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STOCHASTIC PROGRAMMING MODEL FOR PRODUCTION PLANNING WITH STOCHASTIC AGGREGATE DEMAND AND SPREADSHEET-BASED SOLUTION HEURISTICS

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By discretising the stochastic demand, a deterministic nonlinear programming formulation is developed. Then, a hybrid simulation-optimisation heuristic that capitalises on the nature of the problem is designed. The outcome is an evaluation problem that is efficiently solved using a spreadsheet model. The main contribution of the paper is providing production managers with a tractable formulation of the production planning problem in a stochastic environment and an efficient solution scheme. A key benefit of this approach is that it provides quick near-optimal solutions without requiring in-depth knowledge or significant investments in optimisation techniques and software.

Keywords: *production planning, stochastic programming, efficient algorithm, decision-making*

1. Introduction and literature review

Production planning has been the focus of significant research due to its tremendous impact on companies' bottom lines. Over- or under-producing have important negative effects in terms of inventory or backorder costs. The problem has become even more complex due to the increasing uncertainty and dynamism of demand. Consequently, stochastic optimisation methods have been proposed albeit not to their full potential due to their heavy computational burden [2].

Most of the literature focused on the deterministic demand and single item models. For a literature review of the production planning decisions, the reader is referred to Mundi et al. [15] and the references therein. The review captures the essential elements of the production and planning decisions from various marketing, operations management, and engineering design perspectives. A more focused survey by Lattila et al. [12] discusses the various inventory management systems assumptions and how they are handled in the

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operations research context. This would provide the reader with a thorough understanding of the complex nature of the problem at hand and the key characteristics of such systems.

Production planning problems under uncertainty were modelled using Lagrangian relaxation techniques [4]; uncertainties are handled by solving sub-problems using Lagrangian relaxation and mixed integer programming.

Porteus and Porteus derive an optimal simultaneous capacity and production plan for a short-life-cycle item with uncertain demand [17]. They show that despite the fact that the optimal capacity plan for the model is still a target interval policy, the target intervals can depend on the amount of the beginning inventory. In other words, the optimal capacity levels for each month cannot be scheduled in advance, as they may depend on the extent to which left-over inventory is available to meet current needs. Golmohammadi and Cassini expand the model by factoring in the pricing decision. The authors go on to show the existence of optimal schedules for joint production, capacity planning and pricing [5].

Since most manufacturing systems are complex and stochastic, hierarchical decomposition techniques have been used to manage such systems. Setrhi et al. demonstrate through a focused review of research that a hierarchy based on the frequencies of occurrence of different types of events results in decisions that are asymptotically optimal [18]. This finding provides a sufficient stepping stone for the scenario approach used in literature and adopted in this paper. The scenario approach has been utilised in situations where the stochastic demand can be discretised [16]. By formulating a nonlinear program for production planning of petroleum refineries that incorporates uncertainty using scenarios, they report solving real-life problems of several time periods and 5 scenarios to optimality.

The complexity of the problem stems in part from the fact that, as Bradley and Aarntzen point out, capacity and inventory decisions should be considered simultaneously, otherwise there would be an imbalance of capacity and inventory investments [3]. Additionally, indirect decisions, such as budgeting, need to be considered as by Laslo et al., where the problem of determining total budget needs and its distribution among several production facilities is considered [11]. Koberstein et al. extend the financial decisions to incorporate financial hedging into the production planning problem [9]. Multi-period decision variables and multi-stage decision trees are used to formulate and solve the problem and to show that exchange rate uncertainty cannot be eliminated by financial hedging in a stochastic demand environment.

The problem's complexity is further magnified by considering special characteristics of the demand such as seasonality and market growth. Zhang et al. solve a stochastic production planning model and derive managerial insights through parametric analysis [23].

Another manifestation of this augmented complexity is considering sequence-dependent setup times as in the paper by Shaikh et al. [19], where a real-life integrated inventory-production-scheduling problem is solved, resulting in major reductions in inventory levels and significant improvements in service levels.

Rolling horizon procedures generally lead to frequent changes in production decisions. Lin and Uzsoy report a significant reduction in planned changes by analyzing the performance of chance-constrained models with stochastic demand within a rolling horizon environment [13]. Furthermore, as Altendorfer et al. explain, forecast errors would enforce including a planned utilisation factor. The authors go on to show that such a factor has a high impact on optimal costs [1].

