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MODELLING OF INNOVATION DIFFUSION

Since the publication of the Bass model in 1969, research on the modelling of the diffusion of innovation resulted in a vast body of scientific literature consisting of articles, books, and studies of real-world applications of this model. The main objective of the diffusion model is to describe a pattern of spread of innovation among potential adopters in terms of a mathematical function of time. This paper assesses the state-of-the-art in mathematical models of innovation diffusion and procedures for estimating their parameters. Moreover, theoretical issues related to the models presented are supplemented with empirical research. The purpose of the research is to explore the extent to which the diffusion of broadband Internet users in 29 OECD countries can be adequately described by three diffusion models, i.e. the Bass model, logistic model and dynamic model. The results of this research are ambiguous and do not indicate which model best describes the diffusion pattern of broadband Internet users but in terms of the results presented, in most cases the dynamic model is inappropriate for describing the diffusion pattern. Issues related to the further development of innovation diffusion models are discussed and some recommendations are given.

Keywords: *innovation, models of innovation diffusion, ICT market*

1. Introduction

Technological innovations have been recognized as a key input in the process of economic growth ever since Schumpeter [43], Solow [47] and Denison [6]. In the neoclassical approach, innovation was defined as a third factor – besides labour and capital – assumed to be exogenous to the process of growth. The turning point in relation to this approach towards innovation took place when the axiomatics of evolution-

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ary economics emerged and its concepts evolved. Evolutionary economics shifts the point of research interests from the level of macro to the level of micro and concerns innovations as an endogenous factor in econometric models [1], [31], [40]. An endogenous theory of growth takes into account the process of learning, accumulation and dissemination of new knowledge – innovations. The creation and diffusion of innovation in a social system results in positive externalities, and is a *conditio sine qua non* for economic development. Knowledge about the trajectory of the stage of absorbing innovation by potential adopters has significant cognitive benefits for market agents, who are a source of innovations, in terms of assessing their effectiveness, as well as for the institutions establishing the legal and institutional framework for an innovative system aimed at boosting the diffusion of knowledge in the economy. The purpose of this paper is to introduce mathematical models of innovation diffusion and the procedures for estimating their parameters.

The paper presents a practical application of the models discussed to describing the absorption of innovations in the market for information-communication technologies. The conclusions resulting from theoretical considerations, as well as from empirical research, are presented in the summary.

2. Essence of the innovation diffusion process

The theory of economics and management offers many approaches to defining the concept of innovation in functional and material manners [7], [12], [17], [20], [35], [45]. The best known and most popular definition of innovation is the one suggested by OECD, according to which innovation is the implementation of a new or significantly improved product (good or service), or process, a new marketing method, or a new organisational method in business practices, workplace organisation or external relations [34]. This definition, related from the point of view of a producer, points out the most important features of innovation, i.e. the aspect of novelty and the issue of changes in products, processes and organizational methods. Broadening this viewpoint to the consumers' perspective, innovation can mean any product, service or idea that is perceived by someone as a new [20].

Innovations can be classified based on many criteria, including the type of innovation [23], [33], [53], the originality of the changes [11], the impact of a change in consumer behaviour [42] and the degree of novelty [19]. The latter criterion is closely related to the concept of diffusion of innovation which means the temporal process of the diffusion of innovations of the same order, i.e. the same degree of novelty, in a specific economic system [9], [19], [35]. Regarding innovation as scientific or technical information which spreads by contact in the population of potential adopters, one can specify basic properties of the diffusion of innovation, i.e. direct contact with a source of innovation and its self-acting character [13], [48]. The spread of innovation may be vertical or horizontal [32], [35]. The former pertains to the flow of information in scientific and implementation processes and the latter means that the transfer of innovation may be spatial or situational.

