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## INTERACTIVE FUZZY NUMBERS IN THE EVALUATION OF THE EFFECTIVENESS OF INVESTMENT PROJECTS AND THE SELECTION OF EFFICIENT PORTFOLIOS

The paper presents a method for evaluating the effectiveness and risks of investment projects and the selection of efficient portfolios in a situation where parameters in the calculation of effectiveness are expressed in the form of interactive fuzzy numbers. Fuzzy model simulations are used to perform arithmetic operations on interactive fuzzy numbers. The process of selecting investment projects takes into account statistical and economic dependences between projects.

**Keywords:** *portfolio selection, risk, fuzzy numbers, simulation; mathematical programming*

### 1. Introduction

Net present value (*NPV*) is the most commonly used method of evaluating the effectiveness of investment projects. *NPV* is the sum of discounted net benefits (net cash flows) over the whole life cycle of the investment project.

The risk of an investment project or portfolio of investment projects is to be understood as the possibility of an adverse deviation in *NPV* from its anticipated value. Such a definition corresponds to the concept of risk as a negative phenomenon. In assessing risk, in this case, it is recommended to take into account only negative deviations from anticipated *NPV* values. The measure of risk is thus a standard semi-deviation of *NPV*. Risk is a function of the uncertainty of conditions in the implementation and operation of investment projects. From this perspective, the concept of uncertainty concerns forecasted values of parameters in the calculation of effectiveness; e.g. production sales volume, product and raw material prices, capital expenditure levels, material consumption values.

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An efficient investment portfolio is one which provides [6], [25]:

- the highest *NPV* at a given accepted level of risk,
- the lowest risk at a given accepted *NPV* for the portfolio.

The key to define an efficient portfolio is an analysis of statistical dependences between investment projects. A statistical dependence of investment projects is taken into account when a correlation exists between the benefits generated by the projects [6], [25]. When a portfolio contains negatively or weakly positively correlated projects, we obtain a reduced portfolio risk. Besides statistical dependences, there may be economic dependences between investment projects. Economic dependences indicate the manner in which a given project affects the benefits generated by other projects or benefits obtained from the activity of the firm to date. Investment projects may therefore be economically independent (when no such effect exists) or economically dependent (when such an effect exists) [6].

This paper is devoted to the problem of estimating the effectiveness and risk of investment projects and to the problem of selecting an efficient portfolio, where parameters in the calculation of effectiveness are expressed in the form of interactive fuzzy numbers. The problem of selecting efficient portfolios is analyzed under conditions of capital rationing (a limit is set on capital expenditure in a specified period). In the course of selecting investment projects, statistical and economic dependences between the projects are taken into account. The proposed method of estimating the effectiveness and risk of investment projects and of selecting an efficient portfolio is suitable for enterprises with multistage production cycles, e.g. enterprises in the metallurgical or chemical industry.

## 2. Possibility and credibility distributions, fuzzy variables

Possibility distributions were first introduced by Zadeh [29]. The fuzzy set  $\tilde{A}$  generates two functions of the set: a measure of possibility *Pos* and a measure of necessity *Nec*. These measures are defined for every classic set  $X$  in the space  $\mathcal{X}$  ( $X \subset \mathcal{X}$ ) by the formulas:

$$\begin{aligned} Pos(X) &= \sup\{\mu_{\tilde{A}}(x) : x \in X\}, \\ Nec(X) &= \inf\{1 - \mu_{\tilde{A}}(x) : x \notin X\}. \end{aligned} \tag{1}$$

Their definition is associated with a well-known interpretation of a fuzzy set given by Zadeh [29]. It assumes that the fuzzy set  $\tilde{A}$  with a membership function  $\mu_{\tilde{A}}(x)$  is

a fuzzy restriction of a certain variable  $X$ , which takes values in the space  $\mathcal{X}$ . It is assumed that the only information about the variable is that  $X$  is  $\tilde{A}$ . In which case, the variable  $X$  is characterized by the membership function of the fuzzy set, which describes the possibility of  $X$  taking values of  $x \in \mathcal{X}$ , i.e. it induces a possibility distribution in the space  $\mathcal{X}$ . Such a variable  $X$  is called a fuzzy variable with a possibility distribution of:

$$\pi_x(x) = \mu_{\tilde{A}}(x). \quad (2)$$

We can determine the possibility of the event that the value of the variable  $X$  belongs to the set  $X$ , based on the possibility distribution [29]:

$$\pi(X \in X) = \sup(\pi_x : x \in X) = \sup(\mu_{\tilde{A}}(x) : x \in X) = \pi(X). \quad (3)$$

Liu [17], [18] introduces the concept of a credibility measure. The degree of credibility that the value of the variable  $X$  belongs to the set  $X$  can be defined as follows:

$$cr(X \in X) = \frac{\pi(X \in X) + (1 - \pi(X \in X^c))}{2}, \quad (4)$$

where  $X^c$  is the absolute complement of the set  $X$ .

Liu and Liu [16] define the concept of the credibility distribution  $\Phi(x)$ . The distribution function  $\Phi(x)$  determines the degree of credibility that the fuzzy variable  $X$  will have a value equal to or lower than  $x$ . If  $\mu$  is the membership function of the fuzzy variable  $X$ , then the credibility distribution function  $\Phi(x)$  is expressed by:

$$\Phi(x) = \frac{1}{2} \left( \sup_{y \leq x} \mu(y) + 1 - \sup_{y > x} \mu(y) \right) \quad \forall x \in \mathfrak{R}. \quad (5)$$

The concept of the expected value has been introduced for fuzzy variables. Liu and Liu [16] define the expected value  $E(X)$  of the variable  $X$  according to the following formula [16]:

$$E(X) = \int_{-\infty}^{+\infty} x d\Phi(x) \quad (6)$$

if the conditions  $\lim_{x \rightarrow -\infty} \Phi(x) = 0$  and  $\lim_{x \rightarrow \infty} \Phi(x) = 1$  are satisfied.

The standard semi-deviation  $SDev(X)$  of the fuzzy variable  $X$ , which has a finite expected value  $e$ , is defined as:

$$SDev(X) = \sqrt{E((Y)^2)},$$

where:

$$Y = \begin{cases} X - e & \text{if } X < e \\ 0 & \text{if } X \geq e \end{cases}. \quad (7)$$

### 3. Fuzzy numbers in the calculation of the effectiveness of investment projects and in the selection of efficient portfolios

Ward [27] was the first to utilize fuzzy numbers in financial analyses. She presented cash flows with the aid of trapezoidal fuzzy numbers. Buckley used fuzzy numbers for the calculation of net present values [1], [2] of investment projects. Calzi presented principles of expanding financial mathematics to include fuzzy numbers [3]. Choobineh and Behrens present the use of possibility distributions in economic analysis [5]. Esogbue and Hearnese [8] utilize fuzzy numbers in problems concerning the replacement of fixed assets. Their aim was to describe the economic life-cycle of fixed assets. Kahraman, Ruan and Tolga [11], Dimova, Sevastianov and Sevastianov [7] and Kahraman, Tolga and Ulukan [12] present methods for the calculation of selected investment project effectiveness ratios on the assumption that certain parameters are presented in the form of fuzzy numbers. Kuchta [14] presents the use of fuzzy numbers in capital budgeting. She presents a method which enables a decision to be made as to whether to complete an investment project or to abandon it.

Presented in the above-mentioned publications are methods for the calculation of selected investment project effectiveness ratios. These methods are based on the assumption that the basic quantities (cash flows, profit) used to calculate these ratios are presented in the form of fuzzy numbers. Less attention is devoted to the issue of the calculation of these quantities. In the process of their calculation, dependences between the parameters used in the calculation of effectiveness should be taken into account. This is a significant problem which makes the use of these methods difficult in practice, since the literature displays a lack of in-depth studies concerning the practical implementation of arithmetic operations on interactive fuzzy numbers [22], [23].