Multi-item production planning problems with stochastic demand add another layer of complexity due to the increased dependencies between the variables. Shen proposes a two-period nonlinear formulation in a rolling schedule and solved it using a multi-component algorithm [20]. Kazemi et al. integrate fuzzily imprecise and uncertain data to tackle the multi-item, multi-time period production planning problem to propose a profit maximisation fuzzy stochastic linear program [8]. Solyali considers the case where the uncertainty in demand is coupled with uncertainty in returned products [22]. A robust linear programming model is proposed to generate feasible production-disposal policies.

Robust optimisation approaches appear to have an edge over chance-constrained and multi-stage stochastic formulations; however, all models encounter challenging issues in addressing this complex stochastic production planning problem [2].

Quasi-Monte Carlo algorithms have been widely used in connection with solving multistage stochastic programs. In most hybrid simulation-optimisation frameworks, these algorithms are the basis for designing discrete approximations of the models. In the two--stage stochastic programming case, Heitsch et al. demonstrate that near-optimal convergence rates are achieved with normal demand and randomly scrambled point sets [7]. The complexity of the problem requires in most cases heuristic solution approaches. A survey of such methods has been presented by Silver [21].

The model proposed in this paper considers the production planning problem with stochastic aggregate demand. Through the critical lens of the literature, it is apparent that the majority of the developed models require significant knowledge of optimisation techniques coupled with an important investment in solution software. Although this might not pose a problem for large companies with adequate human and financial resources, this would represent a significant hurdle to small and medium enterprises (SMEs). Given the fact that SMEs play a major role in most economies, the small but numerous savings realised at these firms could translate to enormous savings to the production supply chain at large. Therefore, the model developed in this paper is mathematically tractable and computationally, efficient requiring no more than spreadsheet software to obtain a workable solution. The model's performance is tested using simulation to validate its output.

The contribution of this paper is significant in two key dimensions: the model, and the practical solution. The novelty in the model stems from its versatility in the sense that it does not require functional forms for the various variables (demand in particular). The model can be developed based on discrete realisations of the random variables which only requires knowledge of a few of the moments (typically, the mean and the variance). The solution approach also presents a novel mechanism for dealing with complex problems by reducing each instance of the problem to an evaluation problem requiring basic mathematical operations instead of a potentially nonlinear problem requiring sophisticated and costly solution techniques. As such, a key contribution of this paper is trading off complex costly optimal solutions with tractable efficient heuristic solutions.

2. Problem setting

The model is motivated by the situation where SME production managers are faced with preparing an aggregate production plan under significant restrictions on time and resources devoted to model formulation and solution. Given the different costs (processing costs, inventory and backorder costs, and limited production capacity,) the manager's problem is to generate such a production plan that will maximise the net benefit of the product less the various production costs. The typical constraints include lower and upper limits on the production capacity and inventory levels and the inventory continuity constraint.

The following variables have been defined as follows:

I – inventory,
I_0 – beginning inventory,
Q – production quantity,
D – demand,
p – price,
c – production cost,
h – inventory holding costy,
b – backorder cost,
s – shipping cost.

A typical production problem (stochastic nonlinear program SNP) would have the following basic formulation:

$$
\begin{aligned}
\text{SNP: } & \max f(I, Q, D) = pA - \left(cQ + sA + hI^+ + bI^-\right) \\
& \text{st.} \\
& I = Q + I_0 - D \\
& A = \min\,(D, Q + I_0) \\
& Q_{\min} \le Q \le Q_{\max} \\
& I_{\min} \le I \le I_{\max}
\end{aligned}
\tag{1}
$$

where A is the quantity available, $I^+ = A - D$ (inventory) if $A \ge D$ and $I^- = D - A$ (backorder) if $D \ge A$. SNP is a stochastic nonlinear program since the demand D is stochastic and the cost function is potentially nonlinear in the inventory and backorder cost determination.