Among many stylized concepts relating to the process of innovation diffusion, the most prominent ones are: diversified rates of innovation dissemination [13], [39], [41], [43] and diffusion according to the shape of the logistic curve [25], [29], [43]. According to Rogers' concept, there are five features of innovation affecting the rate of its diffusion measured by the length of time required for a certain percentage of the members of an economic system to adopt the innovation. These features are as follows: the relative advantage from use, the compatibility (consistency) with existing values, the complexity and difficulty of use, the testability and visibility of results from using innovations [38]. Rogers' classification is consistent with the concept of Hall, who indicates four factors affecting the rate of diffusion, i.e. the benefits and costs perceived by agents, the market and social environment, as well as problems regarding uncertainty and information [15]. Many empirical studies confirm the impact of these factors on the rate of the diffusion of innovation [4], [14], [18], [51], [54] but the results indicate that factors such as relative benefits of use, compatibility and complexity of an innovation are the most correlated with the rate of the diffusion of innovation.

According to the second stylized concept of the diffusion of innovation, it can be noted that innovations spread slowly in the initial period, next there is a recovery phase and then comes the phase of saturation (Fig. 1). This regularity can be explained by the internationalization of knowledge about an innovation among potential users and the related learning process [15], [48].

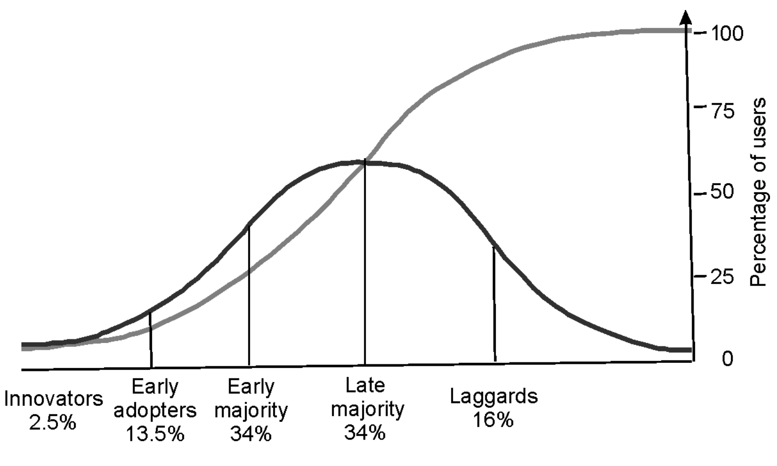

Fig. 1. The curve of innovation diffusion based on [28]

According to Figure 1, the users of innovation can be divided into five categories, namely: innovators, early adopters, early majority, late majority and laggards [38]. This division is the result of the interaction of two factors. The former concerns the heterogeneity of social agents with respect to their level of risk aversion and social and economic characteristics. The latter one results from different rates of acquiring knowledge (learning) by individual agents.

In conclusion, understanding the process of the diffusion of innovation is a crucial issue in determining the extent to which innovative activities undertaken by market agents and government institutions, including alter alia the introduction of new products and processes, financing R&D activity, technology transfer, contribute to economic growth and social welfare and allow various agents in the social and economic system to reduce economic and technological lags.

3. Models of innovation diffusion

To model the diffusion of innovation and thus determine the rate of growth in the number of users of an innovation and predicting their numbers in the future, one can use the mathematical theory of the spread of infections during an epidemic or the theory of information transfer [2], [13]. Using the theory of epidemiology, a fundamental model of innovation diffusion can be expressed by the differential equation:

$$\frac{dN(t)}{dt} = g(t)\left(m - N(t)\right), \tag{1}$$

where: $N(t)$ – the cumulative numbers of adopters at time t, m – ultimate ceiling of potential adopters, $g(t)$ – the coefficient (rate) of diffusion.

This equation points out that the diffusion rate is a function of the number of the potential adopters who have not yet adopted the technology and the rate of diffusion. The rate of diffusion, $g(t)$, reflects the likelihood that potential adopters will adopt the innovation in some small interval of time around time t. The value of $g(t)$ depends on such characteristics of the diffusion process as the type of innovation, communication channels, time and the traits of the social system. Depending on the formula for the coefficient of diffusion, $g(t)$, there are three specific models of innovation diffusion [25]:

- the external-influence model, where the coefficient of diffusion $g(t)$ is a constant p,
- the internal-influence model, where the coefficient of diffusion $g(t)$ is $qN(t)$,
- the mixed-influence model, where the coefficient of diffusion $g(t)$ is $p + qN(t)$.

