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MULTICRITERIA RANKINGS OF OPEN-END INVESTMENT FUNDS AND THEIR STABILITY

For research purposes, three multicriteria outranking methods (PROMETHEE, WSA and TOPSIS) were used to construct rankings of investment funds to assess their performance in the time period from January to July 2008. Nine indicators related to the distributions of return rates, purchase and management costs and to customers' convenience were included in the set of criteria. The weight of each criterion was calculated on the basis of the relative volatility rate of the given criterion.

In order to assess the stability of the rankings, the weight of a single criterion was changed (using each criterion in turn) and new rankings were constructed using the modified weights. The similarity of rankings built before and after these changes was assessed on the basis of the maximum difference between ranks and the Spearman correlation coefficient.

The results obtained enable assessment not only of the stability of each outranking method, but the similarity of results obtained by different methods as well.

All calculations were done using the SANNA software.

Key words: *investment funds, outranking methods, PROMETHEE method, WSA method, TOPSIS method, stability of rankings*

1. Introduction

Investment funds stimulate economic growth, providing capital for the development of enterprises and allowing employment to increase. Their capital allows financing public debt, prevents excessive inflation and provides capital for the needs of banks and local authorities [33]. Investment funds are popular among investors, due to the form of capital allocation provided. Collective investment ensures higher safety than in individual cases, because, due to the policy of diversification, the spectrum of assets is broad enough to reduce risk. Therefore, a decrease in the value of one asset is reflected by only a minimal change in the value of the full portfolio. Funds provide

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other advantages e.g. relatively low transaction costs, the feeling of safety resulting from professional management [1] and the psychological convenience resulting from the delegation of management [23]. They also allow capitalizing a chosen amount of shares at a chosen moment.

Due to the decreasing inflation rate, good economic situation and development of financial markets, together with legal and organizational rules regulating their activity, funds became increasingly popular among investors [35]. The open-end funds analyzed in this study are one of the most homogeneous groups of funds. Their assets are located mainly in shares listed on the Warsaw Stock Exchange and only a small amount is located in instruments of short term debt [7]. Despite some differences in the investment policies declared, the gains of all these funds are strictly connected to the stock indices. Because the WIG index has decreased significantly from the third quarter of 2007, the balances of investment companies were negative for the first time in a very long period. This has resulted not only from the withdrawal of capital but also from the decreasing interest in this form of investment in favor of structured products and bank deposits.

The great number of funds operating on the Polish market requires the elaboration of specialized tools and measures allowing decision-makers to compare them and select the best alternative. Furthermore, because of the financial crisis, both the selection of well managed funds and evaluation of their activities are important. One of the most popular methods of comparing funds is based on rankings constructed using one or more criteria. There are two goals of such analysis. Firstly, it allows investors to compare the funds analyzed. Secondly, it builds the reputation of highly classified funds.

Multicriteria methods [11]–[13], [19] have been successfully used for capital market and investment analysis (see [40], [43]). The multicriteria evaluation of investment funds (and hedge funds) was discussed and presented e.g. in [4], [8], [10], [30], [32], [34]. When selecting criteria, it is considered that the success achieved by a fund depends on such characteristics as: average return rates, risk, additional costs, size of fund, minimum value of the initial investment [7], [27], [29], [36]. Decision-makers are also interested in the quality of service [8] and reputation of managers [30]. In [32] funds are classified e.g. on the basis of their return rates, SHARPE [37], [38], TREYNOR [42] coefficients and quantile based measure VaR [18].

Because of the different relative importance of criteria to different investors, one of the major problems in the approach of multicriteria outranking lies in the determination of weights to guarantee the utility of a ranking to potential investors. In a multicriteria approach, weights and parameters are used in order to reflect and describe a decision-maker's preferences. However, it should be noted that in some situations these preferences are unknown. It is also argued that in cases in which many criteria are simultaneously introduced into a study, the decision-maker is confronted with too much information for such an approach to be used effectively [28]. Determining weights in such cases is complicated and depends on the analyst. This choice can be based e.g. on expert knowledge.

