuristics construction includes splitting into optimization subproblems. The composition of the optimal solution of each subproblem does not necessarily have to translate into an optimal solution of the entire problem. However, the computational speed-up due to the much smaller subproblems outweighs the lack of optimal solution, which would be anyway useless in terms of business. The idea of the proposed heuristic and some details of the implementation are given in the next section of this article.

3. Solution algorithm

Difficulty in solving real-life-sized problems was noticed, among others, in [5], where the optimal solution may be found for a fleet size of up to 9 ships. For larger-scale problems, a heuristic was proposed. Analogously, the optimisation problem of aluminum transportation much exceeds the acceptable computation sizes. Moreover, the existence of a feasible solution is questionable. In this section, we will propose a heuristic algorithm, which allows getting a good approximation of the solution in a reasonable time, on the condition that some assumptions are relaxed, especially those related to orders' deadlines.

The heuristic algorithm may be split into five stages:

Stage I. Filtering the orders and correcting the deadlines.

Stage II. Preliminary order grouping per destination port.

Stage III. Combine group of orders into vessels.

Stage IV. Iterative orders regrouping between vessels (per destination port).

Stage V. Pier allocation.

Each of the stages is described in the remaining part of the section.

Some of the client's expectations are unrealistic and can not be fulfilled. The first stage of the algorithm comes to filtering the set of orders, where orders with deadlines or availability in the loading port exceeding the assumed time horizon are removed. Also, at this stage, the correction of deadlines, which are obviously not feasible, is made. For example, if the first day of order's availability in the loading port increased by the minimal number of days needed for transportation between source and destination port is greater than deadline_to, then deadlines are redefined to meet the realistic loading and transportation time.

Some concessions were made to reduce the dimension of the problem. One of them was the separation of the grouping orders into vessels from the allocation of piers. The second simplification was the parallelization of calculations related to the loading ports – for each of them, stages 2-4 were carried out independently. After filtering and correcting the orders, preliminary grouping for each destination port (and loading port) is performed. It is the second stage of the algorithm. It needs to be emphasized that grouping orders into packages is not equivalent to assigning orders to vessels. The volume of the package is often less than the minimum cost-effective cargo capacity of the vessel. Grouping of orders is done based on the adopted measures of similarity and dissimilarity:

- **similarity index** – number of common possible delivery dates of both orders divided by the total number of potential delivery dates of any of the given orders,
- **dissimilarity index** – one minus similarity index.

The heuristic of a preliminary grouping of orders into packages exploring these measures is to combine orders with a similarity index at least similarity_limit and dissimilarity index lower than dissimilarity_limit.

At this stage, the heuristic could be considered an optimization process because the maximum cargo capacity of vessels must be taken into account. This limitation forces a restriction in grouping into packages, and therefore, in fact, package content optimization is needed. However, another approach was proposed. A greedy combination algorithm on a list of orders sorted in terms of both similarity measures gives a quick solution, which could be changed at further stages of the heuristic. This compromise is based on the speed of the greedy approach allowing one to find an acceptable solution quickly and then potentially iteratively improve it. This stage is extremely sensitive to input data, mainly to order deadlines.

To illustrate this stage, consider the example of five orders with characteristics (volume, first availability day in loading port, beginning and ending deadline for delivery) given in Table 2. For the sake of simplicity, let us assume that there are no holidays in both the loading and unloading ports, and all days are working days. Based on this assumption, the similarity index for orders O1 and O3 is calculated by listing the days on which the vessel's delivery with these two orders can be made. All potential delivery dates are within the range 2022-02-01 and 2022-04-01, but the only possible delivery date is 2022-03-01. Therefore, the similarity index is calculated by dividing the number of common possible delivery dates (1 day) by the total number of potential delivery dates (59 days) equal to 0.02. Similarity indexes for each pair of orders listed in Table 2 are given in Table 3, whereas dissimilarity indexes in Table 4.

Table 2. Basic characteristics of exemplary orders with the same source and destination ports

order ID	volume (VOL)	availability date (AVB)	beginning deadline (FROM)	ending deadline (TO)
O1	500	2022-01-01	2022-02-01	2022-03-01
O2	500	2022-01-01	2022-02-01	2022-03-01
O3	2000	2022-01-01	2022-03-01	2022-04-01
O4	3000	2022-01-01	2022-03-10	2022-04-01
O5	100	2022-01-01	2022-03-01	2022-05-01

Assuming the value of the similarity_limit is equal to 0.7, and the value of dissimilarity_limit is equal to 0.5, we get three packages of orders: in the first package, there are orders O1 and O2 (similarity index equal to 1.00), in the second package orders O3 and O4 (similarity index equal to 0.71), and in the third package there is the single order O5, which has a similarity index with other orders equal only 0.01 (orders O1 and O2), 0.51 (order O3), and 0.42 (order O4).