Fourt and Woodlock formalized the first of these models and used it to analyze the diffusion of innovation on the food product market [10]. According to equation (1), the external-influence model can be expressed as:

$$\frac{dN(t)}{dt} = p(m - N(t)) \tag{2}$$

where: $N(t)$ – the cumulative number of adopters at time t, m – the ceiling, p – the coefficient of innovation.

The constant p in equation (2) is defined as the coefficient of innovation or external influence, emanating from the outside of a social system [5], [37]. Under such a premise, it can be assumed that p depends directly on mass media communication regarding innovation, formulated by market agents, government agencies, etc., and aimed at potential users of innovation. This model is applicable to modelling the diffusion of innovation, where agents of the social system are relatively isolated, when formalized and hierarchical communications dominate the sphere of communication.

Another specific model of innovation diffusion is the internal-influence model, propagated by Mansfield [29]. This model can be expressed using the following differential equation:

$$\frac{dN(t)}{dt} = qN(t)(m - N(t)), \tag{3}$$

where: $N(t)$ – the cumulative number of adopters at time t, m – the ceiling, q – the coefficient of imitation.

The constant q in the internal-influence model, defined as the coefficient of imitation, reflects the interactions of prior adopters with potential adopters. Therefore, the decision by potential users to adopt an innovation depends directly on the information formulated by existing users. The internal-influence model is appropriate to characterize the diffusion of innovation when a social system is relatively small and homogenous and there is a need for legitimizing information prior to adoption. The specific form of this model is the well-known Gompertz function used in forecasting the development of a new technology [22], [30].

The latter of the models analyzed is the mixed-influence model, developed by Bass [3], which subsumes both of the previous models. For the mixed-influence model, the diffusion coefficient $g(t)$ is equal to $p + q\,N(t)$.

In view of its great degree of generality, due to the accommodation of both internal and external influences, mixed-influence models are the most frequently employed in analyses [15], [25], [44]. The mixed-influence model can be expressed using the following equation:

$$\frac{dN(t)}{dt} = \left(p + \frac{q}{m}N(t)\right)(m - N(t)), \tag{4}$$

where: $N(t)$ – the cumulative number of adopters at time t, m – the ceiling, p – the coefficient of innovation, q – the coefficient of imitation.

Assuming $F(t) = N(t)/m$, where $F(t)$ is the fraction of potential adopters who have adopted the technology by time t, the Bass model can be restated as:

$$\frac{dF(t)}{dt} = \left(p + qF(t)\right)\left(1 - F(t)\right). \tag{5}$$

Assuming that the ceiling of potential adopters m is a constant, equation (4) is a first-order differential equation with three parameters p, q, m. Integrating this differential equation yields the diffusion of innovation curve i.e. the cumulative number of adopters at time t, $N(t)$:

$$N(t) = \frac{m - \dfrac{p(m - N_0)}{p + \dfrac{q}{m}N_0}\mathrm{e}^{-(p+q)t}}{1 + \dfrac{\dfrac{q}{m}(m - N_0)}{p + \dfrac{q}{m}N_0}\mathrm{e}^{-(p+q)t}}, \tag{6}$$

where $N_0 = N(0)$.