The randomness of demand can be modelled by a sample of discrete outcomes, generated randomly from the probability distribution. For random deviates of the demand D_l, where l is the index of a realisation of the demand, SNP becomes a deterministic nonlinear program (DNP)

$$\begin{aligned} \text{DNP: } & \max E_D[f(I,Q,D_l) = pA - (cQ + sA + hI^+ + bI^-)] \\ & \text{s.t.} \\ & I = Q + I_0 - D_l \\ & A = \min(D_l, Q + I_0) \\ & Q_{\min} \le Q \le Q_{\max} \\ & I_{\min} \le I \le I_{\max} \end{aligned} \tag{2}$$

where E_D is the expected value relative to the demand. However, the DNP is still difficult to solve to optimality due to nonlinearity.

Nonetheless, by inspecting the DNP it becomes apparent that if values of I_0, Q and D are known, the objective function value and the constraints can be computed explicitly, thus reducing the DNP to an evaluation problem. Consequently, through an adequate discretisation of I_0, Q and D, a three-dimensional grid-search can be performed.

The indices used in the model are as follows: j corresponds to the observed (simulated) production quantity, k refers to the beginning inventory level and l to the realised demand.

For each scenario $(Q_j, I_{0,k})$, the expected value of the net return, Z^*_{jk}, is calculated depending on the realised demand. The maximum of these expected returns represents the best production-inventory policy $Q^* = f(I_0)$ (Fig. 1).

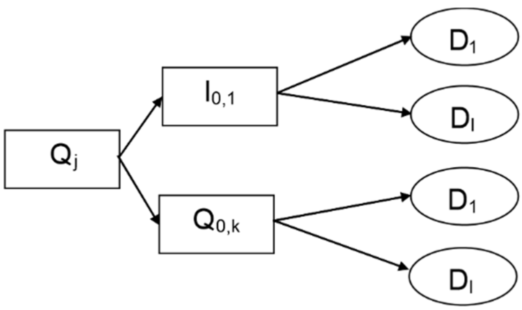

Fig. 1. Scenario tree

The corresponding heuristic model (HM) would therefore be:

$$\text{HM: } \max_{jk} Z^*_{jk} = \sum_l \rho_l \left(pA_{jkl} - \left(cQ_j + sA_{jkl} + hI^+_{jkl} + bI^-_{jkl} \right) \right)$$

$$\begin{aligned} &\text{st.} \\ &I_{jkl} = Q_j + I_{0,k} - D_l \\ &A_{jkl} = \min(D_l, Q_j + I_{0,k}) \\ &Q_{\min} \le Q_j \le Q_{\max} \\ &I_{\min} \le I_{jkl} \le I_{\max} \end{aligned} \tag{3}$$

where ρ_l is the probability of random deviate D_l.

3. Solution procedure

Given a set of discretised demands D_l, a set of discretised production decisions Q_j, and the discretised beginning inventory $I_{0,k}$, the algorithm is described as follows.

Step 0. Initialisation
Set minimum and maximum production capacity, $Q_{\min}$ and $Q_{\max}$, minimum and maximum beginning inventory levels $I_{\min}$ and $I_{\max}$.
Obtain demand data (mean, standard deviation, and a number of outcomes).
Generate discrete outcomes and determine probabilities from the provided distribution.
Step 1. DO WHILE $j \le J$
1. Set $Q^* = 0, Z^* = 0$
Step 2. DO WHILE $k \le K$
1. evaluate Z^*_{jk} as described in (3)
2. if $Z^*_{jk} > Z^*$ then set $Z^* = Z^*_{jk}$ and $Q^* = Q^*_{jk}$ END DO
END DO

The model's solution and simulation results provide the production manager with a decision policy that relates the optimal production quantity to the beginning inventory level.

Being an evaluation model, HM lends itself to an efficient spreadsheet-based solution procedure. The advantage of such a procedure is that it provides the decision-maker with an additional means of "what-if" analysis, particularly with the cost parameters. The solution procedure would consist therefore of constructing J tables (one for each discretised production quantity) with L rows (discretised demand) and K columns (discretised beginning inventory). The step between the discrete values of the production and the beginning inventory is generated by dividing the range (max-min) by the number of desired outcomes as: $\text{Step}_j = \dfrac{Q_{\max} - Q_{\min}}{J}$ and $\text{Step}_k = \dfrac{I_{\max} - I_{\min}}{K}$.

The demand is assumed to range from a maximum of $\mu + 3\sigma$ and a minimum of $\mu - 3\sigma$. The step is then calculated as $\text{Step}_l = \dfrac{6\sigma}{L}$. The probabilities for the discretised values are estimated using the probability of the interval centred at the value and having a width $12\sigma/L$. The probabilities are normalised by dividing the obtained probabilities by their sum to ensure the total is equal to 1.

