OPERATIONS RESEARCH AND DECISIONS
No. 3 2021
DOI: 10.37190/ord210301

FUZZY PROGRAMMING FOR MULTI-CHOICE BILEVEL TRANSPORTATION PROBLEM

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Multi-choice programming problems arise due to the diverse needs of people. In this paper, multi-choice optimization has been applied to the bilevel transportation problem. This problem deals with transportation at both the levels, upper as well as lower. There are multiple choices for demand and supply parameters. The multi-choice parameters at the respective levels are converted into polynomials which transmute the defined problem into a mixed integer programming problem. The objective of the paper is to determine a solution methodology for the transformed problem. The significance of the formulated model is exhibited through an example by applying it to the hotel industry. The fuzzy programming approach is employed to obtain a satisfactory solution for the decision-makers at the two levels. A comparative analysis is presented in the paper by solving the bilevel multi-choice transportation problem with goal programming mode as well as by the linear transformation technique. The example is solved using computing software.

Keywords: *bilevel programming, transportation problem, fuzzy programming, goal programming, tolerance limits, satisfactory solution*

1. Introduction

An individual has several options to travel from one place (origin) to another (destination) through diverse routes, using alternate modes of transportation. Similarly, options also exist for the transportation of goods and materials. The advancement in transportation and logistics has evolved with technology. Different techniques and issues related to transportation problems have been taken up by various authors. Jacobs and Greaves [10] studied the transportation issues in developing and emerging nations. Ji and Chu [13] proposed the dual matrix approach to solve the transportation problem.

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Received 13 February 2021, accepted 28 September 2021

Scott et al. [27] discussed the requirement of public transport in metropolitan cities. Mardani et al. [22] reviewed multiple criteria decision-making techniques in transportation systems. Speranza [29] considered the history and future trends in the area of transportation. Transport is also a source of income as it is used to carry people and raw materials from one place to another. Bastiaanssen et al. [3] discussed the relationship between employment and transport.

The concept of fuzzy mathematical programming and multi-objective programming has been implemented by researchers to solve the transportation problem. Kaur and Kumar [14] represented transportation cost, availability and demand of the product as generalised trapezoidal fuzzy numbers and proposed a new method for solving it. Kaur et al. [15] solved multi-objective multi-index real-life transportation problem by an exponential membership function. Singh et al. [28] studied multi-objective transportation problems and applied a goal programming approach to obtain fuzzy efficient solutions for them. Goswami et al. [8] applied fuzzy programming to a multi-objective transportation problem with varied costs. Abounacer et al. [1] proposed an epsilon-constraint method to three objective programming problem which generates Pareto front. Liang [18] used interactive fuzzy linear programming to solve multi-objective transportation problems. Kumar et al. [17] solved the transportation problem using Pythagorean fuzzy numbers. Pratihar et al. [25] solved the fuzzy transportation problem by a modified Vogel's approximation method.

It is observed from the above that researchers used a multi-objective approach to formulate the transportation problem. However, bilevel programming is another tool to solve them. Bilevel programming problem (BLPP) plays an important role in the field of transportation. BLPP is a hierarchical programming problem that moves sequentially from the upper level to the lower level problem. The bilevel problem in which transportation is defined at both levels is called the bilevel transportation problem. Various researchers used bilevel programming to model transportation problems and proposed methodologies for solving them. Clegg et al. [6] applied the bilevel model to optimise urban transportation. Msigwa et al. [24] formulated a bilevel model for solving transportation problems concerning both road toll pricing and capacity expansions. Zhang and Gao [30] formulated the transportation network design problem as a mixed-integer non-linear bilevel programming problem and solved it. Liu and Zhang [19] solved the bilevel transportation problem by the exact penalty method. Du et al. [7] applied fuzzy bilevel programming to a multi-depot vehicle routing problem for solving the multi-objective multi-item solid transportation problem. Midya et al. [23] employed intuitionistic fuzzy programming to solve multi-stage multi-objective fixed charge solid transportation problems.