For the diffusion of innovation curve (6), the point of inflection, i.e. $(dN(t)/dt)_{\max}$, occurs when:

$$N(t^*) = m\left(\frac{1}{2} - \frac{p}{2q}\right), \tag{7}$$

$$t^* = -\frac{1}{p+q}\log\left(\frac{p}{q}\right), \tag{8}$$

$$n(t^*) = \frac{dN(t^*)}{dt} = m\left(\frac{q}{4} + \frac{p}{2} + \frac{p^2}{4q}\right). \tag{9}$$

In a special case where the coefficient of innovation p is zero, the Bass model simplifies to the following equation:

$$\frac{dN(t)}{dt} = \frac{q}{m}N(t)\left(m - N(t)\right). \tag{10}$$

This model contains two parameters q and m, and is similar to the internal-influence model, except that the coefficient of internal influence is divided by m. This model is referred to as the logistic model. Integrating equation (10) yields the diffusion of innovation curve, $N(t)$:

$$N(t) = \frac{m}{1 + \frac{m - N_0}{N_0} \mathrm{e}^{-qt}}, \tag{11}$$

where $N_0 = N(0)$.

Both the Bass model and the logistic model give S-shaped patterns of the cumulative number of adopters. By definition, an S-shape diffusion initially expands at an increasing rate, the cumulative number of adopters increases over time. As time goes by, the curve reaches a point of inflection, and the adoption rate starts to decrease. Finally, the diffusion reaches saturation level.

All the models analysed assume a constant ceiling for the number of potential adopters *m* but sometimes it increases at an independent rate. Thus, it is necessary to redefine the number of potential adopters *m* in the diffusion model as a function of time $m(t)$. This modification was introduced by Sharif and Ramannathan [47], who proposed an exponential model for the number of potential adopters:

$$m(t) = m_0 \mathrm{e}^{gt}, \tag{12}$$

where $m_0 = m(0)$.

This model is referred to as the dynamic model. It is described by the following two equations:

$$\begin{gathered} \frac{dN(t)}{dt} = \left(p + \frac{q}{m(t)} N(t) \right) \big(m(t) - N(t) \big), \\ m(t) = m_0 \mathrm{e}^{gt}. \end{gathered} \tag{13}$$

where $m_0 = m(0)$.

The dynamic model is the most complex of the four presented in this paper and contains four parameters *p*, *q*, *g* and m_0. The solution of the pair of equations (13) is given below:

$$N(t) = m_0 \mathrm{e}^{gt} \frac{\frac{\phi_1 - \phi_2}{2} - \phi_3 \frac{\phi_1 + \phi_2}{2} \mathrm{e}^{-\phi_1 t}}{q + q\phi_3 \mathrm{e}^{-\phi_1 t}}$$

$$\phi_1 = \sqrt{(g + p - q)^2 + 4pq}, \quad \phi_2 = g + p - q, \quad \phi_3 = \frac{\frac{\phi_1 - \phi_2}{2} - \frac{qN_0}{m_0}}{\frac{\phi_1 + \phi_2}{2} + \frac{qN_0}{m_0}}, \tag{14}$$

$$0 < N_0 = N(t = 0) \le m_0 = m(t = 0).$$

4. Estimating the parameters of diffusion models

The methods of estimating the parameters of models of innovation diffusion play a leading role in fitting models to empirical data and using these models for forecasting [28]. Mahajan, Srinivasan and Mason [26] describe four procedures used to estimate the parameters of diffusion models:

- ordinary least squares (OLS),
- maximum likelihood estimation (MLE),
- nonlinear least squares (NLS),
- algebraic estimation (AE).

The OLS procedure suggested by Bass [3] is one of the earliest procedures for estimating the parameters. This procedure involves estimation of the parameters by discretizing the differential equation (4) as follows:

$$N(t_i) - N(t_{i-1}) = pm + (q-p)N(t_{i-1}) - \frac{q}{m}N^2(t_{i-1}), \tag{15}$$

$$X(i) = \alpha_1 + \alpha_2 N(t_{i-1}) + \alpha_3 N^2(t_{i-1}), \tag{16}$$

where $\alpha_1 = pm$, $\alpha_2 = q - p$, and $\alpha_3 = -q/m$.