Table 3. Similarity indexes for pairs of orders described in Table 2

similarity index	O2	O3	O4	O5
O1	$\frac{28}{28} = 1.00$	$\frac{1}{59} = 0.02$	$\frac{0}{59} = 0.00$	$\frac{1}{89} = 0.01$
O2	—	$\frac{1}{59} = 0.02$	$\frac{0}{59} = 0.00$	$\frac{1}{89} = 0.01$
O3	—	—	$\frac{22}{31} = 0.71$	$\frac{31}{61} = 0.51$
O4	—	—	—	$\frac{22}{52} = 0.42$

Table 4. Dissimilarity indexes for pairs of orders described in Table 2

dissimilarity index	O2	O3	O4	O5
O1	0.00	0.98	1.00	0.99
O2	—	0.98	1.00	0.99
O3	—	—	0.29	0.49
O4	—	—	—	0.58

The third stage concerns the combination group of packages from the second stage into vessels. At this stage, the composition of packages is not changed, and the selected packages to the common destination port are combined. Joining packages is an iterative process carried out in pairs. Two packages that satisfy the constraints (mainly deadlines, aluminum availability in loading port, and vessels limitation of cargo capacity) are combined. The process defined in this way is quick but far from optimal. The reason is an unspecified order of packages' pairs under consideration. Let us consider an example of three packages $package_1$, $package_2$, and $package_3$ with the total size of 1000 t, 2500 t, and 3000 t, respectively. Let us also assume the maximum cargo capacity of vessels / piers is 6000 t. Compare two situations with different order of packages' pairs:

1. $package_1 + package_2$ with volume 3500 t and $package_3$ with volume 3000 t,
2. $package_2 + package_3$ with volume 5500 t and $package_1$ with volume 1000 t.

We have two vessels with comparable volumes (3000 t and 3500 t) in the first case. This is the desired solution in terms of costs. The second case represents a highly disproportionate distribution of the cargo sizes (5500 t and 1000 t). In this case, from a purely economic point of view, the shipment with only 1000 t is unprofitable and would not be shipped. Therefore, in the first case, we are ready to ship all three packages (total of 6500 t), but in the second case, only $package_2$ and $package_3$ will be delivered on time (total of 5500 t). This example shows the possible suboptimality, which – similar to the non-optimal decisions in stage two – could be corrected on the fourth stage of the algorithm. Also, after this stage, some of the orders and packages of orders from the previous stage will not be allocated to vessels.

The next stage of the heuristic is an opportunity to improve the solution obtained in the third stage. Improvement is achieved by iterative regrouping between vessels. This process may be fully parallelized by separate computation for each destination port. In each iteration step, orders on two selected vessels are analyzed. Checking every possible allocation of orders between the two vessels is considered to minimize overall costs (with all constraints fulfilled). In cost estimation, the most advantageous pier allocation is assumed, even if this assumption turns out to be unfulfilled in the last stage of the algorithm. It is also worth noticing that shipping vessels at the first available date are not always the best solution. There may be a situation where shipping a vessel on a given day will mean waiting several days in the destination port due to non-working days. In that case, a cheaper solution would be to send the vessel later to arrive at the port of unloading on a working day following the said non-working days.

The number of possible orders assignments to vessels depends on the number of orders on these vessels and grows exponentially. The exact numbers for selected order quantities on two vessels are given in Table 5. Computations in the case with twenty orders are well within the capabilities of the modern computer machines (1 048 576 possibilities), but with a total of 30 orders on two vessels much more difficult (over billion cases). Therefore, not every possibility is considered at this stage of the heuristics, and a max_allocation_number parameter is defined, denoting the maximum allowable number of possibilities considered. To increase the probability of finding the optimal assignment, possibilities are checked in a specific order, beginning with those with the lowest differences of cargo volumes between vessels.

Table 5. Number of possible orders assignments between the two vessels depending on the number of orders

Total number of orders on two vessels	Number of possible assignments
10	1 024
20	1 048 576
25	33 554 432
30	1 073 741 824
35	34 359 738 368

There are three possible sources of non-optimality at this stage. The first one is similar to an issue raised in the previous stage: pairwise analysis does not have to give the optimal result and strongly depends on the order of pairs. The second source of non-optimality is the simplifying assumption of pier availability. The key argument in favor of using this approach is to reduce the dimension of the problem, which implies a noticeable computational speed-up. The third source of potential lack of optimality is introducing the parameter max_allocation_number. In an obvious way, it shortens the computation time but possibly omits the proper assignment of orders to vessels.