In this paper we assume that a decision-maker's preferences result from a real differentiation between criteria [45]. It should be noted that the values of the variables used in the criteria can be treated as sources of information and therefore the weights should reflect this information. As pointed out in [9], the same argumentation, when reversed, leads to the conclusion that any criterion according to which all alternatives are identically evaluated introduces no additional information into the analysis and therefore is useless in the decision making process. For example, if all investments were characterized by the same return rate and differed only according to the risk criterion, the investor would probably pay greater attention to the risk measure, irrespective of his degree of risk aversion. Weights determined according to the differentiation between alternatives according to criteria can be interpreted as a description of the preferences of an investor who only considers available and comprehensible data on the characteristics of the objects compared. In this paper, a ranking constructed using these weights will be referred to as an *initial ranking*.

Determining weights is the crucial part of the analysis [44], because it is possible that even a small change in these weights leads to substantially different results. Therefore, it is advisable to analyze the stability and sensitivity of the rankings obtained. The problem of sensitivity was discussed e.g. in [3], [22], [24], [41], [44]. Usually, such analysis is focused on the following dimensions:

– changes in ranking resulting from modifications of the data set,

– changes in ranking resulting from a different evaluation of a given alternative according to a chosen criterion,

– minimum modifications of weights required to make a given alternative become ranked first,

– maximum changes in weights not resulting in a change in the ranking.

In this paper stability is defined on the basis of the maximum difference between ranks based on an initial ranking and a ranking built using modified weights.

We constructed synthetic rankings of open-end investment funds operating on the Polish market and assessed their stability by considering changes in weighting vectors. Each multicriteria method used (PROMETHEE – [5, 6], TOPSIS – [15], WSA – [20][1]) is comprehensible to a decision-maker and, as a consequence, he is more ready to accept the results obtained. What is more, each method represents a different approach to the construction of a ranking.

Rankings of alternatives were constructed on the basis of chosen criteria. The weight of each criterion was determined using a taxonomic approach, based on volatility coefficients. In the stability analysis, these initial weights were modified. New rankings, constructed after such a change, can be regarded as results obtained by an individual who assigns greater importance to a given criterion for subjective reasons.

[1] The WSA method presented in this paper is a particular case of the SAW method (Simple Additive Weighting – [20]).

Comparison of rankings gives an overview of the stability of a given multicriteria method.

This paper is organized as follows: In the first section we discuss multicriteria methods used for constructing rankings. Further, we present the set of criteria chosen for the analysis and determine initial weighting vectors. The final section is focused on the obtained rankings of investment funds and assessment of their stability. We comment on the outcome of the comparison and briefly sum up the main results.

2. Outranking methods

Because of the above-mentioned assumption concerning a decision-maker's preferences, we decided to construct rankings using those multicriteria methods that are based on diversity according to a criterion.

The multicriteria methods used in this study were chosen from the best known in such a way that allowed us to create a set of differentiated rules from a mathematical point of view, which are simultaneously algorithmically uncomplicated, comprehensible and easy to interpret for a decision maker without a quantitative background. It should be noted that these features of the methods chosen lead to a higher readiness to accept the results obtained, which is especially important considering the measurable losses and gains from decision problems in the fields of finance and investment. The fact that these methods are intuitive can be considered as an advantage, especially in view of the second part of this study i.e. assessment of stability on the basis of varying the relative importance of criteria.