Given regression coefficients $\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3$, the estimators of the parameters p, q, m can be easily obtained as follows:

$$\hat{p} = \frac{-\hat{\alpha}_2 + \sqrt{\hat{\alpha}_2^2 - 4\hat{\alpha}_1\hat{\alpha}_3}}{2}, \tag{17}$$

$$\hat{q} = \frac{\hat{\alpha}_2 + \sqrt{\hat{\alpha}_2^2 - 4\hat{\alpha}_1\hat{\alpha}_3}}{2}, \tag{18}$$

$$\hat{m} = \frac{-\hat{\alpha}_2 - \sqrt{\hat{\alpha}_2^2 - 4\hat{\alpha}_1\hat{\alpha}_3}}{2\hat{\alpha}_3}. \tag{19}$$

The OLS estimation procedure is easy to implement, but it has three shortcomings. First, when only a few data points are available, due to the likely multicollinearity between regressors, one may obtain parameter estimates that are unstable or possess wrong signs. Second, formulas for the standard errors of the estimates are not available. Third, a time-interval bias is present, since discrete time-series data are used to estimate a continuous-time model.

As a result of the shortcomings of OLS, Mahajan and Schmittlein [46] proposed MLE estimation as an alternative technique of parameter estimation for diffusion models. The unconditional probability for adoption by time t is given by:

$$F(t) = \frac{c\left(1 - \mathrm{e}^{-bt}\right)}{\left(1 + a\mathrm{e}^{-bt}\right)}, \tag{20}$$

where $a \equiv q/p$, $b \equiv (p + q)$ and c is the probability of eventually adopting the innovation. Equation (20) represents the cumulative distribution function of the adoption time for an individual chosen at random from the population.

In order to determine the maximum likelihood estimates of p, q and m, one should first generate the maximum likelihood estimates of a, b and c. It has been well established that under very general regularity conditions, maximum likelihood estimates are asymptotically normal and optimum [36]. The relations between p, q and m and a, b and c are easily obtained as follows:

$$\hat{p} = \frac{\hat{b}}{\hat{a} + 1}, \tag{21}$$

$$\hat{q} = \frac{\hat{a}\hat{b}}{\hat{a} + 1}, \tag{22}$$

$$\hat{m} = \hat{c}M\,. \tag{23}$$

Using equation (20) and assuming x_i to be the number of individuals who adopt the innovation in time interval (t_{i-1}, t_i), $i = 1, 2, \ldots, T$, the likelihood function can be expressed as:

$$L\left(a, b, c, x_i\right) = \left(1 - F\left(t_{T-1}\right)\right)^{x_T} \prod_{i=1}^{T-1} \left(F\left(t_i\right) - F\left(t_{i-1}\right)\right)^{x_i}, \tag{24}$$

and the logarithm of the likelihood function is given by:

$$l\left(a,b,c,x_i\right) = \sum_{i=1}^{T-1} x_i \left(\ln c + \ln\left(\frac{1 - \mathrm{e}^{-bt_i}}{1 + a\mathrm{e}^{-bt_i}} - \frac{1 - \mathrm{e}^{-bt_{i-1}}}{1 + a\mathrm{e}^{-bt_{i-1}}} \right)\right) + x_T \ln\left(1 - c\frac{1 - \mathrm{e}^{-bt_{T-1}}}{1 - a\mathrm{e}^{-bt_{T-1}}}\right). \tag{25}$$

Note that there are no explicit formulas for the parameters a, b and c which maximize $l(a,b,c,x_i)$. Hence the MLEs can be obtained using Hooke–Jeeves' accelerated search pattern [16].

The NLS estimation procedure suggested by Srinivasan and Mason [50] was designed to overcome some of the shortcomings of the maximum likelihood estimation procedure, which itself was designed to overcome the shortcomings of the OLS procedure. Using equation (6), the model for the number of adopters X_i in the time interval (t_{i-1}, t_i) can be expressed as:

$$X_i = N(t_i) - N(t_{i-1}) + \varepsilon_i \tag{26}$$

or

$$X_i = \frac{m - \dfrac{p(m-N_0)\mathrm{e}^{-(p+q)t_i}}{p + \dfrac{q}{m}N_0}}{1 + \dfrac{\dfrac{q}{m}(m-N_0)\mathrm{e}^{-(p+q)t_i}}{p + \dfrac{q}{m}N_0}} - \frac{m - \dfrac{p(m-N_0)\mathrm{e}^{-(p+q)t_{i-1}}}{p + \dfrac{q}{m}N_0}}{1 + \dfrac{\dfrac{q}{m}(m-N_0)\mathrm{e}^{-(p+q)t_{i-1}}}{p + \dfrac{q}{m}N_0}} + \varepsilon_i\,, \tag{27}$$

where ε_i is an additive error term. Based on equation (27), the parameters p, q and m and their asymptotic standard errors can be directly estimated.