The last stage is devoted to the pier allocation. Till this point, for each loading port, the sub-optimal set of vessels with assigned orders has been designated. Some of the orders remained unmerged with other orders or combined in the form of packages from the second stage because they turned out to be too little similar to the other packages to be jointly loaded onto one vessel. A configurable parameter gives the minimum cargo volume that would be profitable to ship. The rest of the orders are assigned to the vessels in an assumed way to be the least costly. This assumption is based on piers' availability in loading and unloading ports. However, this assumption will be perhaps not satisfied because computations for each loading port were carried out independently, and collisions in piers are very likely. Therefore, the obtained solution probably will not be optimal. Still, due to the splitting problem into the assignment of orders to vessels and allocation of piers separately, we have reason to expect that we will get the result in a finite and reasonable time (depending primarily on the value of the parameter max_allocation_number).

Classic optimization of piers allocation takes into account the availability of piers in loading and unloading ports. In case of a lack of matching between available piers and deadlines for orders assigned to the vessel, the result would be no solution. This situation is very likely, therefore instead, optimal allocation of vessels is done in a greedy algorithm by sequential allocation per individual loading port, where optimization is made within the loading port so that the total cost is the lowest. The result may depend on the order of loading ports considered at this stage. Suppose the proper allocation of all vessels does not exist for some loading port. In that case, the deadlines of orders in vessels, for which the available pier was not found are extended for a fixed number of days, and the optimization process for this particular loading port is repeated. This approach must end eventually because a shift of deadlines by the appropriate number of days guarantees piers availability. Also, after extending the deadlines, some vessels may be combined into a single vessel.

To illustrate the stages from III to V, we use orders O1-O4 described in Table 2. After the second stage, we have two packages of orders, which are in stage III combined into vessels:

(A) orders O1 and O2 with a total volume of 1000 t,

(B) orders O3 and O4 with a total volume of 5000 t.

Let us complete the input data for this exemplary problem with the following information:

- 10 days for delivery from source port to destination port (DAYS) regardless of the size of the vessel,
- the maximum speed of loading and unloading (SPEED_LOAD, SPEED_UNLOAD) is equal to 10 000 t, which allows to load and unload all considered orders in one day,
- the price per day of use the vessel is equal to 750 USD for small vessel (cargo volume smaller than 1000 t), 1000 USD for medium vessel (cargo volume between 1000 t and 2000 t), and 1300 USD for big vessel (2000 t cargo volume or more),
- the price per day of cargo storage in the loading port (STORAGE_COST_PER_TONNE) is equal to 0.01 USD per tonne.

Using this input data, we can calculate the cost of shipping two vessels from the exemplary problem, assuming that they are shipped on the first possible day, which, in the considered case, means 2022-01-19 and 2022-02-25 as a beginning of loading vessel (A) and (B), respectively, (variant 1 of shipment). Detail cost are presented in Table 6.

Table 6. Storage cost for variant 1 of the exemplary problem with orders O1-O4 described in Table 2

order IDs	storage begin	storage end	storage days	storage cost
O1	2022-01-01	2022-01-18	17	85
O2	2022-01-01	2022-01-18	17	85
O3	2022-01-01	2022-02-24	54	1080
O4	2022-01-01	2022-02-24	54	1620

Table 7. Shipping cost for variant 1 of the exemplary problem with orders O1-O4 described in Table 2

vessel	loading begin	unloading end	shipping days	shipping cost
(A)	2022-01-19	2022-01-31	12	900
(B)	2022-02-25	2022-03-09	12	15 600

The total cost of orders shipping case in variant 1 equals 27 470 USD including 2 870 USD storage cost and 24 600 USD shipping cost. In the fourth stage, regrouping vessels (A) and (B), should result in the vessel (C) with orders O1-O3 (volume 3000 t) and vessel (D) with single order O4 (volume 3000 t). The best possible loading date begins 2022-02-16 for the vessel (C) and 2022-02-25 for the vessel (D). Denote this as variant 2 of the exemplary problem.

Table 8. Storage cost for variant 2 of the exemplary problem with orders O1-O4 described in Table 2

order IDs	storage begin	storage end	storage days	storage cost
O1	2022-01-01	2022-02-14	44	225
O2	2022-01-01	2022-02-14	44	225
O3	2022-01-01	2022-02-14	44	900
O4	2022-01-01	2022-02-24	54	1620

The total cost of orders shipping case in variant 2 equals 26 970 USD including 2 970 USD storage cost and 24 000 USD shipping cost. Comparing to variant 1 of shipment, storage costs are slightly higher (2 870 vs. 2 970 USD), but shipping costs have decreased from 24 600 to 24 000 USD. Therefore, total costs are lower in variant 2, so this variant will result from stage IV of the heuristics. In the last stage, pier allocation is to be done. To illustrate possible non-optimality of the heuristics result despite the optimality at every stage, let us assume that there is no free pier in the loading port on the day 2022-02-16, which is the only date when orders O1-O3 can be shipped subject to deadlines constraints. The only option is to load orders the day before at 2022-06-15. However, this means that the vessel has to be used for one day longer, and the demurrage at the destination port will be incurred. Therefore, the total cost would increase to 27 940 USD, which is more than the costs of variant 1.