The methods for building an efficient investment portfolio most often use probability distributions to describe uncertainties regarding parameters in the calculation of effectiveness [6], [21], [24], [25], [30]. Presented in the literature are also methods for the selection of investment portfolios based on the assumption that uncertainty regarding parameters in the calculation of effectiveness is expressed in the form of fuzzy

numbers. Such methods have been proposed by Huang [9], [10] and by Liu and Iwamura [15]. Here the capital budget is developed so that intended values of effectiveness ratios are achieved within a given degree of credibility or possibility. Kahraman [13] uses dynamic programming under conditions of fuzziness to describe the capital budget of a company. Chan et al. [4] and Mohamed and McKown [20] present multicriteria methods for determining a capital budget in a situation where uncertainty regarding parameters in the calculation of effectiveness is characterized by fuzzy numbers.

The concepts presented above for the selection of an efficient portfolio under conditions of fuzziness do not take into account the appearance of statistical dependences between projects. In the selection process, projects are generally treated as economically independent. This limits the possible practical use of these models to a considerable degree.

#### 4. Arithmetic operations on interactive fuzzy numbers

Fuzzy model simulations can be used to perform arithmetic operations on interactive fuzzy variables [23]. The rules of fuzzy model simulation were developed by Iwamura and Liu [15]. The simulation consists of random sampling on models with fuzzy parameters. Suppose that  $f$  is a function and  $\hat{\mathbf{X}} = (X_1, X_2, \dots, X_m)$  a vector of interactive fuzzy variables, for which values are limited by the fuzzy numbers  $\tilde{X}_1, \tilde{X}_2, \dots, \tilde{X}_m$ , respectively. In this case, fuzzy model simulation can be used to determine the possibility distribution and credibility distribution  $f(\hat{\mathbf{X}})$ . The following algorithm is proposed for estimating the possibility distribution and credibility distribution while taking into account the interaction of the fuzzy numbers  $\tilde{X}_1, \tilde{X}_2, \dots, \tilde{X}_m$ .

START

Step 1. Define  $\alpha_0, \wp, \ddot{n}, \ddot{l}$

Step 2.  $\alpha = \alpha_0$

Step 3. Define  $\alpha$  levels for the fuzzy numbers  $\tilde{X}_1, \tilde{X}_2, \dots, \tilde{X}_m$

Step 4.  $n = 1$

Step 5. Randomly generate the vector  $[x_1, x_2, \dots, x_m]$

Step 6. Define  $f_n = f(x_1, x_2, \dots, x_m)$  and the degree of membership  $\mu(f_n)$

Step 7.  $n = n + 1$

Step 8. If  $n \leq \ddot{n}$  go to step 5

Step 9.  $\alpha = \alpha + \wp$

Step 10. If  $\alpha \leq 1$  go to step 3

Step 11. Define the possibility distribution  $f(\hat{\mathbf{X}})$

- Divide the sample of values obtained  $f_n$  into  $\ddot{l}$  disconnected classes
- For every class  $l = 1, 2, \dots, \ddot{l}$  define,  $\mu(f_l) = \max_{f_n \in \text{class } l} (\mu(f_n))$  and  $f_l$  to equal the average of the  $f_n$  belonging to class  $l$ .

Step 12. Define the credibility distribution  $f(\hat{\mathbf{X}})$  using formula (5)

STOP

The vector  $[x_1, x_2, \dots, x_m]$  is generated randomly from the  $\alpha$  levels of the fuzzy numbers  $\tilde{X}_1, \tilde{X}_2, \dots, \tilde{X}_m$ . In order to set the components of the vector  $[x_1, x_2, \dots, x_m]$ , random numbers are first generated with a uniform distribution on the interval  $[0, 1]$ . Successive random numbers correspond to successive components of the vector  $\hat{\mathbf{X}}$ . These numbers are randomly selected taking into account their dependence determined by the correlation matrix of the vector components. For this purpose, a method presented by Yang [28] based on the Choleski decomposition of the correlation matrix is utilized. Then based on the random numbers generated, the value of the vector  $[x_1, x_2, \dots, x_m]$  is set. Afterwards, the degree of membership  $\mu(f_n)$  is calculated. The degree of membership is determined by the formula:

$$\mu(f_n) = \min[\mu(x_1), \mu(x_2), \dots, \mu(x_m)]. \quad (8)$$

This method of simulation allows the interaction between the fuzzy parameters to be taken into account when calculating the possibility distribution and credibility distribution  $f(\hat{\mathbf{X}})$ . Thanks to the credibility distribution  $f(\hat{\mathbf{X}})$  defined in this way, the expected value  $E(f(\hat{\mathbf{X}}))$  and standard semi-deviation  $SDev(f(\hat{\mathbf{X}}))$  can be determined using formulas (6), (7).

## 5. Formulating the problem of selecting efficient portfolios of investment projects

An investment project is understood to be a variant of modernization or the construction of a production department along with the possible construction or modernization of facilities in auxiliary departments. In order to simplify the notation of the model, it is assumed that not changing the current state of a production department is one of its investment projects. In the algorithm for the selection of efficient portfolios

of investment projects presented below, it is assumed that a project that does not change the current state of a department is included in the implementation if no alternative project developed for implementation is accepted.

The optimization horizon is the number of years for which the net cash flows of the company are being forecasted. It is equal to the sum of the period of capital budgeting and the longest life cycle for the investment projects analyzed. The capital budgeting period is the time interval for which the capital budget is developed.

The mathematical model used for the selection of efficient portfolios of investment projects consists of two groups of equations. The first group includes equations defining the production capacity of the company, depending on the portfolios of projects qualified for implementation. This group also includes equations regarding a company's material balance and equations determining the conditions for selecting portfolios of investment projects. The other group of equations are financial equations. They enable determination of a company's financial forecasts based on a selected portfolio of investment projects and given forecast for the parameters in the calculation of effectiveness. The following notation is formally used for the model:

- $Pr_{ijw}^{t\tau}$  – variable denoting the total output of product  $i$  produced in department  $j$  in year  $t$  when project  $w$  qualifies for implementation in year  $\tau$ ,
- $G_i^t$  – variable denoting sales of product  $i$  in year  $t$ ,
- $KC^t$  – variable denoting the cost of the goods sold in year  $t$ ,
- $KRK^t$  – variable denoting the value of short-term credit in year  $t$ ,
- $KRD^t$  – variable denoting the value of long-term credit in year  $t$ ,
- $ZKD^t$  – variable denoting the value of a long-term loan taken out in year  $t$ ,
- $ZKK^t$  – variable denoting the value of a short-term loan taken out in year  $t$ ,
- $SKK^t$  – variable denoting the value of short-term loan repayments in year  $t$ ,
- $ZO^t$  – variable denoting operating profit in year  $t$ ,
- $ZB^t$  – variable denoting gross profit in year  $t$ ,
- $ZKO^t$  – variable denoting change in net working capital in year  $t$ ,
- $ZN^t$  – variable denoting net profit in year  $t$ ,
- $SP^t$  – variable denoting cash value in year  $t$ ,
- $\overline{NCF}_{Pz}^t$  – variable denoting a company's net cash flow in year  $t$ ,
- $NPV_{WP}$  – variable denoting the expected value of the  $NPV$  of investment project portfolio  $WP$ ,
- $s\sigma_{WP}$  – variable denoting the standard semi-deviation of the  $NPV$  of investment project portfolio  $WP$
- $W_{jw}^\tau$  – binary variable denoting acceptance for implementation or rejection of project  $w$  developed for department  $j$  in year  $\tau$ ,
- $W_j$  – index set of development projects for production department  $j$ ,
- $WW_n$  – index set of the  $n$ th set of mutually exclusive projects,
- $N$  – index set of distinguished mutually exclusive projects,