The last of the discussed methods of estimation of parameters for diffusion models is the algebraic procedure – AP. This procedure requires knowledge of the time of occurrence of the inflection point in the innovation diffusion curve. This knowledge can be based on actual or historical data, analogues or expert judgments [27].

For the Bass model, let N^* be the cumulative number of innovation adopters at time t^*, i.e. at the point of inflection in the diffusion curve, and n^* is the rate of increase in the number of innovation adopters at time t^*, equations (7)–(9) can be restated, assuming $F^* = N^*/m$, $f^* = n^*/m$, as:

$$t^* = -\frac{1}{p+q}\log\left(\frac{p}{q}\right), \tag{28}$$

$$\frac{N^*}{m} = \frac{1}{2} - \frac{p}{2q}, \tag{29}$$

$$\frac{n^*}{m} = \frac{q}{4} + \frac{p}{2} + \frac{p^2}{4q}. \tag{30}$$

If n^*, N^*, and t^* are known, equations (28)–(30) can be solved to yield:

$$p = \frac{n^*(m-2N^*)}{(m-N^*)^2}, \tag{31}$$

$$q = \frac{n^* m}{\left(m - N^*\right)^2}, \tag{32}$$

$$t^* = \frac{\left(m - N^*\right)}{2n^*} \ln\left(\frac{m}{m - 2N^*}\right). \tag{33}$$

Consequently, equation (33) can be used to find m numerically or by trial and error. Once m is known, equations (31) and (32) can be used to estimate p and q.

It is worth noting that the algebraic procedure can be used to find starting values for the estimates when using the NLS and MLE procedures.

5. Innovation diffusion in the information and communication technologies market

The concept of information and communication technology – ICT refers to a family of technologies that process, collect and send information in electronic form. The main segment of the ICT market is the market for internet access, especially broadband access [21] (i.e. access to connections of speed 144 kbit/s or greater). The development of the ICT market in Poland and other EU countries is regulated and supported by official institutions, since the level of advancement in ICT technology creates technical, economic and educational conditions for more effective and efficient transfer of information in product manufacturing and offering services.

The internet access market is interesting for modelling the diffusion of innovation for several reasons. First, there is a network effect, which assumes that the value of this new technology to the user depends largely on the extent to which it is used by the other members of a social system [12]. Second, the internet access market is rich in proactive advertising-informative initiatives taken up by internet providers and institutions supporting the development of an information society [8]. Third, the high rate of development of this market results from the fact that there is an exhaustive set of internet access technologies, which include many protocols from the DSL family (ADSL, SDSL, etc.), cable television networks (cable modems), 3G cell phones (UMTS, EDGE etc.) and others.

Due to these factors, three models have been used: the Bass model, the logistic model and the dynamic model for modelling the diffusion of broadband internet services, which are innovations on the ICT market. The data included the percentage of broadband internet users (i.e. the number of users per 100 inhabitants) in 29 countries and within OECD countries as a whole at six month intervals over the period 2000 –2009. For the models considered, the estimates of parameters and their significance are summarized in Table 1 (source: author's calculation).