Table 9. Shipping cost for variant 2 of the exemplary problem with orders O1-O4 described in Table 2

vessel	loading begin	unloading end	shipping days	shipping cost
(C)	2022-02-16	2022-03-01	12	13 000
(D)	2022-02-25	2022-03-09	12	12 000

The goal of the heuristic is to reduce the problem complexity and speed up the computations. The idea behind the proposed algorithm is, on the one hand, to separate assignments of orders to vessels from allocation of piers, and on the other, to parallelize the computations by executing them per loading port. The stages of the algorithm described in this section allow obtaining solutions that compromise the speed of calculations and optimality. This approach has some limitations but is also flexible regarding ease of generalization. These issues will be discussed in more detail in the next section.

4. Limitations and possibility of generalizations

A large number of presented assumptions were introduced to, on the one hand, increase the readability of the heuristics description, and on the other hand, to show the multitude of aspects that may be taken into account during the calculations. From the nine assumptions presented in Section 2, only the last one should be considered as a limitation that can be easily lifted. The approach presented in this article allows adding additional restrictions or features easily. Some of the most common or desired requirements are listed in this section.

- Restriction to one destination port may be lifted. For instance, we can allow for vessels with two destination ports. It is a realistic assumption because shipping orders by two different vessels simultaneously to two ports that are relatively close to each other is not economically justified. One significant change in the heuristic is needed to consider such a modification. Stage III should be split into two substages: the first is combining per destination port groups of orders into vessels, and the second is trying to combine groups of orders to different destination ports into common vessels. The second substage is not obvious because the order in which successive pairs are dealt with impacts the result firmly.
- Besides storage and transportation costs, founding costs may be added to the optimization problem. Founding costs are calculated based on order volumes and handling time and are charged until the order is delivered. This kind of change is easily implemented in the heuristic by modifying the objective function.
- In the basic problem formulation, aluminum cargo is homogenous. Still, there are different types of materials collectively referred to as aluminum, e.g. primary aluminum, billets, rolling slabs, primary foundry alloys, or wire rods. Adding additional restrictions or requirements is possible and extends the method's applicability. Also, the heuristic should be appropriate not only for aluminum but for all homogeneous types of cargo like wood pellets, gas, or oil.
- It is possible to differentiate according to the client's requirements. For example, different costs for delays in delivery per client or specific preferences for joint or separate shipments can be added. These new constraints have to be included at each stage of the heuristic.
- Most of the parameters are typical for the ports but can be differentiated per port or even pair of ports (loading-unloading). This kind of generalization can be used for the parameter max_allocation_number. Depending on the order volumes for some ports, setting this parameter higher to avoid shipping small vessels would be reasonable. In contrast, only small vessels have a chance to be shipped for others due to the lower volumes of aluminum ordered by clients.
- Cost parameter $VESSEL\_COST\_PER\_DAY$ depends only on a vessel, but prices go up in periods of high demand. Therefore, it would be wise to add this relationship to the problem. This

change is easy in terms of implementation and will have a marginal effect on the speed of calculations. Analogously, dependence from the date, besides the port, can be added for $STORAGE\_COST\_PER\_TONNE$.

It needs to be emphasised that the proposed heuristics is dedicated to the homogeneous types of cargo. Theoretically, it could be extended to all kinds of cargo. Still, its original form neglects many issues related to the practical operation during loading in the port like vessels stability, distribution of load along the vessel, or cargo separation due to carriage of cargo with different chemical and physical properties. All mentioned issues affect the cargo loading rate and assignments of orders to vessels, but in the case of homogeneous types of cargo, their impact on loading is relatively low. In this context, the proposed heuristics do not address all aspects of loading marine cargo, although it does offer the possibility of integrating additional restrictions or requirements.

In the remaining part of this section, some remarks about the possibility to improve or speed up the heuristic are given. First of all, postponement of delivery deadlines for selected orders may provide better solutions. There is no clear indication of which orders should be subject to this procedure, but a careful analysis of historical data can be provided as a guide.

Secondly, at the pier allocation stage, shifted order deadlines may create the possibility of merging vessels, which was impossible for the original deadlines. Therefore, an additional stage may be added after the pier allocation stage, with vessel merging and reallocating piers. Theoretically, this stage should be repeated until no improvement by merging vessels is possible.

Also, many calculations can be done in parallel. It is essential in the most complex, in terms of computation time, the fourth stage of the heuristic. Each allocation between the two vessels may be considered independently. Including the parallelization libraries in the implementation could be challenging due to the need to consider the parameter max_allocation_number. Using multiprocessor machines we would like to terminate the computations in all related threads or processes (depending on the operating system).