Multi-choice optimisation has been studied by various authors due to its relevance in day-to-day life. It can be seen in a wide range of problems like production planning, assignment problem, inventory transportation, knapsack problem, logistic distribution, etc. Researchers have considered multi-choice optimisation for modelling multi-objective programming problems in the transportation industry. Ho et al. [9] dealt with the problem of location selection by obtaining weights using the analytic hierarchy process and implementing it to each goal using a multi-choice goal programming model. Maiti and Roy [21] applied the intuitionistic fuzzy method to solve multi-choice bilevel programming for Stackelberg games. Cao et al. [4] developed a tri-objective bi-level model for post-disaster fuzzy supplies. Aghababaei et al. [2] proposed a bi-level model to manage the deficient supply of drugs and planning of ration for difficult pandemic situations. Jalil et al. [11] presented a multi-level model for the solid transportation problem. Chakraborty et al. [5] discussed a solution approach for a multi-objective multi-choice fuzzy transportation problem using the chance operator. Ranarahu et al. [26] considered the probabilistic transportation problem with uncertain parameters in the supply and demand constraints. Mahapatra et al. [20] contemplated a multi-choice environment to a stochastic transportation problem.

It follows that the multi-choice programming so far has been applied to multi-objective programming problems. Different solution techniques have been proposed by researchers for multi-objective transportation problems from time to time. This motivates the present authors to propose a methodology for multi-choice optimisation to a bilevel transportation problem.

In the present paper, a mathematical model is defined for multi-choice bilevel transportation problem (BTPMCP) and a technique for the same problem is developed. In this problem, supply and demand parameters are in multiple choices. The multi-choice parameters are dealt with by converting them into polynomials using the Lagrange interpolation. This regenerates our defined problem into a mixed integer programming problem. The mixed-integer problem is solved by the fuzzy programming approach. The solution thus obtained is the satisfactory solution for the defined problem. BTPMCP is also solved by the goal programming method. A comparative analysis is done in the paper by comparing the solutions obtained from fuzzy programming and goal programming with the technique proposed by Khalil et al. [16]. An algorithm is developed and then explained by applying it to the hotel industry.

Section 2 of the paper presents the mathematical model for the bilevel transportation problem in which supply and demand parameters are of multi-choice. Section 3 describes the solution method for the defined problem by fuzzy programming and goal programming approach. Section 4 envisages an algorithm for BTPMCP. Section 5 presents an illustrative example of a hotel industry depicting the importance of the algorithm. The conclusion and prospective work are included in section 6, followed by references.

2. Bilevel transportation problem with multi-choice parameters

Mathematically, the bilevel transportation problem with multi-choice parameters (BTPMCP) is defined as

$$\min_{Z_1} F_{11}(Z_1, Z_2) = c_1^T Z_1 + c_2^T Z_2$$

where Z_2 solves for a given Z_1

$$\min F_{12}(Z_1, Z_2) = d_1^T Z_1 + d_2^T Z_2$$

subject to

$$\begin{aligned}
&\sum_{t\in T'} z_{st} \le (p_s'^{(1)}, p_s'^{(2)}, \ldots, p_s'^{(m_s)}); \; s\in S' \\
&\sum_{s\in S'} z_{st} \ge (q_t'^{(1)}, q_t'^{(2)}, \ldots, q_t'^{(n_t)}), \; t\in T' \\
&\sum_{t\in T''} z_{st} \le (p_s''^{(1)}, p_s''^{(2)}, \ldots, p_s''^{(x_s)}), \; s\in S'' \\
&\sum_{s\in S''} z_{st} \ge (q_t''^{(1)}, q_t''^{(2)}, \ldots, q_t''^{(y_t)}), \; t\in T'' \\
&z_{st} \ge 0, \; \forall s\in S, \; t\in T
\end{aligned}$$

where S is the total number of sources, T – total number of destinations, S' and T' are the numbers of sources and destinations at the upper level, S'' and T'' are the numbers of sources and destinations at the lower level.

$$S = S' \cup S'', \; S': (s = 1, 2, \ldots, k_1), \; S'': (s = k_1 + 1, \ldots, k)$$

$$T = T' \cup T'', \; T': (t = 1, 2, \ldots, j_1), \; T'': (t = j_1 + 1, \ldots, j)$$

$$c_1 = [c'_{st}], \; d_1 = [d'_{st}], \; s\in S', \; t\in T'$$

$$c_2 = [c''_{st}], \; d_2 = [d''_{st}], \; s\in S'', \; t\in T''$$

Here, $c'_{st} > 0$, $d'_{st} > 0$ and $c''_{st} > 0$, $d''_{st} > 0$ are the cost parameters for the upper level and lower problem, respectively.