- $I$  – index set of products,  
 $I_j$  – index set of products produced in department  $j$ ,  
 $J$  – index set of production departments,  
 $B$  – index set of raw materials,  
 $pd^t$  – interest income tax rate in year  $t$ ,  
 $v_j^t$  – manufacturing capacity of department  $j$  in year  $\zeta$  of the life cycle,  $\zeta = t - \tau$ ,  
 $g_i^t$  – forecasted sales for product  $i$  in year  $t$ ,  
 $m_{ijzw}^\zeta$  – per unit consumption of product  $i$  used to produce product  $z$  in department  $j$ , after implementing project  $w$  in year  $\zeta$  of the life cycle,  $\zeta = t - \tau$ ,  
 $m_{biw}^\zeta$  – per unit consumption of raw material  $b$  used to produce product  $i$ , after implementing project  $w$  in year  $\zeta$  of the life cycle,  $\zeta = t - \tau$ ,  
 $\dots$   
 $\ddot{t}_{jw}$  – life cycle of project  $w$  implemented in department  $j$ ,  
 $\ddot{t}$  – time horizon of the optimization,  
 $\ddot{\tau}$  – capital budgeting period,  
 $c_i^t$  – selling price of product  $i$  in year  $t$ ,  
 $c_b^t$  – price of raw material  $b$  in year  $t$ ,  
 $kz_{ijw}^\zeta$  – variable processing cost for product  $i$  manufactured in production department  $j$ , after implementing project  $w$  in year  $\zeta$  of the life cycle,  $\zeta = t - \tau$ ,  
 $rk^t$  – interest rate on short-term credit in year  $t$ ,  
 $rd^t$  – interest rate on long-term credit in year  $t$ ,  
 $\eta_0^t$  – limit on investment outlays in the year  $t$ ,  
 $\eta_{jw}^\zeta$  – forecasted investment outlay in year  $t$  for project  $w$  in department  $j$  whose implementation started in year  $\tau$ ,  $\zeta = t - \tau$ ,  
 $s^{t\tau}$  – index representing the value of loan repayments in year  $t$  for a loan taken out in year  $\tau$ ,  
 $bp^t$  – current ratio in year  $t$ ,  
 $sq^t$  – cash ratio in year  $t$ ,  
 $cz^t$  – inventory turnover ratio in year  $t$ ,  
 $cna^t$  – accounts receivable turnover ratio for year  $t$ ,  
 $czb^t$  – current liabilities turnover ratio for year  $t$ ,  
 $\xi^t(WP)$  – function assigning a company's fixed costs without amortization in year  $t$  to any project portfolio,  
 $\chi^t(WP)$  – function assigning the value of amortization in year  $t$  to any project portfolio.

The equations from the first group are as follows:

- Limits on manufacturing capacities in production departments

$$\sum_{i \in I_j} Pr_{ijw}^{t\tau} \leq v_{jw}^\zeta W_{jw}^\tau \quad (9)$$

for  $\tau = 0, 1, 2, \dots, \ddot{\tau}$ ,  $j \in J$ ,  $w \in W_j$ ,  $t = \tau, \tau + 1, \tau + 2, \dots, \tau + \ddot{t}_{jw}$ ,



$$W_{jw}^\tau = \begin{cases} 1 & \text{if } w \in WP \\ 0 & \text{if } w \in W - WP \end{cases} \quad (10)$$

$$\text{for } \tau = 0, 1, 2, \dots, \ddot{\tau}, \quad j \in J, \quad w \in W_j, \quad t = \tau, \tau+1, \tau+2, \dots, \tau + \ddot{t}_{jw}.$$

$$\sum_{\tau=0}^{\ddot{\tau}} W_{jw}^\tau \leq 1, \quad \text{for } j \in J, w \in W_j, \quad (11)$$

$$\sum_{j \in W W_n} W_{jw}^\tau \leq 1 \quad \text{for } n \in N, \quad (12)$$

$$\sum_{j \in J} \sum_{w \in W_j} \sum_{\tau=0}^{\ddot{\tau}} \eta_{jw}^\tau W_{jw}^\tau \leq \eta_0^t \quad \text{for } t = 0, 1, \dots, \ddot{t}. \quad (13)$$

- Equations for the company's material balance

$$\sum_{j \in J} \sum_{w \in W_j} \sum_{\tau \leq t} P r_{ijw}^{t,\tau} - \sum_{j \in J} \sum_{w \in W_j} \sum_{z \in I} \sum_{\tau \leq t} m_{zjw}^\tau P r_{zjw}^{t,\tau} = G_i^t \quad \text{for } i \in I; t = 0, 1, 2, \dots, \ddot{t}. \quad (14)$$

$$G_i^t \leq g_i^t \quad \text{for } i \in I, \quad t = 0, 1, 2, \dots, \ddot{t}. \quad (15)$$

Equations (9), (10) determine the quantity and structure of production of each department in consecutive years of the optimization horizon depending on the portfolio of projects qualified for implementation. The value of the binary variable  $W_{jw}^\tau$  denotes whether a project has qualified for implementation. This variable also determines the year of the budgeting period in which the project qualifies for implementation. After establishing the values of the binary variables  $W_{jw}^\tau$ , production capacity is determined for each individual stage of the company's technological cycle. Equations (11)–(13) determine the feasible sets of projects qualified for implementation. Equation (11) expresses the conditions under which investment projects become mutually exclusive. Equation (12) expresses the condition that each project can qualify for implementation only in one year of the budgeting period. Equation (13) reflects that outlays on projects qualifying for implementation in year  $t$  are lower than the limit on investment  $\eta_0^t$ . Equation (14) determines the company's material balance. It reflects the distribution of the production of specific products for sales and for internal production. Equation (15) reflects the limits on sales of specific products.

Changes in production capacity resulting from undertaking selected investment projects can cause step changes in fixed costs. The  $\xi$  function takes this fact into account. It determines a company's fixed costs without amortization according to the

portfolio of qualified projects. This function is defined by indicating which projects or sets of project will increase or decrease fixed costs and by how much. Undertaking investment projects is connected with changing the value of a company's tangible fixed assets. These changes result from bringing new facilities into use or liquidating existing ones. Such modifications incur changes in amortization. The  $\chi$  function allows amortization to be determined for consecutive years of the optimization period for each set of qualified investment projects. Similarly to the previous function, it is defined by determining in which years and by how much the implementation of a project or set of projects will increase or decrease amortization.

Equations (9)–(15) determine feasible sets of projects qualified for implementation, values of the total production of specific departments, amount of sales, external supplies of products for further processing and investment outlays for the implementation of projects.

The other set of equations in the model are financial equations. They are linear equations, which for all the above mentioned parameters in equations (9)–(15) determine the individual items on the financial balance sheet, profit and loss statement and cash flow reports of the company. They also ensure the preservation of appropriate relations between selected and projected items of a company's financial statements. These relations are determined by set values for selected financial ratios used in the financial analysis of a company. Examples of equations in the second group are presented below. These equations determine the cost of goods sold, operating profits, gross profit, net profit, changes in net working capital, cash value, long-term and short-term loans and net cash flow (*NCF*). A condition which requires that the established value of the current ratio is not exceeded is also presented. These equations express dependences that are well-known in financial studies. Therefore, a detailed presentation and discussion of these equations has been omitted.