Methods from the PROMETHEE family rank alternatives on the basis of pairwise comparisons. Differences between the evaluations of two different alternatives according to a given criterion are taken into consideration when determining the weights. We assume that the smaller the difference between variants according to a chosen criterion, the weaker the preference for the best alternative according to this criterion. It should be noted, that the parameters in this method can be used not only to describe preferences, but also to reflect the quality of data [11]. The choice of one of six types of generalized criteria (see: [11]) allow modeling preferences with a given function. A rich set of generalized criteria, enabling elastic modeling of the decision-makers' preferences has turned out to be an attractive approach, which has been successfully implemented in optimization tasks of different types (e.g. goal programming, see: [26]). We should also note that this method is easy to interpret, as all the parameters used in the analysis are significant and their meaning is clear to the decision-maker.

In this study we used the PROMETHEE II method, because of the possibility to construct a complete ranking, and the generalized Gaussian criterion, due to the fact that it does not require additional assumptions concerning a decision-maker's preferences, but highlights the natural diversity between alternatives according to the criteria.

The TOPSIS method (Technique for Order Preference by Similarity to an Ideal Solution) is based on the distance of the given alternative from two reference points (ideal and worst case solutions to the multicriteria problem). The decision maker's preferences are introduced into the analysis in the form of the weights and parameters used to calculate these distances. The algorithm proposed is comprehensible and intuitive. However, the lack of the possibility of introducing the equivalence of criteria within the given boundaries can be considered as a disadvantage of this method. Secondly, this method does not include relative weights for the two reference points [31]. It is assumed that in all cases the avoidance of failure is as important to the decision-maker as the achievement of success.

The WSA method (Weighted Sum Approach) is one of the less complicated methods of constructing rankings. The comparability of evaluations according to the criteria is guaranteed by standardization. In the next step, a ranking is constructed on the basis of the weighting vector. WSA is a compensatory approach, which allows an alternative which was highly evaluated only according to one criterion to obtain a high ranking, even though the evaluations according to other criteria were low. When the ranking of investment funds is considered, this feature can be regarded as an advantage, due to the fact that gains from investment (i.e. a high return rate) may compensate the costs resulting from fees required by the fund.

The simplicity of this method is one of its main advantages. Moreover, because the ranking is based to a great degree on the differentiation of alternatives according to the criteria, the weight of criteria according to which there is little differention is lower. This method can be regarded as an algorithm ranking alternatives with the minimum contribution from the decision-maker.

It has been noted that in some cases the WSA and PROMETHEE methods lead to very similar results [14]. Because of the possibility of constructing similar rankings, the question arises as to how those rankings would react to changes in the weights, in particular, whether their similarity would be preserved in this case.

3. Criteria

We considered two groups of criteria for the comparison of investment funds[2].

[2] We used data from the period January – June 2008.

The first set of criteria is associated with the investment risk which results from the volatility of the return rate[3]. Because of the asymmetric distribution of return rates, apart from the *expected value* and *standard deviation* calculated as square root of variance [25], we considered risk measures based on higher central moments.

Skewness indicates the relative likelihood of achieving gains significantly higher or lower than the mean. A rational decision-maker chooses investments characterized by a high value of this measure (see also [2]). *Kurtosis* is a measure of the concentration around the mean. It informs us about the possibility of extreme values of return rates. A high kurtosis means that the variance is strongly influenced by infrequent, but extreme, deviation. Therefore, a rational decision-maker chooses investments characterized by a low value of this measure. Both these measures are used to assess investment risk not only in the case of funds but shares as well [30], [36]. The last criterion included in this group is the *0.05 percentile of the return rate*.

The second group consists of criteria associated with investment costs: the minimum required value of the first input, minimum required value of following inputs and management fee.

These criteria are regarded as constraints for the clients of investment funds. A high *minimum required value of the first input* can be regarded as a barrier to entry for some investors. The alternatives were most differentiated according to this criterion. The *minimum required values of following inputs* were less differentiated, but a high value here can also make it impossible for investors to invest a chosen amount of capital at a given moment. Although a fund offers shares, the decision-maker has to wait until he gathers an appropriate sum.