$(p_s'^{(1)}, p_s'^{(2)}, \ldots, p_s'^{(m_s)})$, $(q_t'^{(1)}, q_t'^{(2)}, \ldots, q_t'^{(n_t)})$ are multi-choice parameters for the upper level problem and $(p_s''^{(1)}, p_s''^{(2)}, \ldots, p_s''^{(x_s)})$, $(q_t''^{(1)}, q_t''^{(2)}, \ldots, q_t''^{(y_t)})$ are multichoice parameters for the lower level problem, respectively.

$Z_1 = [z_{st}]$, $s\in S'$, $t\in T'$ and $Z_2 = [z_{st}]$, $s\in S''$, $t\in T''$ are the decision variables describing the quantity transported from the sth origin to the tth destination.

Also, $p_s'^{(m_s)} \ge 0$, $s\in S'$, $q_t'^{(n_t)} \ge 0$, $t\in T'$, $p_s''^{(x_s)} \ge 0$, $s\in S''$, $q_t''^{(y_t)} \ge 0$, $t\in T''$.

Feasibility condition for (BTPMCP):

$$\sum_{s\in S} p_s \geq \sum_{t\in T} q_t; \quad \sum_{s\in S'} p'_s \geq \sum_{t\in T'} q'_t \text{ and } \sum_{s\in S''} p''_s \geq \sum_{t\in T''} q''_t$$

3. Technique for solving the problem BTPMCP

The supply and demand parameters in the problem BTPMCP are of multiple choices. These parameters are transformed into interpolating polynomials by Lagrange's interpolation [12]. The multi-choice supply parameters $p_s'^{(m_s)}$ $(s \in S')$ are replaced by assigning the integer variables u_s, which takes m_s number of values ($u_s = 0, 1, ..., m_s - 1$). The interpolating polynomial for sth multi-choice supply parameter is composed as:

$$\begin{aligned} R^1_{p'_s}(u_s) = & \frac{(u_s-1)(u_s-2)\ \dots\ (u_s-m_s+1)}{(-1)^{m_s-1}(m_s-1)!}\, p_s'^{(1)} \\ & + \frac{u_s(u_s-2)\ \dots\ (u_s-m_s+1)}{(-1)^{m_s-2}(m_s-2)!}\, p_s'^{(2)} \\ & + \frac{u_s(u_s-1)\ \dots\ (u_s-m_s+1)}{(-1)^{m_s-3}2!(m_s-3)!}\, p_s'^{(3)} \\ & + \dots + \frac{u_s(u_s-1)\ \dots\ (u_s-m_s+2)}{(m_s-1)!},\ p_i'^{(m_s)},\ s = 1, 2, ..., k_1 \end{aligned} \tag{1}$$

Similarly, to interpolate the multi-choice supply parameters $p_s''^{(x_s)}(s \in S'')$, allocate integer variables v_s which takes x_s number of values ($v_s = 0, 1, ..., x_s - 1$), the polynomial is defined as

$$\begin{aligned} R^2_{p''_s}(v_s) = & \frac{(v_s-1)(v_s-2)\ \dots\ (v_s-x_s+1)}{(-1)^{x_s-1}(x_s-1)!}\, p_s''^{(1)} \\ & + \frac{v_s(v_s-2)\ \dots\ (v_s-x_s+1)}{(-1)^{x_s-2}(x_s-2)!}\, p_s''^{(2)} + \dots \\ & + \frac{v_s(v_s-1)\ \dots\ (v_s-x_s+2)}{(x_s-1)!}\, p_s''^{(x_s)},\ s = k_1+1, ..., k \end{aligned} \tag{2}$$

Assign integer variables a_t ($t = 1, ..., j_1$) and b_t ($t = j_1 + 1, ..., j$) for the multi-choice demand parameters at upper and lower level respectively, the interpolating polynomials are defined as follows:

$$R^3_{q'_t}(a_t) = \frac{(a_t-1)(a_t-2)\ \dots\ (a_t-n_t+1)}{(-1)^{t_\ell-1}(n_t-1)\,!}\, q'^{(1)}_t$$
$$+\ \frac{a_t(a_t-2)\ \dots\ (a_t-n_t+1)}{(-1)^{n_t-2}\ (n_t-2)\,!}\, q'^{(2)}_t$$
$$+\ \dots +\ \frac{a_t(a_t-1)(a_t-2)\ \dots\ (a_t-n_t+2)}{(n_t-1)\,!}\, q'^{(n_t)}_t,\ t=1,\ \dots,\ j_1 \tag{3}$$