5. Conclusions

The logistics of homogeneous marine cargo loading and transportation due to high complexity in terms of an optimization problem should be considered problematic. Still, the potential for reducing costs is enormous. The heuristic presented in this article addresses the issue of high computational complexity and the need to relax the assumptions related to unrealistic customer expectations. Following all restrictions, the optimization problem would not have any feasible solution. Allowing to shift deadlines of selected orders makes it possible to get the satisfactory assignment of orders to vessels and vessels to piers in loading and unloading ports. This separation of the stages with these allocations is the main reason behind reducing the problem's complexity. The second speed-up idea is the approach where some rough approximation of the solution is found, and then many tries to improve it are performed.

The critical aspect of the heuristic is the answer to how far the result from the optimal solution is. There are two main reasons for the non-optimality of the solution. The first reason stems from an idea of separation of assignment of orders for vessels from the allocation of piers to vessels. The composition of optimal outcomes of each subproblem does not necessarily translate into the optimality of a whole problem. The second reason is the incredible computational complexity of the fourth stage of the algorithm, where potentially not all combinations can be checked. Therefore, depending on the value of the max_allocation_number parameter, we can be closer or further from the optimal solution of this subproblem.

The presented solution to the cargo loading problem is easily parameterizable and prone to generalization in the form of additional restrictions or features. Even exploring parallelization potential, sufficiently large computing power is still needed for real-life situations. By properly calibrating the heuristic parameters, a solution that allows for significant savings for the manufacturer can be obtained in a reasonable time. The future works in this area can be focused on the preliminary steps where a more advanced approach is used to correct the order deadlines. It should translate into much faster computations and provide a starting point for developing validation rules.

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A. Appendix

Let us introduce the following definitions:

- $VOL(o)$ – volume of order $o \in \mathcal{O}$ (in tonnes),
- $LP(o)$ – loading (source) port of order $o \in \mathcal{O}$,
- $UP(o)$ – unloading (destination) port of order $o \in \mathcal{O}$,
- $AVB(o)$ – first day of availability of order $o \in \mathcal{O}$ in the loading port $LP(o)$,
- $FROM(o)$ – beginning deadline for delivery of order $o \in \mathcal{O}$,
- $TO(o)$ – ending deadline for delivery of order $o \in \mathcal{O}$,
- $DAYS(P1, P2, v)$ – number of transport days between ports $P1$ and $P2$, if the route is traveled by vessel $v \in \mathcal{V}$,
- $UPPER\_LIMIT(k)$ – maximum capacity of pier k (in tonnes),
- $SPEED\_LOAD(k, v)$ – maximum speed of loading vessel $v \in \mathcal{V}$ on pier k (in tonnes per day),
- $SPEED\_UNLOAD(k, v)$ – maximum speed of unloading vessel $v \in \mathcal{V}$ on pier k (in tonnes per day),
- $WORKING\_DAY(d, P) \in \{0, 1\}$ – indication whether day $d \in \mathcal{T}$ is a working day (value 1) in the port $P \in \mathcal{P}$ or not (value 0),
- $MAX\_CARGO(v)$ – maximum load capacity of the vessel $v \in \mathcal{V}$ (in tonnes),
- $VESSEL\_COST\_PER\_DAY(v)$ – price per day of use of the vessel $v \in \mathcal{V}$ (in US dollars),
- $STORAGE\_COST\_PER\_TONNE(P)$ – price per day of cargo storage in the port $P \in \mathcal{P}$ (in US dollars per tonne).

Let us move on to the definition of variables:

- $x^d_{o,v,k} \geq 0$ – how many tonnes of the order $o \in \mathcal{O}$ are to be loaded on the day $d \in \mathcal{T}$ at the pier $k \in \mathcal{R}_{LP(o)}$ on the vessel $v \in \mathcal{V}$,
- $\alpha^d_{o,v,k} \in \{0,\ 1\}$ – equal to 1 if the order $o \in \mathcal{O}$ is being loaded on the day $d \in \mathcal{T}$ at the pier $k \in \mathcal{R}_{LP(o)}$ on the vessel $v \in \mathcal{V}$ going from source port $LP(o)$ to the destination port $UP(o)$, and 0 otherwise,
- $y^d_{o,v,k} \geq 0$ – how many tonnes of the order $o \in \mathcal{O}$ are to be unloaded on the day $d \in \mathcal{T}$ at the pier $k \in \mathcal{R}_{UP(o)}$ from the vessel $v \in \mathcal{V}$,
- $\beta^d_{o,v,k} \in \{0,\ 1\}$ – equal to 1 if the order $o \in \mathcal{O}$ is being unloaded on the day $d \in \mathcal{T}$ at the pier $k \in \mathcal{R}_{LP(o)}$ from the vessel $v \in \mathcal{V}$ going from the source port $LP(o)$ to the destination port $UP(o)$, and 0 otherwise,
- $c^d_{o,P} \geq 0$ – how many tonnes of order $o \in \mathcal{O}$ are stored on the day $d \in \mathcal{T}$ at the port $P \in \mathcal{P}$,
- $\delta^d_{v,k} \in \{0,\ 1\}$ – equal to 1 if the vessel $v \in \mathcal{V}$ is being during the loading process on the day $d \in \mathcal{T}$ at the pier k, and 0 otherwise,
- $\varepsilon^d_{v,k} \in \{0,\ 1\}$ – equal to 1 if the vessel $v \in \mathcal{V}$ is being during the unloading process on the day $d \in \mathcal{T}$ at the pier k, and 0 otherwise,
- $\gamma^d_{v,P1,P2} \in \{0,\ 1\}$ – equal to 1 if the vessel $v \in \mathcal{V}$ is being during the transportation process between ports $P1$ and $P2$ on the day $d \in \mathcal{T}$, and 0 otherwise.