$$\begin{aligned}
 KC^t = & \sum_{\tau \leq t} \sum_{w \in W_j} \sum_{j \in J} \sum_{i \in I_j} k_{ijw}^s Pr_{ijw}^{i,\tau} + \sum_{b \in B} \sum_{\tau \leq t} \sum_{w \in W_j} \sum_{j \in J} \sum_{i \in I} c_b^t m_{bijw}^s Pr_{ijw}^{i,\tau} \\
 & + \chi^t(WP) + \xi^t(WP) \quad \text{for } t = 0, 1, \dots, \ddot{t},
 \end{aligned} \tag{16}$$

$$ZO^t = \sum_{i \in I} c_i^t G_i^t - KC^t \quad \text{for } t = 0, 1, \dots, \ddot{t}, \tag{17}$$

$$ZB^t = ZO^t + rk^t KRK^t + rd^t KRD^t \quad \text{for } t = 0, 1, \dots, \ddot{t}, \tag{18}$$

$$ZN^t = \begin{cases} ZB^t & \text{if } ZB^t \leq 0 \\ pd^t ZB^t & \text{if } ZB^t \geq 0 \end{cases} \quad \text{for } t = 0, 1, \dots, \ddot{t}, \tag{19}$$

$$\begin{aligned}
ZKO^t = & \left( sq^t \left( \frac{czb^t}{360} (KC^t - \chi^t(WP)) + KRK^t \right) \right. \\
& \left. - sq^{t-1} \left( \frac{czb^{t-1}}{360} (KC^{t-1} - \chi^{t-1}(WP)) + KRK^{t-1} \right) \right) \\
& + \left( \frac{cna^t}{360} \times \sum_{i \in I} c_i^t G_i^t - \frac{cna^{t-1}}{360} \times \sum_{i \in I} c_i^{t-1} G_i^{t-1} \right) + \left( \frac{cz^t}{360} \times KC^t - \frac{cz^{t-1}}{360} \times KC^{t-1} \right) \\
& - \left( \frac{czb^t}{360} \times (KC^t - \chi^t(WP)) - \frac{czb^{t-1}}{360} (KC^{t-1} - \chi^{t-1}(WP)) \right) \\
& - (KRK^t - KRK^{t-1}) \quad \text{for } t=0, 1, \dots, \bar{t},
\end{aligned} \tag{20}$$

$$\begin{aligned}
SP^{t-1} + ZN^t + & \left( \frac{czb^t}{360} (KC^t - \chi^t(WP)) - \frac{czb^{t-1}}{360} (KC^{t-1} - \chi^{t-1}(WP)) \right) \\
- & \left( \frac{cz^t}{360} KC^t - \frac{cz^{t-1}}{360} KC^{t-1} \right) - \left( \frac{cna^t}{360} \sum_{i \in I} c_i^t G_i^t - \frac{cna^{t-1}}{360} \sum_{i \in I} c_i^{t-1} G_i^{t-1} \right) + ZKK^t - SKK^t \\
+ & ZKD^t - \sum_{\substack{t=1 \\ \tau < t}}^{\bar{t}} s^{t\tau} ZKD^t - \sum_{j \in J} \sum_{w \in W_j} \sum_{\tau=0}^t \eta_{jw}^s W_{jw}^\tau + \chi^t(WP) - SP^t = 0 \quad \text{for } t=0, 1, \dots, \bar{t},
\end{aligned} \tag{21}$$

$$KRK^{t-1} + ZKK^t - SKK^t - KRK^t = 0 \quad \text{for } t=0, 1, \dots, \bar{t}, \tag{22}$$

$$KRD^{t-1} + ZKD^t - \sum_{\substack{t=1 \\ \tau < t}}^{\bar{t}} s^{t\tau} ZKD^t - KRD^t = 0 \quad \text{for } t=0, 1, \dots, \bar{t}, \tag{23}$$

$$\begin{aligned}
& \frac{cz^t}{360} \times KC^t + \frac{cna^t}{360} \times \sum_{i \in I} c_i^t G_i^t + SP^t - bp^t \times \\
& \times \left( \frac{czb^t}{360} \times (KC^t - \chi^t(WP)) + KRK^t \right) \geq 0 \quad \text{for } t=0, 1, \dots, \bar{t}
\end{aligned} \tag{24}$$

$$\begin{aligned}
NCF_{Prz}^t = & ZN^t + \chi^t(WP) - ZKO^t - \sum_{j \in J} \sum_{w \in W_j} \sum_{\tau=0}^t \eta_{jw}^s W_{jw}^\tau \\
& + ZKK^t - SKK^t + ZKD^t - \sum_{\substack{t=1 \\ \tau < t}}^{\bar{t}} s^{t\tau} ZKD^t \quad \text{for } t=0, \dots, \bar{t}-1
\end{aligned} \tag{25a}$$

$$\begin{aligned}
NCF_{prz}^t &= ZN^t + \chi^2(WP) - ZKO^t - \sum_{j \in J} \sum_{w \in W_j} \sum_{\tau=0}^t \eta_{jw}^\tau W_{jw}^\tau \\
&+ ZKK^t - SKK^t + ZKD^t - \sum_{\substack{t=1 \\ \tau < t}}^{\bar{t}} s^{\tau t} ZKD^t \\
&+ \frac{cz^t}{360} KC^t + \frac{cna^t}{360} \sum_{i \in I} c_i^t G_i^t \quad \text{for } t = \ddot{t} \quad (25b) \\
&+ sq^t \left( \frac{czb^t}{360} (KC^t - \chi^t(WP)) + KRK^t \right) \\
&- \frac{czb^t}{360} (KC^t - \chi^t(WP)) - KRD^t - KRK^t
\end{aligned}$$

Most of the parameters found in these equations:  $v_{jw}^\tau, g_i^t, \eta_{jw}^\tau, \eta_0^t, m_{bijw}^\tau, m_{izjw}^\tau, \ddot{t}, \ddot{t}_{jw}, c_i^t, c_b^t, kz_{ijw}^\tau, rk^t, rd^t$  are usually marked by uncertainty. They are expressed in the form of fuzzy numbers.

The multicriteria problem of choosing efficient portfolios of investment projects is defined as follows: find non-dominated portfolios of investment projects by assuming equations and inequalities (9)–(25) as constraints in the model and the following as criteria:

$$\overline{NPV}_{WP} \rightarrow \max \quad (26)$$

$$s\sigma_{WP} \rightarrow \min \quad (27)$$

The method of estimating  $\overline{NPV}_{WP}$  and  $s\sigma_{WP}$  is presented in Section 6 when discussing the algorithm for selecting efficient portfolios of investment projects.

## 6. Algorithm for selecting efficient portfolios of investment projects

The algorithm utilized for selecting efficient portfolios of investment projects is presented below:

START

Define  $\bar{l}\bar{j}$

Define the set of non-dominated solutions  $PE = \emptyset$

$lp = 0$

Repeat

    Generate  $WP$

Determine the possibility distribution and credibility distribution of  $NPV_{WP}$ . Calculate the expected value of  $\overline{NPV}_{WP}$  ( $\overline{NPV}_{WP}$ ) and standard semi-deviation of  $NPV_{WP}$  ( $s\sigma_{WP}$ ) (*Algorithm: estimating the credibility distribution of NPV and expected value of NPV and standard semi-deviation of NPV*)

If  $\overline{NPV}_{WP} \geq 0$  and  $WP$  is not dominated by any solution in  $PE$

then

Add  $WP$  to  $PE$

Remove from  $PE$  all solutions dominated by added solutions

$lp = 0$

else

$lp = lp + 1$

End if

Until:  $lp = l\ddot{p}$

Present set  $PE$

STOP

The parameter  $l\ddot{p}$  denotes the number of iterations run by the algorithm without finding a solution that dominates existing solutions after which the calculation process is terminated. A genetic algorithm is used to generate  $WP$  sets. The selection of  $WP$  sets is made by taking into account the criteria described by formulas (26) and (27) and the inequalities (11)–(13). According to the algorithm presented, only those investment portfolios which are characterized by a non-negative expected value, i.e.  $\overline{NPV}_{WP} \geq 0$ , are included the set  $PE$ . The credibility distribution of  $NPV$ , expected value of  $NPV$  and standard semi-deviation of  $NPV$  are estimated with the aid of fuzzy model simulation. The algorithm for estimating these quantities is presented below.

The algorithm for estimating the credibility distribution of  $NPV$ , expected value of  $NPV$  and standard semi-deviation of  $NPV$ .