The numerical case study is performed, assuming the demand is normally distributed with a mean 200 and standard deviation 50 units and using the data in Table 1.

Table 1. Data for the case study. Price and cost parameters

<table>
<tr><td>p</td><td colspan="2">5</td><td>h</td><td>0.5</td><td>b</td><td>1</td><td>c</td><td>2</td><td>s</td><td>0.5</td><td></td></tr>
<tr><td colspan="12">Beginning inventory and production levels</td></tr>
<tr><td>$I_{0,k}$</td><td colspan="2">0</td><td>10</td><td>20</td><td>30</td><td>40</td><td>50</td><td>60</td><td>70</td><td>80</td><td>90</td></tr>
<tr><td>Q_j</td><td>50</td><td>80</td><td>110</td><td>140</td><td>170</td><td>200</td><td>230</td><td>260</td><td>290</td><td>320</td><td>350</td></tr>
<tr><td colspan="12">Randomly generated demand realisations and corresponding probabilities</td></tr>
<tr><td>D_i</td><td colspan="2">80</td><td>110</td><td>140</td><td>170</td><td>200</td><td>230</td><td>260</td><td>290</td><td>320</td><td>350</td></tr>
<tr><td>$\wp_i$</td><td colspan="2">0.016189</td><td>0.051898</td><td>0.119017</td><td>0.195655</td><td>0.230877</td><td>0.195655</td><td>0.119017</td><td>0.051898</td><td>0.016189</td><td>0.003604</td></tr>
</table>

After computing the corresponding expected values for each scenario, the solution obtained is shown in Fig. 2.

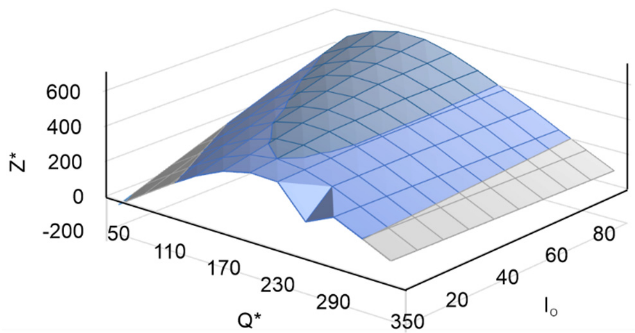

Fig. 2. Net return

Note that the net return peaks around 140–170 units irrespective of the beginning inventory level. Additionally, the net return increases as the beginning inventory level increases. This is due to the reduction in the number of backorders and their corresponding cost.

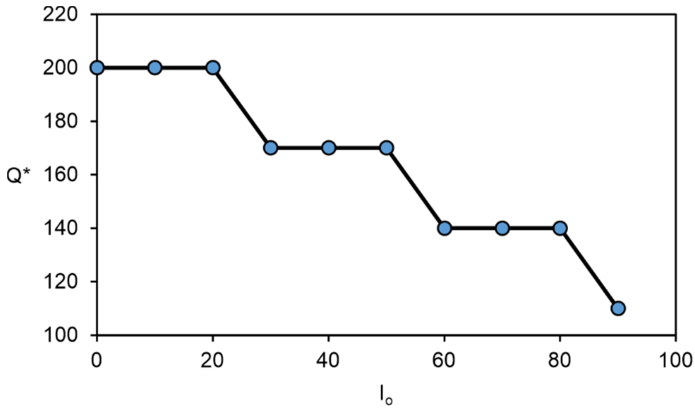

Fig. 3. Decision policy $Q^* = f(I_0)$

A decision policy $Q^* = f(I_0)$ is generated and presented in Fig. 3. The optimal production quantity decreases, as expected, as the level of the beginning inventory increases. However, note that the maximum production quantity is equal to the mean demand. This would seem counterintuitive, given the relatively high backorder cost used in the case study.

4. Simulation analysis and managerial insights

The simulation study consists of randomly generating the demand and the beginning inventory level. The random demand is generated from the normal distribution with a mean 200 and standard deviation of 50; whereas the beginning inventory is randomly generated from a uniform distribution ranging from 0 to 100. Using the optimal decision policy obtained from the model, the optimal production quantity corresponding to the beginning inventory level is selected and the net return is computed. The simulation involved 250 replicas. The average net return is computed, corresponding to each production quantity level. The results presented in Fig. 4 show that the maximum return occurs for production lots between 140 and 170 units.