It is assumed that on non-working days a vessel is still during the loading, unloading or transportation process, respectively. Also, the names would be repeated in the optimization process, covering a long time period. Therefore, vessels are defined not by name or id number but by piers in source and destination ports and the date of shipment. The constraints and objective function are given in the mathematical formulas but not necessarily in the linear form. There are two reasons for that. The first is the clarity of the formulation. The main goal of this article was to present a heuristic approach to find the approximation of the solution and not the usage of the classical linear optimization method. It is the second reason for presentation in the mathematical but simplified form.

- Each order must be delivered before deadline_to.

$$\forall_{o\in\mathcal{O}}\; c_{o,UP(o)}^{TO(o)} = VOL(o) \tag{1}$$

- There is a restriction for daily loading and unloading (different for each pier and vessel size).

$$\forall_{d\in\mathcal{T}}\forall_{v\in\mathcal{V}}\forall_{P\in\mathcal{P}}\forall_{k\in\mathcal{R}_P}\; \sum_{o\in\mathcal{O}} x_{o,v,k}^{d} \leq SPEED\_LOAD(k,v) \tag{2}$$

$$\forall_{d\in\mathcal{T}}\forall_{v\in\mathcal{V}}\forall_{P\in\mathcal{P}}\forall_{k\in\mathcal{R}_P}\; \sum_{o\in\mathcal{O}} y_{o,v,k}^{d} \leq SPEED\_UNLOAD(k,v) \tag{3}$$

- Loading and unloading takes place only on working days. The storage does not change in non-working days.

$$\forall_{d\in\mathcal{T}}\forall_{P\in\mathcal{P}}\; WORKING\_DAY(d,P) = 0 \implies \forall_{o\in\mathcal{O}}\forall_{v\in\mathcal{V}}\forall_{k\in\mathcal{R}_P}\; \alpha_{o,v,k}^{d} = x_{o,v,k}^{d} = 0 \tag{4}$$

$$\forall_{d\in\mathcal{T}}\forall_{P\in\mathcal{P}}\; WORKING\_DAY(d,P) = 0 \implies \forall_{o\in\mathcal{O}}\forall_{v\in\mathcal{V}}\forall_{k\in\mathcal{R}_P}\; \beta_{o,v,k}^{d} = y_{o,v,k}^{d} = 0 \tag{5}$$

$$\forall_{d\in\mathcal{T}}\forall_{P\in\mathcal{P}}\; WORKING\_DAY(d,P) = 0 \implies \forall_{o\in\mathcal{O}}\; c_{o,P}^{d} = c_{o,P}^{d-1} \tag{6}$$

- There is no need to store cargo before the day the order is available, and after that cargo storage only changes from the previous day after loading or unloading.

$$\forall_{o\in\mathcal{O}}\forall_{d\in\mathcal{T}}\forall_{P\in\mathcal{P}}\; d \leq AVB(o) - 1 \implies c_{o,P}^{d} = 0 \tag{7}$$

$$\forall_{o\in\mathcal{O}}\forall_{d\in\mathcal{T}}\forall_{P\in\mathcal{P}}\; d \geq AVB(o) \implies c_{o,P}^{d} = c_{o,P}^{d-1} - \sum_{\substack{v\in\mathcal{V}\\ k\in\mathcal{R}_P}} x_{o,v,k}^{d} + \sum_{\substack{v\in\mathcal{V}\\ k\in\mathcal{R}_P}} y_{o,v,k}^{d} \tag{8}$$

- Vessels capacity is limited.

$$\forall_{v\in\mathcal{V}}\forall_{P\in\mathcal{P}}\; \sum_{\substack{o\in\mathcal{O}\\ d\in\mathcal{T}\\ k\in\mathcal{R}_P}} x_{o,v,k}^{d} \leq MAX\_CARGO(v) \tag{9}$$

$$\forall_{v\in\mathcal{V}}\forall_{P\in\mathcal{P}}\; \sum_{\substack{o\in\mathcal{O}\\ d\in\mathcal{T}\\ k\in\mathcal{R}_P}} y_{o,v,k}^{d} \leq MAX\_CARGO(v) \tag{10}$$

- Piers capacity is limited.