$$R^4_{q''_t}(b_t) = \frac{(b_t-1)(b_t-2)\ \dots\ (b_t-y_t+1)}{(-1)^{y_t-1}(y_t-1)\,!}\, q''^{(1)}_t$$
$$+\ \frac{b_t(b_t-2)\ \dots\ (b_t-y_t+1)}{(-1)^{y_t-2}(y_t-2)\,!}\, q''^{(2)}_t$$
$$+\ \dots\ +\ \frac{b_t(b_t-1)\dots(b_t-y_t+2)}{(y_t-1)\,!}\, q''^{(y_t)}_t,\ t = j_1+1,\ \dots,\ j \tag{4}$$

Introducing interpolating polynomials from equations (1)–(4) in BTPMCP, the problem is transmuted to a mixed integer programming problem. It is denoted as (BMIPP):

$$\min_{Z_1} F_{11}(Z_1,\ Z_2) = c_1^T Z_1 + c_2^T Z_2$$

where Z_2 solves for a given Z_1

$$\min_{Z_2}\ F_{12}(Z_1,\ Z_2) = d_1^T Z_1 + d_2^T Z_2$$

subject to

$$\begin{aligned} &\sum_{t\in T'} z_{st} \le R^1{}_{p'_s}(u_s),\ s\in S' \\ &\sum_{s\in S'} z_{st} \ge R^3_{q'_r}\ (\mathrm{a}_t),\ t\in\ T' \end{aligned} \tag{5}$$

$$\begin{aligned} &\sum_{t\in T''} z_{st} \le R^2_{p''_s} v_s),\ s\in S'' \\ &\sum_{s\in S''} z_{st} \ge R^4_{q''_t}(b_t),\ t\in\ T'' \end{aligned} \tag{6}$$

$$z_{st} \geq 0, \ \forall \, s \in S, \quad t \in T \tag{7}$$

$$0 \leq u_s \leq m_s - 1, \, s = 1, 2, ..., k_1, \, 0 \leq a_t \leq n_t - 1, \, t = 1, 2, ..., j_1$$

$$0 \leq v_s \leq x_s - 1, \, s = k_1 + 1, ..., k, \, 0 \leq b_t \leq y_t - 1, \, t = j_1 + 1, ..., j \tag{8}$$

$$u_s, v_s \in Z^+ \cup \{0\}, \, s \in S, \, a_t, b_t \in Z^+ \cup \{0\}, \, t \in T$$

In order to solve BMIPP and to obtain the satisfactory solution for the problem BTPMCP, fuzzy programming method is employed.

3.1. Solution method for BTPMCP: Fuzzy programming method

In order to apply fuzzy approach, the fuzzy membership functions for the decision makers at two levels are constructed. The upper level problem $F_{11}(Z_1, Z_2)$ is solved subject to the constraint set (5)–(8). Let its minimum and maximum values be denoted by $F_{11}^{\min}$ and $F_{11}^{\max}$. Using this data, the membership function for the upper level problem is defined as

$$\mu\left(F_{11}(Z_1, Z_2)\right) = \begin{cases} 1 & F_{11}(Z_1, Z_2) \leq F_{11}^{\min} \\ \dfrac{F_{11}^{\max} - F_{11}(Z_1, Z_2)}{F_{11}^{\max} - F_{11}^{\min}} & F_{11}^{\min} < F_{11}(Z_1, Z_2) < F_{11}^{\max} \\ 0 & F_{11}(Z_1, Z_2) \geq F_{11}^{\max} \end{cases} \tag{9}$$

Let the minimum accepted degree of satisfaction for the upper level problem be ρ'.

Similarly, solving the lower level problem $F_{12}(Z_1, Z_2)$ subject to the constraint set (5)–(8), let $F_{12}^{\min}$ and $F_{12}^{\max}$ be the minimum and maximum values for $F_{12}(Z_1, Z_2)$. The membership function for $F_{12}(Z_1, Z_2)$ is defined as

$$\mu\left(F_{12}(Z_1, Z_2)\right) = \begin{cases} 1 & F_{12}(Z_1, Z_2) \leq F_{12}^{\min} \\ \dfrac{F_{12}^{\max} - F_{12}(Z_1, Z_2)}{F_{12}^{\max} - F_{12}^{\min}} & F_{12}^{\min} < F_{12}(Z_1, Z_2) < F_{12}^{\max} \\ 0 & F_{12}(Z_1, Z_2) \geq F_{12}^{\max} \end{cases} \tag{10}$$

Let the minimum accepted degree of satisfaction for the lower problem be ρ''.