Table 1. Estimates of the parameters and their significance for the diffusion models considered (based on calculations od the author)

Country \ Model	BASS				Logistic			Dynamic				
	P	Q	M	Adj. R^2	Q	M	Adj. R^2	P	Q	m_0	G	Adj. R^2
Australia	–0.0033***	0.4912***	25.6559***	0.998	0.425***	26.5843***	0.9967	–0.003	0.4848***	26.5505***	–0.0023	0.9978
Austria	0.011***	0.2299***	25.0376***	0.9976	0.2989***	23.2635***	0.9959	0.0078*	0.1737***	55.7036	–0.0496	0.9978
Belgium	0.0385***	0.0888**	39.4898***	0.9955	0.3976***	27.029***	0.9574	0.0273	0.0804*	56.5118	–0.0182	0.9952
Canada	0.0156***	0.1808***	32.9216***	0.9939	0.5334***	25.3067***	0.9262	0.016***	0.4336***	10.8684***	0.0495***	0.9972
Czech Republic	–0.0018**	0.5522***	18.1053***	0.993	0.4561***	19.3747***	0.9911	–0.017***	0.9813***	5.0758***	0.0807***	0.9983
Denmark	0.0178***	0.2813***	40.6913***	0.9958	0.4285***	37.5658***	0.9864	0.0062***	0.1815***	202.288**	–0.1068***	0.9986
Finland	0.0027	0.4544***	31.5128***	0.9954	0.4909***	31.147***	0.9953	0.0052***	0.3455***	59.4922***	–0.0441**	0.9973
France	0.0084***	0.3141***	32.1226***	0.9992	0.4632***	28.448***	0.9901	0.0085***	0.3958***	18.3342***	0.035***	0.9997
Germany	0.0098***	0.2342***	40.1414***	0.9952	0.4503***	29.5989***	0.9737	0.0004	0.1909***	1914.29	–0.2458***	0.9981
Greece	0.0002**	0.5246***	20.5689***	0.9936	0.745***	16.6699***	0.9869	–0.0005	1.0171***	1.0315**	0.1752***	0.9975
Hungary	0.0036***	0.3892***	19.4427***	0.9936	0.6036***	16.7533***	0.9796	0.0026	0.3508***	34.5817	–0.0363	0.9933
Iceland	0.0264***	0.2982***	34.2418***	0.9939	0.5347***	31.5444***	0.9736	0.0225***	0.2435***	48.5499**	–0.0236	0.994
Ireland	0.0009***	0.5267***	22.4401***	0.999	0.6918***	20.8203***	0.9937	0.0009***	0.5429***	19.5561***	0.0088	0.9989
Italy	0.0086***	0.3532***	20.5329***	0.9973	0.505***	18.8016***	0.9894	0.0022	0.5434***	9.9342***	0.0465***	0.9994
Japan	0.035***	0.2757***	24.6552***	0.9973	0.5843***	22.2693***	0.9695	0.0292***	0.4566***	15.9779***	0.0287***	0.9992
Korea	0.4096***	–0.4096**	35.4457***	0.9656	0.6128***	29.4379***	0.8592	–0.0784	1.1402**	23.4239***	0.0203***	0.9909
Luxembourg	0.0117***	0.354***	34.1025***	0.9945	0.5681***	29.8885***	0.9803	0.0124**	0.4598***	19.1753**	0.0409	0.9947
Netherlands	0.0033*	0.3573***	38.6794***	0.9988	0.3842***	38.1249***	0.9986	0.0021	0.3743***	35.8756***	0.005	0.9987
New Zealand	0.0012	0.3595***	26.3852***	0.9959	0.3875***	25.4454***	0.9958	0.0013	0.3505***	29.745	–0.0074	0.9955
Norway	0.0083***	0.3855***	35.4431***	0.9987	0.5201***	33.421***	0.9928	0.0074***	0.4298***	28.8413***	0.0137	0.9989
Poland	0.0013	0.503***	11.7085***	0.9871	0.5837***	11.1833***	0.9867	0.0008	0.5602***	8.0627	0.0253	0.9863
Portugal	0.0127***	0.3491***	16.8325***	0.9958	0.513***	15.6495***	0.9874	0.0076	0.4894***	10.3684***	0.0321***	0.9975
Slovak Republic	0.003***	0.3942***	16.4696***	0.9936	0.6925***	12.0654***	0.9727	0.0012	0.79***	1.7715***	0.1456***	0.9964
Spain	0.0157***	0.2572***	24.1696***	0.9985	0.5364***	19.6983***	0.9654	0.0083**	0.215***	56.8891**	–0.0543*	0.9988
Sweden	0.0189***	0.2312***	37.0112***	0.9893	0.3775***	33.0094***	0.98	0.0011	0.1482***	1271.83	–0.2339***	0.9967
Switzerland	0.0097***	0.3685***	34.6508***	0.9975	0.4674***	33.2667***	0.9941	0.0095***	0.3745***	33.7331***	0.0018	0.9973
Turkey	0.0006***	0.4796***	9.8672***	0.9982	0.5753***	9.067***	0.996	–0.0003	0.6388***	2.638***	0.0807***	0.999
United Kingdom	0.0073***	0.3907***	30.3601***	0.9997	0.6119***	27.4893***	0.9846	0.0075***	0.4277***	24.2142***	0.0148***	0.9998
United States	0.0124***	0.1959***	32.6285***	0.9989	0.2766***	29.0788***	0.9964	0.004**	0.1414***	158.966**	–0.0974***	0.9995
OECD	0.0122***	0.2103***	28.5137***	0.9981	0.3117***	24.713***	0.9936	0.002**	0.154***	294.797**	–0.1473***	0.9995