$$\forall_{o\in\mathcal{O}}\forall_{d\in\mathcal{T}}\forall_{v\in\mathcal{V}}\forall_{P\in\mathcal{P}}\forall_{k\in\mathcal{R}_P}\; \alpha_{o,v,k}^{d} \cdot MAX\_CARGO(v) \leq UPPER\_LIMIT(k) \tag{11}$$

$$\forall_{o\in\mathcal{O}}\forall_{d\in\mathcal{T}}\forall_{v\in\mathcal{V}}\forall_{P\in\mathcal{P}}\forall_{k\in\mathcal{R}_P}\; \beta_{o,v,k}^{d} \cdot MAX\_CARGO(v) \leq UPPER\_LIMIT(k) \tag{12}$$

- There can be only one vessel loading and unloading on the pier each day.

$$\forall_{d\in\mathcal{T}}\forall_{P\in\mathcal{P}}\forall_{k\in\mathcal{R}_P}\; \sum_{v\in\mathcal{V}} \delta_{v,k}^{d} \leq 1 \tag{13}$$

$$\forall_{d\in\mathcal{T}}\forall_{P\in\mathcal{P}}\forall_{k\in\mathcal{R}_P}\; \sum_{v\in\mathcal{V}} \varepsilon_{v,k}^{d} \leq 1 \tag{14}$$

- Each order must be loaded and unloaded completely.

$$\forall_{o\in\mathcal{O}} \sum_{\substack{d\in\mathcal{T}\\ v\in\mathcal{V}\\ P\in\mathcal{P},\ k\in\mathcal{R}_P}} x^d_{o,v,k} = VOL(o) \tag{15}$$

$$\forall_{o\in\mathcal{O}} \sum_{\substack{d\in\mathcal{T}\\ v\in\mathcal{V}\\ P\in\mathcal{P},\ k\in\mathcal{R}_P}} y^d_{o,v,k} = VOL(o) \tag{16}$$

- Full order on single vessel.

$$\forall_{o\in\mathcal{O}}\forall_{d_1,d_2\in\mathcal{T}}\forall_{P1,P2\in\mathcal{P}}\forall_{k_1\in\mathcal{R}_{P1}}\forall_{k_2\in\mathcal{R}_{P2}}\forall_{\substack{v_1,v_2\in\mathcal{V}\\ v_1\neq v_2}}\ \alpha^{d_1}_{o,v_1,k_1} + \alpha^{d_2}_{o,v_2,k_2} \leq 1 \tag{17}$$

$$\forall_{o\in\mathcal{O}}\forall_{d_1,d_2\in\mathcal{T}}\forall_{P1,P2\in\mathcal{P}}\forall_{k_1\in\mathcal{R}_{P1}}\forall_{k_2\in\mathcal{R}_{P2}}\forall_{\substack{v_1,v_2\in\mathcal{V}\\ v_1\neq v_2}}\ \beta^{d_1}_{o,v_1,k_1} + \beta^{d_2}_{o,v_2,k_2} \leq 1 \tag{18}$$

- Dependencies between quantitative and binary variables.

$$\forall_{o\in\mathcal{O}}\forall_{d\in\mathcal{T}}\forall_{P\in\mathcal{P}}\forall_{k\in\mathcal{R}_P}\forall_{v\in\mathcal{V}}\ \alpha^d_{o,v,k} = 0 \Leftrightarrow x^d_{o,v,k} = 0 \tag{19}$$

$$\forall_{o\in\mathcal{O}}\forall_{d\in\mathcal{T}}\forall_{P\in\mathcal{P}}\forall_{k\in\mathcal{R}_P}\forall_{v\in\mathcal{V}}\ \beta^d_{o,v,k} = 0 \Leftrightarrow y^d_{o,v,k} = 0 \tag{20}$$

- Dependencies between vessels loading, transportation and unloading.

$$\forall_{v\in\mathcal{V}}\forall_{d\in\mathcal{T}}\forall_{P1,P2\in\mathcal{P}}\ \gamma^d_{v,P1,P2} = 1 \implies \forall_{\substack{d_1\in\mathcal{T}\\ d1>d}}\forall_{k\in\mathcal{R}_{P1}}\ \delta^{d_1}_{v,k} = 0 \tag{21}$$

$$\forall_{v\in\mathcal{V}}\forall_{d\in\mathcal{T}}\forall_{P1,P2\in\mathcal{P}}\ \gamma^d_{v,P1,P2} = 1 \implies \forall_{\substack{d_1\in\mathcal{T}\\ d1<d}}\forall_{k\in\mathcal{R}_{P2}}\ \varepsilon^{d_1}_{v,k} = 0 \tag{22}$$