A look at the scatter plot of the net return relative to the production quantity depicted, reveals a similar pattern (Fig. 5). The highest values of net return are at the 140 and 170 production levels. Additionally, the variation of the net returns for these two production levels is lower compared to the other situations.

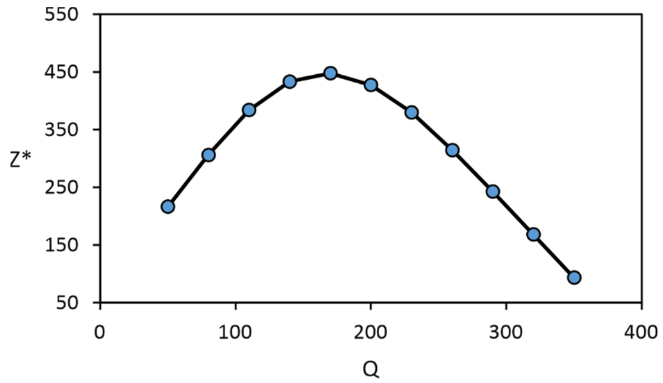

Fig. 4. Simulated $Z = f(Q)$

This validates the model's solution discussed in the previous section. As evidenced by the model's solution and the simulation analysis, the production lot size do not exceed the mean demand even though the backorder cost represents 25% of the total unit cost. This implies that in a stochastic demand environment, as shown in Fig. 4, under-producing outperforms over-producing even in the case where the backorder cost is double the inventory holding cost. Additionally, the optimal production quantity does not vary significantly as the beginning inventory level changes. This robustness in the solution means that level production schedules can be prepared for different beginning inventory levels, and thus reducing what is known as "nervousness" in the production system (frequent changes to production schedules). A further insight from Fig. 5 is that the net return from a production lot size of 110 has lesser variation and lies entirely in the positive quadrant, so it might be a viable option for risk-averse decision makers. However, it is a less-likely scenario than the lot size of 140 which represents a more balanced alternative in terms of likelihood and expected return.

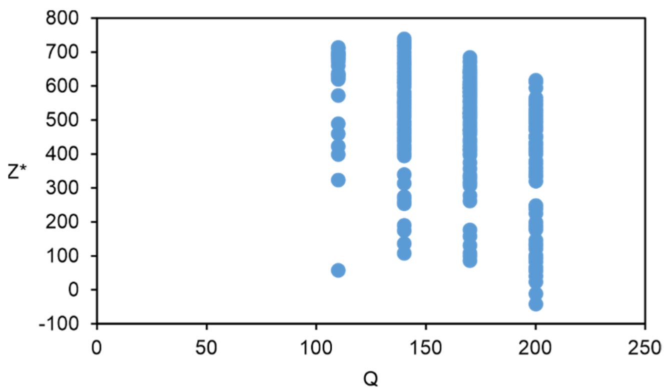

Fig. 5. Scatter plot of simulated $Z = f(Q)$

For a production manager of a small or medium enterprise, obtaining a solution cost-effectively is a tremendous advantage even if the solution is near-optimal. Most of the models presented in the literature develop utterly complex models requiring costly solution procedures which is not available to most of SMEs. The model presented in this paper explicitly incorporates the randomness in the demand in a mathematically-tractable fashion. Furthermore, the solution procedure is easily implemented on spreadsheet software that is a common fixture in most, if not all, SMEs.

5. Concluding remarks and future research

An aggregate production planning model with stochastic demand is developed. The model discretises the random demand, the beginning inventory, the production lot size, and performs a three-dimensional search. The model is implemented in spreadsheet software which enables the decision-maker to easily modify the various parameters and to observe the effect on the solution. The model's solution is validated by a quasi-Monte Carlo simulation, showing the accuracy and the robustness of the solution proposed.

The model is flexible to accommodate any fineness of discretisation desired by the decision-maker. Although the model represents an important efficient aggregate planning tool, it falls short of providing operational production plans. The model can be further extended to address production planning for a planning horizon of several periods and for using the current model to fine-tune the production decision for each period. These venues and others are considered subjects for future research.

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