A separate criterion is the *number of methods of placing orders* as factor reflecting convenience to investors. The funds analyzed offered various methods of placing orders: not only directly at a customer service point, but also by transfers, phone, fax or the Internet. It is clear that the use of remote methods allows more rapid placement of orders. The majority of the funds offered 4 or 5 methods.

The multicriteria methods presented in the previous section require determining the weighting vector essential in determining the relative importance of criteria. In this paper, we used a taxonomic approach and on the basis of [21] derived the weights by taking into consideration the natural differentiation according to each criteria as measured by the volatility coefficient defined by formula (1).

$$v_i = \frac{s_i}{\bar{f}_i}, \tag{1}$$

[3] A 5-day investment period was used to reduce the impact of spurious deviations on the results of the analysis.

where:

s_i – standard deviation according to criterion i,

$\bar{f}_i$ – mean according to criterion i,

$i = 1, ..., 9$.

The values of the weights were derived as follows:

$$w_i = \frac{|v_i|}{\sum_{i=1}^{9} |v_i|}. \tag{2}$$

In consequence, criteria according to which alternatives were more diversified were assigned higher weights. This method is intuitive as when variants are only slightly differentiated according to a given criterion, the decision-maker should not ascribe great importance to it [9]. The weights suggested by us are a good starting point for stability analysis, because subsequent changes can be interpreted as reflecting a stronger (or weaker) preference for a given criterion as compared with the level used for the initial ranking. Therefore, the modified ranking is the result obtained by an investor for whom a given criterion is more (less) important than stems purely from differentiation according to it. The similarity of rankings before and after a change in weights can be considered to be the similarity of the results obtained by two individuals: an investor considering only differentiation according to the criteria and one assessing funds in a way described by modified weighting vectors.

The initial weighting vector is presented in table 1.

Table 1. Initial weighting vector

Expected value	Standard deviation	Skewness	Kurtosis	Minimum required value of the first input	Minimum required value of following inputs	Number of methods of placing orders	Management fee	0.05 percentile of return rate
0.02065	0.01362	0.11812	0.07186	0.50966	0.21579	0.01803	0.01753	0.01474

Source: author's own calculation.

The minimum values of inputs were the factors that most greatly differentiated open-end investment funds. In the period analyzed, the effectiveness of investments measured by the moments of 5-day return rates was more similar. The greatest differences in the first group of criteria were noted in the cases of the *skewness* and *kurtosis* criteria, which emphasizes the necessity of including measures based on higher moments in the study.

4. Stability of rankings

In this section we present the initial rankings of open-end investment funds and assess their similarity. In the next step we describe the method of modifying weighting vectors in the analysis of stability and comparison of the results obtained.

The initial rankings were constructed using the three abovementioned outranking methods and initial weighting vector presented in table 1. The set of alternatives was restricted to those open-end investment funds for which the data was complete. Therefore, we constructed rankings of 47 objects. The results are presented in table 2.

It should be noted that a set of funds tend to obtain high ranks regardless of the method used. Moreover, the same funds were ranked in the last 3 positions in all cases.