Let z_{11}, z_{12}, z_{13} and z_{14} be the decision variables controlled by the decision maker at the upper level. Let θ_i^1 and θ_i^2 be the maximum and minimum tolerance limits for z_{1i} (i = 1, 2, 3, 4). Here, i denotes the number of variables controlled by the upper level problem.

Define the membership functions for z_{1i} (i = 1, 2, 3, 4) as

$$\mu(z_{1i}) = \begin{cases} \dfrac{z_{1i} - \left(z_{1i}^F - \theta_i^2\right)}{\theta_i^2}, & z_{1i}^F - \theta_i^2 \le z_{1i} \le z_{1i}^F \\ \dfrac{\left(z_{1i}^F + \theta_i^1\right) - z_{1i}}{\theta_i^1}, & z_{1i}^F \le z_{1i} \le z_{1i}^F + \theta_i^1 \end{cases} \tag{11}$$

Let the minimum accepted degree of satisfaction for the decision variables z_{1i} be denoted by ρ_3^i $(i = 1, 2, 3, 4)$. Let $\eta_1 = \min(\rho', \rho'', \rho_3^i)$.

Utilising the fuzzy membership functions defined above ((9)–(11)), the Fuzzy programming problem (FPBTPP) is determined as follows:

$$\max \eta_1$$

subject to

$$\mu\left(F_{11}(Z_1, Z_2)\right) \ge \eta_1 \tag{12}$$

$$\mu\left(F_{12}(Z_1, Z_2)\right) \ge \eta_1 \tag{13}$$

$$\mu(z_{1i}) \ge \eta_1, \ i = 1, 2, 3, 4 \tag{14}$$

$$\begin{aligned} &\sum_{t \in T'} z_{st} \le R_{p_s'}^1(u_s), \ s \in S' \\ &\sum_{s \in S'} z_{st} \ge R_{q_t'}^3(a_t), \ t \in T' \end{aligned} \tag{15}$$

$$\begin{aligned} &\sum_{t \in T''} z_{st} \le R_{p_s''}^2(v_s), \ s \in S'' \\ &\sum_{s \in S''} z_{st} \ge R_{q_t''}^4(b_t), \ t \in T'' \end{aligned} \tag{16}$$

$$z_{st} \ge 0, \ \forall\, s \in S, \ t \in T \tag{17}$$

$$0 \le u_s \le m_s - 1,\ s = 1, 2, ..., k_1,\ 0 \le a_t \le n_t - 1,\ t = 1, 2, ..., j_1$$

$$0 \le v_s \le x_s - 1,\ s = k_1 + 1, ..., k,\ 0 \le b_t \le y_t - 1,\ t = \mathrm{j}_1 + 1, ..., j \tag{18}$$

$$u_s, v_s \in Z^+ \cup \{0\},\ s \in S,\ a_t, b_t \in Z^+ \cup \{0\},\ t \in T,\ \eta_1 \in [0, 1]$$

Solve fuzzy programming problem FPBTPP by LINGO 17.0. It determines the satisfactory solution for the defined problem BTPMCP.

3.1.1. Goal programming method

BMIPP can also be solved by the method of goal programming. To determine the goal programming model, the aspiration level for the objective functions at two levels is elucidated. It is given by $F_{11}^g = \dfrac{F_{11}^{\max} + F_{11}^{\min}}{2}$ and $F_{12}^g = \dfrac{F_{12}^{\max} + F_{12}^{\min}}{2}$. Let d_1^+ and d_1^- be the positive and negative deviational variables for the objective function at the upper level, and d_2^+ and d_2^- be the positive and negative deviational variables for the objective function at the lower level, respectively. The goal programming model is denoted as GPBTPMCP:

$$\max \eta_2$$

subject to

$$\begin{aligned} F_{11} - d_1^+ + d_1^- &= F_{11}^t \\ F_{12} - d_2^+ + d_2^- &= F_{12}^t \end{aligned} \tag{19}$$