START

Define  $\alpha, \beta, l\ddot{r}$

Repeat

Define  $\alpha$  levels for the parameters used in the calculation of effectiveness that is affected by uncertainty

Define  $l\ddot{r}_\alpha$

For  $l = 1$  to  $l = l\ddot{r}_\alpha$

Generate values of the parameters used in the calculation of effectiveness that is affected by uncertainty

Define the right hand side of inequality (9) in accordance with the set  $WP$ . Solve the linear programming problem, for which the constraints are specified by equations and inequalities (9), (14)–(25). The objective function is expressed by:

$$\sum_{t=0}^T \frac{1}{(1+r_{dys})^t} NCF_{Pr=(WP)}^t(l) \rightarrow \max$$

Define the right hand side of inequality (9) assuming that no investment project qualifies for execution. Solve the linear programming problem, for which the constraints are specified by equations and inequalities (9), (14)–(25). The objective function is expressed by:

$$\sum_{t=0}^{\ddot{T}} \frac{1}{(1+r_{dis})^t} NCF_{Prz(\emptyset)}^t(l) \rightarrow \max$$

Calculate  $NPV_{WP}(l) = \sum_{t=0}^{\ddot{T}} \frac{1}{(1+r_{dys})^t} NCF_{WP}^t(l)$ ,  $NCF_{WP}^t(l)$  is calculated according

to formula (28)

Next  $l$

$$\alpha = \alpha + \wp$$

Until  $\alpha > 1$

Determine the possibility distribution and credibility distribution of  $NPV_{WP}$ . Calculate the expected value of  $NPV_{WP}$  ( $\overline{NPV_{WP}}$ ) and its standard semi-deviation  $NPV_{WP}(s\sigma_{WP})$

STOP

According to the algorithm presented,  $\alpha$  levels are set for each parameter used in the calculation of effectiveness that is affected by uncertainty. Parameter values are randomly generated from these levels. The process takes into account the correlation of these parameters. Then, the production and sales quantities and also the financial results of a company are optimized. Optimization is carried out for two variants: assuming implementation of a selected portfolio of investment projects and assuming it is not implemented. For this purpose, the mathematical model formulated in Section 5 is utilized. A linear optimization problem is solved. This linear optimization problem is defined by equations and inequalities (9) and (14)–(25). In the first variant, the optimization criterion is given by

$$\sum_{t=0}^{\ddot{T}} \frac{1}{(1+r_{dis})^t} NCF_{Prz(WP)}^t \rightarrow \max$$

where  $NCF_{Prz(WP)}^t$  denotes the forecasted net cash flows of a company, assuming the implementation of an investment project,  $r_{dis}$  denotes the discount rate) and in the second variant

$$\sum_{t=0}^{\ddot{T}} \frac{1}{(1+r_{dis})^t} NCF_{Prz(\emptyset)}^t \rightarrow \max$$

where  $NCF_{Prz(\emptyset)}^t$  denotes the forecasted net cash flows of a company in the case an investment project is not implemented).  $NCF_{Prz(WP)}^t$  and  $NCF_{Prz(\emptyset)}^t$  are calculated using formula (25). The net cash flows associated with the portfolio analyzed ( $NCF_{WP}^t$ ) are calculated using the formula:

$$\begin{aligned}
NCF_{WP}^t &= (1 - pd^t)ZO_{WP}^{*t} + \chi^t(WP) \\
&\quad - ZKO_{WP}^{*t} - \sum_{j \in J} \sum_{w \in W_j} \sum_{\tau=0}^t \eta_{jw}^\zeta W_{jw}^{*\tau} \quad \text{for } t = 0, \dots, \ddot{i} - 1 \quad (28a) \\
&\quad - (1 - pd^t)ZO_{(\emptyset)}^{*t} + \chi^t(\emptyset) - ZKO_{(\emptyset)}^{*t}
\end{aligned}$$

$$\begin{aligned}
NCF_{WP}^t &= (1 - pd^t)ZO_{(WP)}^{*t} + \chi^t(WP) \\
&\quad - ZKO_{(WP)}^{*t} - \sum_{j \in J} \sum_{w \in W_j} \sum_{\tau=0}^t \eta_{jw}^\zeta W_{jw}^{*\tau} \\
&\quad + sq^t \left( \frac{czb^t}{360} (KC_{(\emptyset)}^{*t} - \chi^t(\emptyset)) + KRK^t \right) \\
&\quad + \frac{cz^t}{360} KC_{(WP)}^{*t} + \frac{cna^t}{360} \sum_{i \in I} c_i^{ta} G_{i(WP)}^{*ta} \\
&\quad - \frac{czb^t}{360} (KC_{(WP)}^{*t} - \chi^t(WP)) \\
&\quad - (1 - pd^t)ZO_{(\emptyset)}^{*t} + \chi^t(\emptyset) - ZKO_{(\emptyset)}^{*t} \\
&\quad + sq^t \left( \frac{czb^t}{360} (KC_{(\emptyset)}^{*t} - \chi^t(\emptyset)) + KRK^t \right) + \frac{cz^t}{360} KC_{(\emptyset)}^{*t} \\
&\quad + \frac{cna^t}{360} \times \sum_{i \in I} c_i^{ta} G_{i(\emptyset)}^{*ta} - \frac{czb^t}{360} (KC_{(\emptyset)}^{*t} - \chi^t(\emptyset)) \quad \text{for } t = \ddot{i} \quad (28b)
\end{aligned}$$

Individual quantities in formula (28) with the superscript \* are the optimum values of the respective variables, obtained from solving the above-mentioned optimization tasks. The subscripts  $WP$  and  $\emptyset$  correspond the respective values of variables assuming first a selected portfolio of investment projects is implemented, and secondly in the case it is rejected. The  $NPV$  of a selected portfolio is calculated using the following formula:

$$NPV_{WP} = \sum_{t=0}^{\ddot{i}} \frac{1}{(1 + r_{dys})^t} NCF_{WP}^t \quad (29)$$

At the same time, the grade of membership is calculated for the  $NPV$  according to the method presented in Section 4 when discussing fuzzy model simulation. The procedure for randomly generating values of parameters used in the calculation of effectiveness that are affected by uncertainty and for calculating  $NPV$  and its grade of membership is repeated several times for various values of  $\alpha$ . As a result, the possibility distribution and credibility distribution for the  $NPV$  of the analyzed investment project portfolio are found. They are defined according to the rules discussed in steps 11 and 12 of the algorithm presented in Section 4. The expected value and standard semi-deviation of the  $NPV$  are determined according to the estimated credibility distribution.

In order to reduce the required number of simulations, the number of random selections is different for individual levels of a fuzzy number. If, for example, the total number of random selections is  $l\ddot{r}$ , then the number of random selections for each  $\alpha$  level is proportional to the value of  $\log_{\alpha} = (1/(1 + nr_{\alpha}))^{\lambda}$ , where  $nr_{\alpha}$  is the level number and  $\alpha$  and  $\beta$  are experimentally defined parameters controlling the number of random selections for individual levels. The number of random selections from each level can be presented in the form  $l\ddot{r}_{\alpha} = (\log_{\alpha} / \sum_{\alpha} \log_{\alpha}) l\ddot{r}$ .

The parameter  $\beta$  was selected by varying its value in the range 0.5 to 2.0. The value chosen was the one which gave stability of the solution with the smallest number of random selections  $l\ddot{r}$ . The solutions obtained were considered to be stable when the difference between the highest and lowest value of the standard semi-deviation of the  $NPV$  obtained from 20 repetitions of the calculations was not greater than 5% of the lowest value.

The expected value and standard semi-deviation calculated in the manner discussed above take into account economic dependences between investment projects. Let us suppose that one of the investments results in reducing costs in a hot rolled sheet mill. As a result, the transfer of low cost  $HRC$  to successive phases of the technological process affects the forecasted financial performance of the company. These benefits will therefore be taken into consideration in existing departments, as well as in other projects included in the portfolio.

The statistical dependences between investments result from the correlation between the parameters used in the calculation of effectiveness, mainly the prices of steel and raw materials and the sales volume of individual ranges of metallurgical products. In each replication,  $NPV_{WP}$  is calculated for randomly generated values of the parameters used in the calculation of effectiveness. This procedure takes into account the correlation between these parameters. As a result, the possibility distributions  $NPV_{WP}$  estimated in accordance with this procedure take into account the correlation between the benefits generated from individual projects. This correlation is due to the correlation of the parameters used in the calculation of effectiveness.