$$\forall_{v\in\mathcal{V}}\forall_{d\in\mathcal{T}}\forall_{P1\in\mathcal{P}}\forall_{k\in\mathcal{R}_{P1}}\ \delta^d_{v,k} = 1 \implies \forall_{\substack{d_1\in\mathcal{T}\\ d1\leq d}}\forall_{P2\in\mathcal{P}}\ \gamma^{d_1}_{v,P1,P2} = 0 \tag{23}$$

$$\forall_{v\in\mathcal{V}}\forall_{d\in\mathcal{T}}\forall_{P1\in\mathcal{P}}\forall_{k\in\mathcal{R}_{P1}}\ \delta^d_{v,k} = 1 \implies \forall_{\substack{d_1\in\mathcal{T}\\ d1\leq d+DAYS(P1,P2,v)}}\forall_{P2\in\mathcal{P}}\forall_{k_2\in\mathcal{R}_{P2}}\ \varepsilon^{d_1}_{v,k_2} = 0 \tag{24}$$

$$\forall_{v\in\mathcal{V}}\forall_{d\in\mathcal{T}}\forall_{P2\in\mathcal{P}}\forall_{k\in\mathcal{R}_{P2}}\ \varepsilon^d_{v,k} = 1 \implies \forall_{\substack{d_1\in\mathcal{T}\\ d1\geq d}}\forall_{P1\in\mathcal{P}}\ \gamma^{d_1}_{v,P1,P2} = 0, \tag{25}$$

$$\forall_{v\in\mathcal{V}}\forall_{d\in\mathcal{T}}\forall_{P2\in\mathcal{P}}\forall_{k\in\mathcal{R}_{P2}}\ \varepsilon^d_{v,k} = 1 \implies \forall_{\substack{d_1\in\mathcal{T}\\ d1\geq d-DAYS(P1,P2,v)}}\forall_{P1\in\mathcal{P}}\forall_{k_1\in\mathcal{R}_{P1}}\ \delta^{d_1}_{v,k_1} = 0 \tag{26}$$

- Dependencies between order and vessel loading. Non-working days between loading days are treated as days on which the vessel is loaded.

$$\begin{aligned}\forall_{v\in\mathcal{V}}\forall_{d\in\mathcal{T}}\forall_{P\in\mathcal{P}}\forall_{k\in\mathcal{R}_P}\ \delta^d_{v,k} = 1 \Leftrightarrow & \left(\max_{o\in\mathcal{O}}\ \alpha^d_{o,v,k} = 1\right) \vee \\ & \Bigg(\max_{o\in\mathcal{O}} \alpha^d_{o,v,k} = 0 \wedge \\ & \max_{o\in\mathcal{O}}\ \alpha^{\max(\{d_1\in\mathcal{T}:\ d_1<d\ \wedge\ WORKING\_DAY(d_1,P)=1\})}_{o,v,k} = 1 \wedge \\ & \max_{o\in\mathcal{O}}\ \alpha^{\min(\{d_1\in\mathcal{T}:\ d_1>d\ \wedge\ WORKING\_DAY(d_1,P)=1\})}_{o,v,k} = 1\Bigg)\end{aligned} \tag{27}$$

- Dependencies between order and vessel unloading. Non-working days between unloading days are treated as days on which the vessel is unloaded.

$$\forall_{v\in\mathcal{V}}\forall_{d\in\mathcal{T}}\forall_{P\in\mathcal{P}}\forall_{k\in\mathcal{R}_P}\ \varepsilon^d_{v,k}=1 \Leftrightarrow \left(\max_{o\in\mathcal{O}}\ \beta^d_{o,v,k}=1\right)\vee \left(\max_{o\in\mathcal{O}}\beta^d_{o,v,k}=0\ \wedge \max_{o\in\mathcal{O}}\ \beta^{\max(\{d_1\in\mathcal{T}:\ d_1<d\ \wedge\ WORKING\_DAY(d_1,P)=1\})}_{o,v,k}=1\ \wedge \max_{o\in\mathcal{O}}\ \beta^{\min(\{d_1\in\mathcal{T}:\ d_1>d\ \wedge\ WORKING\_DAY(d_1,P)=1\})}_{o,v,k}=1\right) \tag{28}$$

The objective function is to minimize the overall cost, including storage and transportation costs. It can be defined by the formula:

$$\sum_{P\in\mathcal{P}}\left(STORAGE\_COST\_PER\_TONNE(P)\cdot\sum_{o\in\mathcal{O}}\ \sum_{\substack{d\in\mathcal{T}\\ d<FROM(o)}}\ c^d_{o,P}\right)+ \sum_{v\in\mathcal{V}}\left(VESSEL\_COST\_PER\_DAY(v)\cdot\sum_{d\in\mathcal{T}}\ \sum_{P1,P2\in\mathcal{P}}\left(\gamma^d_{v,P1,P2}+\sum_{k\in\mathcal{R}_{P1}}\ \delta^d_{v,k}+\sum_{k\in\mathcal{R}_{P2}}\ \varepsilon^d_{v,k}\right)\right) \tag{29}$$