$$\begin{aligned} \eta_2 - d_1^+ &\ge 0 \\ \eta_2 - d_2^+ &\ge 0 \end{aligned} \tag{20}$$

$$\begin{aligned} \sum_{t\in T'} z_{st} &\le R_{p_s'}^1(u_s),\ s \in S' \\ \sum_{s\in S'} z_{st} &\ge R_{q_t'}^3(a_t),\ t \in T' \end{aligned} \tag{21}$$

$$\begin{aligned} \sum_{t\in T''} z_{st} &\le R_{p_s''}^2(v_s),\ s \in S'' \\ \sum_{s\in S''} z_{st} &\ge R_{q_t''}^4(b_t),\ t \in T'' \end{aligned} \tag{22}$$

$$d_1^+ d_1^- = 0, \quad d_2^+ d_2^- = 0, \quad z_{st} \geq 0, \quad \forall\, s \in S, \quad t \in T$$

$$0 \leq u_s \leq m_s - 1,\ s = 1, 2, ..., k_1,\ 0 \leq a_t \leq n_t - 1,\ t = 1, 2, ..., j_1$$

$$0 \leq v_s \leq x_s - 1,\ s = k_1 + 1, ..., k,\ 0 \leq b_t \leq y_t - 1,\ t = j_1 + 1, ..., j \tag{23}$$

$$u_s, v_s \in Z^+ \cup \{0\},\ s \in S,\ a_t, b_t \in Z^+ \cup \{0\},\ t \in T,\ \eta_2 \in [0, 1]$$

GPBTPMCP is solved by computing software LINGO 17.0 and satisfactory solution is obtained for BTPMCP.

The following section describes the algorithm for procuring satisfactory solution for BTPMCP by the methods explained above, fuzzy and goal programming. A comparative analysis of the solutions so obtained from two methods can thus be contrived.

4. An algorithm for solving multi-choice bilevel transportation problem (BTPMCP)

Step 1. Consider a bilevel transportation problem, BTPMCP, with multiple choices in supply and demand parameters.

Step 2. The multi-choice supply and demand parameters in BTPMCP are transformed into interpolating polynomials. The polynomials are formulated by Lagrange's interpolation.

Step 3. Interpolation remodels the problem BTPMCP into a mixed integer programming problem BMIPP.

Step 4. To solve BMIPP, the membership functions are defined for the decision-makers at both levels as well as for the variables controlled by the upper-level decision-maker. The procured problem is a fuzzy programming problem, FPBTPP.

Step 5. FPBTPP is solved using LINGO 17.0. It establishes a satisfactory solution for the problem BTPMCP.

Step 6. The goal programming model, GPBTPMCP, is defined for BTPMCP by introducing the deviational variables for the objective functions at two levels. The aspiration levels are also described by exemplifying the maximum and minimum values of the objective functions at both levels. A satisfactory solution for the problem is obtained using LINGO 17.0.

Step 7. A comparative analysis is done for the satisfactory solutions obtained from two methods, FPBTPP and GPBTPMCP.

5. An illustrative example for the bilevel transportation problem with multi-choice parameters (BTPMCP)

Example. Supersonic, Ltd., having its registered office in Mumbai, is organising its Annual Summit in Bangalore. The shareholders attending this summit shall be coming from within the country and from outside. The company has arranged their boarding at resorts 1, 2, 3 and 4 with different categories of rooms available on the ground floor, first floor, and the second floor. The shareholders shall be coming from the international flights, national flights and AC first-class trains. Let z_{11} be the number of delegates taken from origin 1 (international airport) to resort 1, z_{12} be the number of delegates being taken from origin 1 to resort 2 and so on. Let z_{21} be the number of delegates taken from origin 2 (national airport) to resort 1 and so on. Let z_{31} be the number of delegates taken from origin 3 (railway station) to resort 1 and so on. The cost incurred in taking the delegates other than the company's employees from different origins to different resorts denoted by F_{11} is shown in Table 1.

Table 1. Cost at the upper level

Origin	Resort			
	1	2	3	4
1	15	25	10	28
2	22	17	20	45
3	22	28	12	32

Let F_{12} be the cost incurred in taking employees of the company coming from branches located nationally as well as internationally, shown in Table 2.