### 7. Numerical examples

The results of calculations made for a simple example to illustrate the effectiveness of the proposed methods are presented below. The effective project portfolio was determined for the production setup presented in Fig. 1.

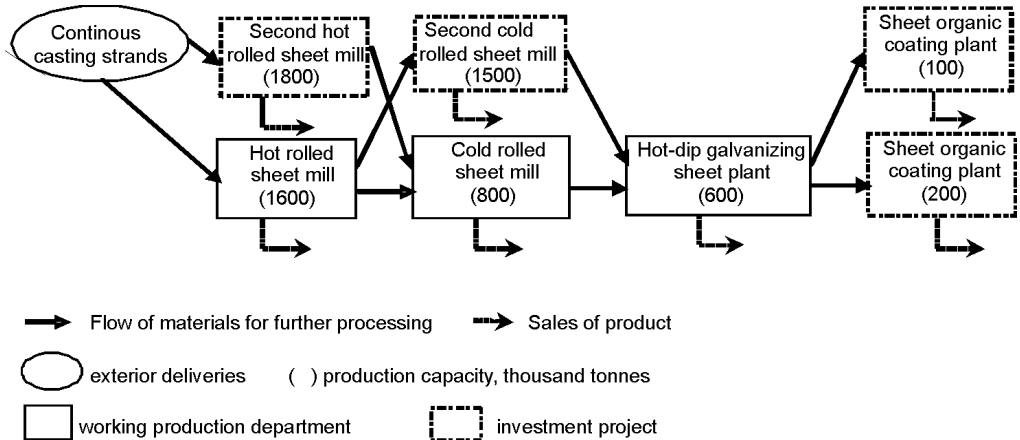


Fig. 1. Diagram of the production setup analyzed

Table 1. Investment projects taken into consideration

No.	Investment project	Construction period [months]	Investment outlay [thousand USD]
1	Construction of a second hot rolled sheet mill	(18; 20; 24)	(115 120.7; 128 333.3; 143 339.0)
2	Construction of a second cold rolled sheet mill	(18; 20; 22)	(144 073.7; 160 416.7; 179 164.0)
3	Construction of a new department of sheet organic coating, production capacity 100 thousand tonnes	(18; 20; 22)	(9247.5; 9916.7; 10 886.0)
4	Construction of a new department of sheet organic coating, production capacity 200 thousand tonnes	(18; 20; 22)	(14 657.5; 15 750.0; 16 996.2)

Table 1 presents the investment projects considered, together with triangular fuzzy numbers representing the construction period and investment outlay for the projects analyzed. The construction of a new department of sheet organic coating with a production capacity of 100 thousand tonnes and of a new department of sheet organic coating with a production capacity of 200 thousand tonnes were mutually exclusive projects. A budgeting period of three years is assumed for the construction of the

model. In Table 1, projects 1 and 2 could qualify for implementation in any year of the budgeting period and projects 3 and 4 could qualify in the second and third years of the budgeting period. A four-year life cycle is assumed for each project. This assumption required the use of 10 binary variables. These binary variables determine whether the projects listed in Table 1 qualify or are rejected in consecutive years of the budget period. Besides these binary variables, there were also 218 continuous variables used in the model, which allowed the equations to be formulated in both groups. There were 216 constraints in the model.

**Table 2.** Triangular fuzzy numbers presenting sales forecasts for the products produced by the company

Year	Sales [thousand t]			
	HRC	CRC	HDG strip	Organic coated strip
0	(502.3; 1 093.0; 1 712.3)	(702.7; 1 430.6; 2 243.7)	(117.5; 238.8; 384.0)	–
1	(448.2; 1 125.1; 1 933.9)	(501.8; 1 464.6; 2 433.9)	(121.0; 284.0; 462.5)	–
2	(473.2; 1 156.7; 1 817.2)	(619.2; 1 499.9; 2 399.6)	(168.8; 366.8; 606.3)	(139.8; 308.5; 489.8)
3	(513.1; 1 190.2; 1 916.6)	(662.0; 1 530.2; 2 416.7)	(174.3; 381.1; 625.5)	(138.1; 319.1; 493.9)
4	(583.3; 1 221.3; 1 935.7)	(667.1; 1 547.3; 2 513.4)	(187.9; 398.2; 648.5)	(120.4; 332.7; 536.8)
5	(491.7; 1 255.3; 2 042.8)	(730.0; 1 567.3; 2 612.4)	(195.3; 413.8; 666.2)	(148.4; 330.8; 533.2)
6	(563.6; 1 282.9; 2 051.1)	(770.2; 1 600.3; 2 624.9)	(202.5; 429.4; 700.2)	(161.2; 344.2; 549.8)
7	(534.2; 1 307.0; 2 050.4)	(704.7; 1 641.2; 2 613.7)	(223.3; 449.2; 713.5)	(149.9; 357.4; 574.9)

**Table 3.** Triangular fuzzy numbers presenting forecasts of prices of the products produced by the company and continuous casting stands

Year	Price [USD/t]				
	Continuous casting stands	HRC	CRC	HDG strip	Organic coated strip
0	(399.3; 491.0; 587.0)	(667.5; 821.8; 977.9)	(855.9; 1 052.3; 1243.4)	(910.3; 1093.1; 1322.2)	
1	(386.6; 480.0; 567.8)	(635.9; 803.9; 955.9)	(806.5; 1023.4; 1239.2)	(907.1; 1078.7; 1273.8)	
2	(370.2; 452.6; 537.3)	(609.5; 758.6; 890.7)	(809.2; 970.2; 1158.0)	(877.4; 1054.5; 1317.3)	(972.4; 1216.0; 1453.5)
3	(350.4; 437.5; 512.0)	(611.1; 735.2; 875.9)	(765.8; 937.7; 1122.6)	(854.5; 1035.2; 1235.1)	(977.2; 1193.9; 1429.1)
4	(352.4; 432.0; 528.4)	(593.5; 721.2; 865.5)	(750.0; 920.6; 1077.9)	(831.0; 1006.0; 1203.7)	(968.4; 1163.7; 1389.0)
5	(338.7; 423.7; 519.2)	(565.1; 707.4; 839.1)	(734.8; 908.6; 1063.8)	(820.0; 981.9; 1162.5)	(936.0; 1129.6; 1353.7)
6	(338.4; 413.5; 499.2)	(575.9; 693.1; 830.6)	(732.8; 885.8; 1060.7)	(761.9; 961.9; 1159.8)	(921.7; 1111.3; 1334.4)
7	(337.6; 410.7; 490.4)	(560.0; 685.7; 811.3)	(725.0; 876.1; 1061.8)	(764.3; 929.2; 1136.2)	(855.3; 1073.9; 1256.8)

The computational simulations took into consideration the uncertainty of the possible quantity of sales for each of the ranges of products produced by the company, prices of these products, prices of semi-finished steel (prices of continuous casting stands) and consumption per unit indicators for all products, construction period and investment outlay for the projects analyzed. It was assumed, that the remaining parameters used in the effectiveness calculations were determined.

Tables 2 and 3 present the appropriate triangular fuzzy numbers representing sales forecasts for the individual products produced by the company based on analysis and forecasts of product prices and continuous casting stands. Table 4 presents the triangular fuzzy numbers presenting material consumption indicators for individual stages of processing in the company under analysis.

**Table 4.** Material consumption indicator t/t

Material consumption indicator t/t	Fuzzy numbers
Continuous casting semi-product – HRC	(1.061; 1.067; 1.072)
HRC – CRC	(1.099; 1.114; 1.120)
CRC – HDG strip	(1.010; 1.023; 1.031)
HDG strip – organic coated strip	(1.003; 1.005; 1.008)

The prices of individual assortments of steel products and continuous casting stands are strongly correlated. Similarly, the sales of each assortment of steel products are correlated. Table 5 presents the correlation matrix for the sales of individual ranges of products. Table 6 presents the correlation matrix for the prices of individual products and continuous casting stands.