Table 2. Initial rankings of open-end investment funds

	PROMETHEE	TOPSIS	WSA
1	2	3	4
AIG FIO Akcji	29	25	31
AIG FIO Małych i Średnich Spółek	37	22	37
AIG SFIO Parasol Pod. - SUB Akcji	34	36	34
Allianz FIO – subfundusz Allianz Akcji	9	20	9
Allianz FIO – subfundusz Allianz Akcji MiŚ Spółek	5	11	6
Allianz FIO – subfundusz Allianz Akcji Plus	7	15	7
Allianz FIO – subfundusz Allianz Budownictwo 2012	8	12	10
Arka BZ WBK Akcji FIO	18	21	20
BPH SUB Akcji	27	27	26
BPH SUB Akcji Dynamicznych Spółek (PLN)	40	32	40
BPH SUB Akcji Dynamicznych Spółek (USD)	44	18	44
Commercial Union FIO - SUB CU Polskich Akcji	32	39	32
DWS Polska FIO Akcji	14	16	13
DWS Polska FIO Akcji Plus	26	10	28
DWS Polska FIO Akcji Spółek Eksportowych	6	1	2
DWS Polska FIO Top 25 Małych Spółek	21	7	23
Fortis FIO – subfundusz Fortis Akcji	4	5	5
Idea Akcji FIO	3	8	4
ING FIO Akcji	11	13	11
ING FIO Średnich i Małych Spółek	12	6	12
ING Parasol SFIO – SUB Akcji Plus	24	14	25
ING SFIO Akcji 2	47	47	47
KBC Akcji Małych i Średnich Spółek FIO	39	37	39
KBC Parasol FIO – subfundusz Akcyjny	33	41	35
KBC Portfel VIP SFIO Akcyjny	45	45	45
KBC Portfel VIP SFIO-Akcji Średnich Spółek	46	46	46
Legg Mason Akcji FIO	22	35	21
Millennium FIO SUB Akcji	31	26	30

Table 2 continued

1	2	3	4
Millennium FIO SUB Małych i Średnich Spółek	23	9	22
Noble Funds FIO – subfundusz Noble Fund Akcji MSP	38	42	38
Pioneer Akcji Polskich FIO	28	24	27
Pioneer Małych i Śred. Spółek Rynku Polskiego FIO	41	33	41
Pioneer Średnich Spółek Rynku Polskiego FIO	42	34	42
PKO/CS Akcji Małych i Średnich Spółek – fio	10	4	8
PZU FIO Akcji KRAKOWIAK	35	38	33
PZU FIO Akcji Małych i Średnich Spółek	43	44	43
SKARBIEC FIO – subfundusz AKCJA	30	43	29
SKARBIEC TOP – mBank Agresywny	16	29	16
SKARBIEC TOP – MultiBank Agresywny	15	30	15
SKARBIEC TOP – SFIO BRE Private Banking Agresywny	13	28	14
SKARBIEC-MAŁYCH I ŚREDNICH SPÓŁEK FIO	25	19	24
SKARBIEC-TOP Funduszy Akcji SFIO	17	31	17
SKOK FIO Akcji	36	40	36
UniFundusze FIO - SUB UniAkcje ME 2012	1	2	1
UniFundusze FIO - SUB UniAkcje MiŚS	2	3	3
UniFundusze FiO - SUB UniKorona Akcje	19	17	18
UniFundusze FIO - SUB UniMAXAkcje	20	23	19

Source: author's own calculations.

The similarity of rankings was assessed on the basis of Spearman's rank correlation coefficient [39], defined as:

$$r_s = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2-1)}, \tag{3}$$

where n denotes the number of alternatives and d_i the difference between the ranks of alternative *ai* in the pair of rankings compared.

A comparison of rank correlation coefficients is presented in table 3[4]. The most similar results were obtained by the PROMETHEE and WSA methods. This similarity is a consequence of the choice of the generalized Gaussian criterion, which resulted in both methods using the information originating from the differentiation according to criteria in a similar way. Therefore, we may assume that the results of the analysis of stability for these two methods will be similar as well.

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*** – coeffcients signifficant for $\alpha = 0.01$,
** – coeffcients signifficant for $\alpha = 0.05$,
* – coeffcients signifficant for $\alpha = 0.1$.

Table 3. Correlations between initial rankings

Method	PROMETHEE/TOPSIS	PROMETHEE/WSA	TOPSIS/WSA
r_s	0.7894***	0.9963***	0.7864***

Source: author's own calculations

In order to assess the stability and sensitivity of rankings to changes in weights (preferences), in the next step we gradually modified weighting vectors and iteratively increased weights in the following way:

$$w_k^+ = \frac{|v_k^+|}{\sum_{i=1}^{9} |v_i^+|}\ , \tag{4}$$

where:

$$v_i^+ = \begin{cases} v_i \cdot \left(1 + \dfrac{j}{10}\right), & i = k, \\ v_i, & i \neq k, \end{cases} ,$$

$j \in \overline{1,90}$ – iteration number.