Table 2. Cost at the lower level

Origin	Resort			
	1	2	3	4
1	16	2	9	14
2	6	15	12	8
3	10	21	4	7

Different categories of rooms are available for guests in resorts 1–4. The rooms have been categorised based on their cost from highest to lowest as I to IV, respectively. The data is shown in Table 3.

Table 3. Different categories of rooms in four resorts

Floor	Category			
	IV	III	II	I
Ground	60	`62	64	68
First	–	45	47	50
Second	–	70	71	73

In Table 3, the total number of rooms of category I available on the ground floor in four resorts taken together is 68. Similarly, the total number of rooms of category I available on the first floor in all the four resorts is 50, and so forth.

Table 4 depicts different food packages for the guests resort wise. The packages include distinct popular food options.

Table 4. Available food packages in four resorts

Food package	Resort			
	1	2	3	4
A	18	15	20	24
B	20	12	21	26
C	17	18	24	28
D	22	–	22	27

The aim of the company is to minimize the cost of transportation of guests. They also intend

- to choose rooms and food packages in such a manner that the stay of guests' is comfortable,
- the overall expense of the company related to this meeting comes out to be minimum.

Solution. A bilevel transportation problem with multi-choice parameters (BTPMCP) is:

$$\min_{Z_1} F_{11}(Z_1, Z_2) = 15z_{11} + 25z_{12} + 10z_{13} + 28z_{14} + 22z_{21} + 17z_{22} + 20z_{23} + 45z_{24} + 22z_{31} + 28z_{32} + 12z_{33} + 32z_{34}$$

where Z_2 solves

$$\min_{Z_2} F_{12}(Z_1, Z_2) = 16z_{11} + 2z_{12} + 9z_{13} + 14z_{14} + 6z_{21} + 15z_{22} + 12z_{23} + 8z_{24} + 10z_{31} + 21z_{32} + 4z_{33} + 7z_{34}$$

subject to

$$z_{11} + z_{12} + z_{13} + z_{14} \le \{60, 62, 64, 68\}$$
$$z_{21} + z_{22} + z_{23} + z_{24} \le \{45, 47, 50\}$$
$$z_{31} + z_{32} + z_{33} + z_{34} \le \{70, 71, 73\}$$
$$z_{11} + z_{21} + z_{31} \ge \{18, 20, 17, 22\}$$
$$z_{12} + z_{22} + z_{32} \ge \{15, 12, 18\}$$

$$z_{13}+z_{23}+z_{33}\ge\{20,\ 21,\ 24,\ 22\}$$
$$z_{14}+z_{24}+z_{34}\ge\{24,\ 26,\ 28,\ 27\}$$
$$z_{k\ell}\ge 0,\ k=1,\ 2,\ 3,\ \ell=1,\ 2,\ 3,\ 4$$

Here, Z_1 = {z_{11}, z_{12}, z_{13}, z_{14}) are the variables controlled by the upper level and Z_2 = {z_{21}, z_{22}, z_{23}, z_{24}, z_{31}, z_{32}, z_{33}, z_{34}} are the variables controlled by the lower level. Using Lagrange's interpolating polynomials and fuzzy programming approach, the above problem (FPBTPP) is defined as

$$\max \eta_1$$

subject to

$$F_{11}\left(Z_1,\ Z_2\right)+4593\eta_1\le 5924$$
$$F_{12}\left(Z_1,\ Z_2\right)+2883\eta_1\le 3257$$

$$z_{11}+17\eta_1\le 17$$

$$z_{12}+12\eta_1\le 12$$

$$z_{13}+20\eta_1\le 20$$

$$z_{14}+8\ \eta_1\le 36$$

$$z_{11}+z_{12}+z_{13}+z_{14}\le\frac{1}{3}u_1^3-u_1^2+\frac{8}{3}u_1+60$$

$$z_{21}+z_{22}+z_{23}+z_{24}\le\frac{1}{2}v_1^2+\frac{3}{2}v_1+45$$

$$z_{31}+z_{32}+z_{33}+z_{34}\le\frac{1}{2}v_2^2+\frac{1}{2}v_2+70$$

$$z_{11}+z_{21}+z_{31}\ \ge\frac{13}{6}a_1^3-9a_1^2+\frac{53}{6}a_1+18$$

$$z_{12}+z_{22}+z_{32}\ \ge\frac{9}{2}a_2^2-\frac{15}{2}a_2+15$$

$$z_{13}+z_{23}+z_{33}\geq -\frac{7}{6}b_1^3+\frac{9}{2}b_1^2-\frac{7}{3}b_2+20$$