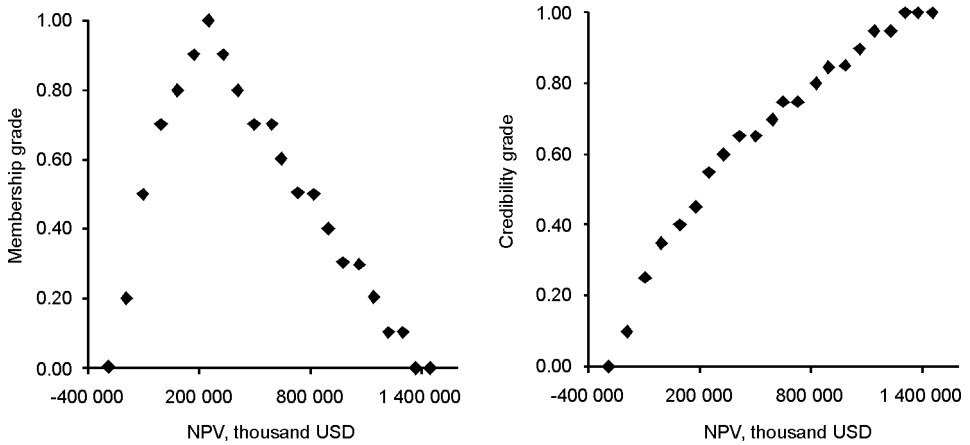
**Table 5.** Correlation matrix for the sales of individual ranges of products

Material	HRC	CRC	HDG strip	Organic coated strip
HRC	1.0000	0.6947	0.6460	0.6713
CRC	0.6947	1.0000	0.8850	0.6643
HDG strip	0.6460	0.8850	1.0000	0.5432
Organic coated strip	0.6713	0.6643	0.5432	1.0000

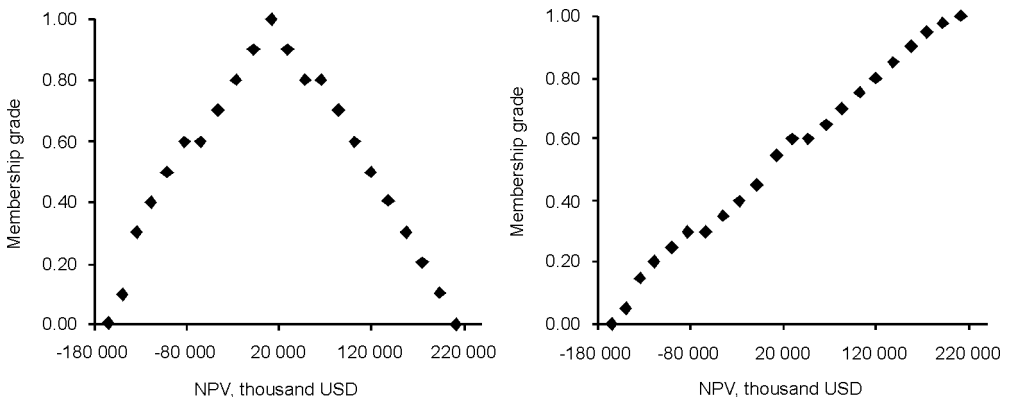
**Table 6.** Correlation matrix for the prices of individual products and continuous casting stands

Material	HRC	CRC	HDG strip	Organic coated strip	Continuous casting stands
HRC	1.0000	0.9822	0.9246	0.7136	0.8250
CRC	0.9822	1.0000	0.9452	0.7171	0.7907
HDG strip	0.9246	0.9452	1.0000	0.8248	0.6513
Organic coated strip	0.7136	0.7171	0.8248	1.0000	0.2793
Continuous casting stands	0.8250	0.7907	0.6513	0.2793	1.0000

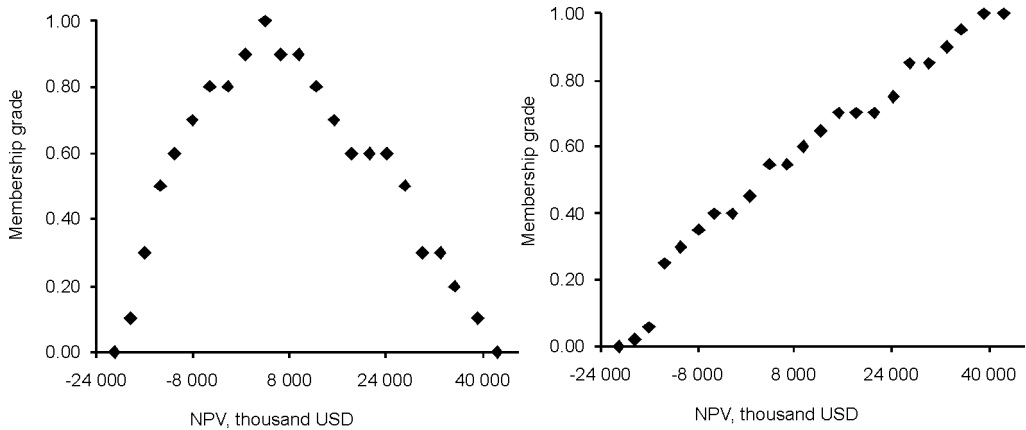
In the calculations, it is assumed that  $\varrho = 0.1$ ,  $\beta = 1$  and  $l\ddot{r} = 8500$ ,  $l\ddot{p} = 200$ . This allows the calculation of the expected value and semi-standard deviation of  $NPV$ . To realize the computer simulations, a program written in the C# programming language was used. The program allows calculations to be made for two variants. One calculation is carried out for effectiveness and risk estimation of a chosen investment portfolio (using the algorithm for the estimation of the credibility distribution of  $NPV$ , as well as its expected value and standard semi-deviation). The other calculation is carried out for the selection of efficient investment portfolios (using the algorithm for the selection of efficient investment portfolios).



**Fig. 2.** Possibility distribution (left) and credibility distribution (right) of the project: construction of a second hot rolled sheet mill – commencement of construction: year 0. Average value of  $NPV = 373\,224.1$  thousand USD; standard semi-deviation of  $NPV = 279\,864.5$  thousand USD

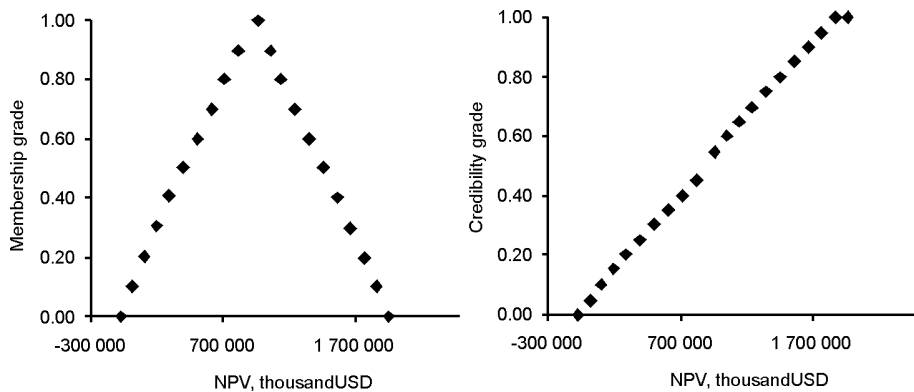


**Fig. 3.** Possibility distribution (left) and credibility distribution (right) of the project: construction of a second cold rolled sheet mill – commencement of construction: year 1. Average value of  $NPV = 16\,383.7$  thousand USD; standard semi-deviation of  $NPV = 76\,158.5$  thousand USD



**Fig. 4.** Possibility distribution (left) and credibility distribution (right) of the project: Construction of a new department of organic sheet coating – commencement of construction: year 2. Average value of  $NPV = 7028.0$  thousand USD; standard semi-deviation of  $NPV = 1952.0$  thousand USD