* if $p < 0.1$; ** if $p < 0.01$; *** if $p < 0.001$.

Of the models which assume a constant number of potential adopters, in all cases the logistic model shows that the parameters are all highly significant ($p < 0.001$) and the adjusted coefficient of determination is close to one., This is true for the Bass model in most cases, excluding Finland, Holland, New Zealand and Poland. For these four countries using the Bass model, only the estimate of the parameter p turns out to be insignificant due to its value close to zero, which indicates the advantage of using the logistic model in this situation. In other cases, the most appropriate model is the Bass model, due to the higher value of the adjusted R^2, together with the high significance of all the parameter estimates.

In the case of the dynamic model, it seems inappropriate for many countries, due to the insignificance of the parameters and their unrealistic values. Only in the cases of Japan and the UK, can the dynamic model can be used effectively to analyze changes in the percentage of inhabitants using the Internet, because of the significance of the parameters and the relatively low value of the coefficient g. The low level of usefulness of the dynamic model may well result from the type of dependent variable. The object of research is the percentage of inhabitants using the Internet, and not their absolute number, so the results of the analyses indicate that the ceiling on the number of potential adopters does not undergo any statistically significant changes and should thus be defined as a constant.

Note that the estimates of the parameters p and q are consistent with the conclusions of the meta-analysis of Juland, who stated that the value of p was often lower than 0.01, and the value of q ranged from 0.3 to 0.5 [24]. Sultan, Farley and Lehmann drew similar conclusions. On the basis of 213 research papers applying the Bass model and its modifications, they reported that the average value of the p and q parameters came to 0.03 and 0.38, respectively [52].

6. Conclusion

The application of three fundamental models of innovation diffusion has been presented, i.e. the internal-influence model, the external-influence model and the mixed-influence model, as well as the modified (dynamic) model, which relaxes the assumption of a constant number of potential users for an innovation. Next, the models were used to describe the process of innovation diffusion in the markets for broadband internet access in 29 OECD countries. The parameters of the models were estimated using the nonlinear least squares method. The results of this research do not enable the authors to unambiguously evaluate the usefulness of particular models in describing the process of innovation absorption in markets for broadband internet access due to the high diversity of the estimates of the parameters and the coefficients of determination according to the market (country) considered. Nevertheless, it is worth noting that the dynamic model is inappropriate for many countries, since the estimates of the parameters are insignificant and their values are unrealistic. This aberration may result from the type of independent variable, which relates to the percentage of internet users and not the absolute number.

Undoubtedly, further research on the modelling of innovation diffusion should be aimed at taking into consideration the influence of internal and external factors on the rate of innovation absorption, which may result in optimizing the combination of these factors.

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