$$z_{14}+z_{24}+z_{34}\geq -\frac{1}{2}b_2^3+\frac{3}{2}b_2^2+b_2+24$$

$$0\leq u_1\leq 3,\ 0\leq v_1\leq 2,\ 0\leq v_2\leq 2$$

$$0\leq a_1\leq 3,\ 0\leq a_2\leq 2,\ 0\leq b_1\leq 3,\ 0\leq b_2\leq 3$$

$$u_1, v_1, v_2\in Z^+\cup\{0\},\ a_1, a_2, b_1, b_2\in Z^+\cup\{0\},\ \eta_1\in[0, 1]$$

Solving the problem (FPBTPP) by LINGO 17.0, we get $\eta_1 = 0.944$, $z_{11} = 0$, $z_{12} = 0$, $z_{13} = 0$, $z_{14} = 0$, $z_{21} = 17$, $z_{22} = 12$, $z_{23} = 0$, $z_{24} = 0$, $z_{31} = 0$, $z_{32} = 0$, $z_{33} = 20$, $z_{34} = 24$, $u_1 = 1$, $v_1 = 1$, $v_2 = 1$, $a_1 = 2$, $a_2 = 1$, $b_1 = 0$, $b_2 = 0$, $F_{11}(Z_1, Z_2) = 1586$ and $F_{12}(Z_1, Z_2) = 530$.

The most sought after rooms in the four resorts are: category III rooms on the ground floor (62), category II rooms on the first floor (47), and second floor (71). The food packages opted by the company for guests are: package C in resort 1, package B in resort 2, and package A in resorts 3 and 4. Thus, the cost of transportation of guests at the upper level is 1586 and at the lower level is 530, provided the company adopts the combination of rooms and food packages derived as above.

The numerical example is also solved by goal programming and the technique described in Khalil's paper [16]. A comparative analysis is done (Table 5).

Table 5. Comparative analysis

Method of solving	Objective function values		Elapsed run time (LINGO 17.0)	Total number of iterations
	Upper level: F_{11}	Lower level: F_{12}		
Fuzzy programming	1586	530	1.73 sec	1823
Goal programming	3627	1815	0.05 sec	104
By Khalil [16]	1605	540	0.28 sec	121

From Table 5, it can be observed that the minimum cost of the objective functions at two levels is obtained by the proposed fuzzy programming approach.

Remarks. The above example deals with the guests of Super Sonic, Ltd. The lodgers in four resorts other than company's delegates or employees are not taken into consideration. The actual data has not been considered in the above example.

6. Conclusions

Multi-choice optimisation emerges as a result of various parameters, such as variation in climatic conditions, options of freight transport, driving conditions, differential labour cost, to name a few. This paper aims to solve the bilevel transportation problem having varied choices in every aspect of life. In today's world, variety is in every nook and corner, be it food, clothing, accessories or the mode of transport. Thus, it results in multiple choices in supply and demand parameters. The present model has been applied to the hotel industry providing several attractive options to the consumers. This scenario can be exercised by a hotel/resort which has to cater to the lodging preferences of different people and their dining tastes. Different people have different tastes and peculiar residing preferences. The paper proposes an analytical approach for multi-choice problems for the hotel industry. This approach procures a satisfactory solution for both the decision-makers at two levels. Therefore, this novel approach is presented by the authors. The computing software LINGO 17.0 is used for calculations. The problem stated in this paper is also solved by the goal programming approach and the method proposed by Khalil et al. [16]. It has been observed from a comparative analysis that the cost at two levels is minimum when the problem is solved by the procedure of proposed fuzzy programming.

Although interpolating polynomials are used in this paper by Lagrange interpolation, the multi-choice parameters can also be dealt with the binary variable approach and linear least square approximation approach. Also, cost coefficients in the objective functions as multi-choice parameters could be examined for future study. The concept of multi-choice programming has been applied to the bilevel transportation problem. It can also be extended to the multi-level transportation problem. The example described in the paper is solved by LINGO 17.0, whereas other readily available software can also be used to determine the satisfactory solution for BTPMCP. Uncertainty is another aspect that can be taken care of in the transportation problem.

Acknowledgements

The authors express their gratitude to the reviewers for their valuable suggestions, which has helped us in improving the quality of the paper to a great extent.

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