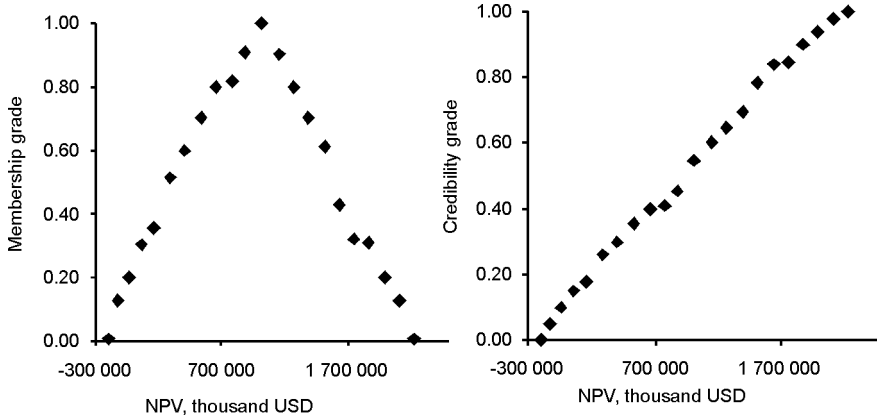
Figures 2–4 present the possibility distributions and credibility distributions for the following projects: construction of a second hot rolled sheet mill – commencement of construction: year 0, construction of a second cold rolled sheet mill – commencement of construction: year 1, construction of a new department of organic sheet coating – commencement of construction: year 2. Figure 5 presents the possibility distribution and the credibility distribution for a project portfolio which includes the above-mentioned projects.



**Fig. 5.** Possibility distribution (left) and credibility distribution (right) for an investment portfolio. Average value of  $NPV = 959\,107.7$  thousand USD; standard semi-deviation of  $NPV = 396\,998.9$  thousand USD

The data presented in Figures 2–5 illustrate well the effects of economic dependences between projects. The sum of the expected values of the  $NPV$  of projects under

analysis is equal to 396 635.2 thousand USD. The *NPV* of the portfolio, taking into account the effects of economic dependences between the investment projects, is almost 2.5 times greater than the sum of the *NPV* of the projects analyzed separately.



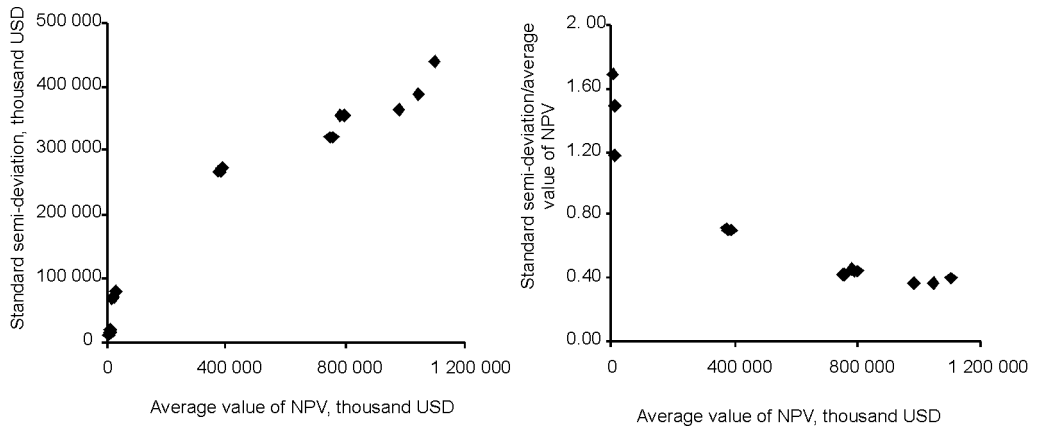
**Fig. 6.** Possibility distribution (left) and credibility distribution (right) for an investment portfolio assessed under the assumption of a lack of correlation between prices and sales volumes. Average value of *NPV* = 935 349.9 thousand USD; standard semi-deviation of *NPV* = 493 469.6 thousand USD

The effect of statistical dependences between individual projects may be analyzed similarly. Figure 6 presents the possibility distribution and credibility distribution functions for an investment portfolio, as discussed above. This possibility distribution is defined under the assumption that there are no statistical dependences between the prices of individual products and the sales of individual ranges of products (the respective random numbers in the fuzzy model simulations were generated as independent quantities). The data collected indicate that when the correlation between parameters is not taken into account in the calculation of effectiveness, the semi-standard deviation of the *NPV* of the investment portfolio under analysis obtained is by 24.3% higher than in the case where these values are treated as mutually correlated. Overlooking statistical dependences in the calculation of effectiveness using fuzzy numbers would lead to incorrect assessment of the risks of investment projects.

Figure 7 presents a non-dominated solution and the relation of semi-standard deviation to the expected value of *NPV* for the portfolios obtained using the presented algorithm for the selection of efficient investment portfolios.

The data presented in Figure 7 allow us to state that despite the existence of a strong correlation between the prices of steel products and their sales volumes, the effects of diversification of the portfolio are clearly visible. As the expected value of the *NPV* of an investment portfolio increases, so does its semi-standard deviation. The dynamics of change in the expected value of *NPV* and its semi-standard deviation vary. As a result, the dependence between the expected value of *NPV* and its semi-

standard deviation takes the form of a logarithmic curve. The ratio of the standard semi-deviation to the expected value of the  $NPV$  for the portfolio presented in Figure 7 varies between 0.37 and 1.69. Diversification of the investment portfolio allows the risk of investment to be significantly reduced. The reduction in the risk of efficient portfolios compared with the risk of investment projects examined separately is a result of the varied strength of correlation between the prices of various ranges of steel products and their sales volumes.



**Fig. 7.** A non-dominated solution and the ratio of the semi-standard deviation to the expected value of  $NPV$  for the portfolios obtained using the presented algorithm for the selection of efficient investment portfolios

Furthermore, the reduction in risk is a result of the appearance of independent parameters. We may mention here, for example, indicators of material consumption and investment outlay on projects. The effect of the number of projects in the portfolio is also significant. That is, the more investment projects in a portfolio, the lower the portfolio risk. It is also worth pointing out the significant effect of economic dependences between projects on the relation between the semi-standard deviation of the  $NPV$  of the portfolio and the expected value of its  $NPV$ .

## 8. Conclusion

The parameters used in the calculation of effectiveness are usually correlated. The use of fuzzy model simulation allows us to take into account dependences between parameters in the calculation of effectiveness expressed in the form of fuzzy numbers.

Overlooking such dependences in the calculation of effectiveness using fuzzy numbers would lead to an incorrect assessment of the risks of investment projects.

In the literature devoted to issues of the risk of investment projects, probability approaches are dominant. The calculations made and results obtained allow us to state that the credibility distribution describes the effectiveness and risk of an investment project equally well as a probability distribution. On the basis of a credibility distribution, the standard semi-deviation of  $NPV$  may be calculated. This can be equally easily interpreted and useful in the decision-making process as a standard semi-deviation calculated on the basis of a probability distribution.

The paper presents a new method for the selection of efficient investment portfolios, where parameters used in the calculation of effectiveness are expressed in the form of interactive fuzzy numbers. The selection of efficient investment portfolios is formulated as a multicriteria optimization problem. Therefore, a set of non-dominated investment portfolios is sought. The example presented indicates that statistical and economic dependences significantly affect the effectiveness and risk of investment projects. As the expected value of the  $NPV$  of an investment portfolio increases, so does its semi-standard deviation. The dynamics of change in the expected value and semi-standard deviation of  $NPV$  vary. As a result, the dependency between the expected value and standard semi-deviation of  $NPV$  takes the form of a logarithmic curve. Thus the ratio of the standard semi-deviation to expected value of  $NPV$  varies significantly, in the example given over the range of 0.37 to 1.69. Diversification of an investment portfolio in the metallurgical industry allows the risk of investment to be significantly reduced.

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