Due to the fact that the weights were normalized after each iteration, as the importance of the i-th criterion increased the importance of the others gradually decreased. However, the structure of the weights resulting from differentiation according to criteria was still preserved. After each change, a new ranking was constructed and the correlation with the initial ranking assessed.

In an analogous way, we studied the similarity of rankings in the situation in which the decision-maker ascribes a weaker relative importance to the given criterion than results from formula (2). The iterative formula in this case is the following:

$$w_k^- = \frac{|v_k^-|}{\sum_{i=1}^{9} |v_i^-|}\ , \tag{5}$$

where:

$$v_i^- = \begin{cases} v_i \cdot \left(1 - \dfrac{j}{10}\right), & i = k, \\ v_i, & i \neq k, \end{cases}$$

$j \in \overline{1,9}$.

The range of weights was determined so that it would be possibly broad, but not lead to excessive overrating (underrating) of weights. Both the highest and lowest weights used for each criterion are presented in table 4.

Table 4. Range of weights

	Minimum	Initial value	Maximum
Expected value	0.00210	0.02093	0.17414
Standard deviation	0.00138	0.0138	0.1213
Skewness	0.01322	0.1197	0.57253
Kurtosis	0.00768	0.07282	0.43637
Minimum required value of the first input	0.09415	0.51652	0.91224
Minimum required value of following inputs	0.02678	0.21869	0.73345
Number of methods of placing orders	0.00183	0.01828	0.15516
Management fee	0.00178	0.01777	0.15143
0.05 percentile of return rate	0.00149	0.01474	0.13017

Source: author's own work.

For the purpose of this paper we define stability as follows:

Ranking *RB* has stability of order *s* with respect to changes in weights if $\max_i(d_i) = s$, where $d_i = d_{i,B} - d_{i,Z}$ denotes the difference between the ranks of alternative a_i in the initial ranking R_B and a ranking R_Z constructed using modified weights. A ranking with stability of order 0 is referred to as *strictly stable*.

Which values of the abovementioned measure *s* are undesired is a decision-maker's individual choice. This might depend on the number of objects ranked and the values of the correlation coefficients or result from individual preferences or constraints on the investor. Rankings with a low stability of order generate very similar rankings for different decision-makers, whereas those of a high stability of order are more prone to changes due to modifications of the weights (and therefore, according to the assumptions made – changes of preferences). In the case of business problems, where a decision should not depend significantly on the personal traits of the decision-maker, but ought to be based on objective economic data, rankings with a low stability of order should be used. If an individual investor knows that the ranking of investment funds has a low stability of order, he would be more ready to trust such a hierarchy because it is very likely that in the case of small changes in preferences, he will obtain a very similar outcome.

For example, in the case discussed in this study the value of the *s* measure after an increase in the weight of the *standard deviation* criterion can be interpreted as follows: an investor with a greater aversion to risk ranked funds to within *s* places of the initial ranking.

This approach can be used for different types of rankings.

4.1. Results for increasing weights

In the first step we assessed the stability of rankings using weights modified according to formula (4).

The results presented in figure 1 indicate that the stability of order of the PROMETHEE and WSA methods is similar. The differences between the *s* measures for these two methods do not exceed 8 with an average of 3.756. It should be noted that for up to 30 iterations the TOPSIS method has a lower stability of order than WSA and PROMETHEE. For modifications of the weights within these boundaries, the positions of funds in a TOPSIS ranking do not change as much as for rankings constructed using other methods. For the *expected value* criterion this feature is preserved below a 70% increase in the weight of this criterion, whereas for the *skewness*, *0.05 percentile* and *standard deviation* criteria no ranking is strictly stable.

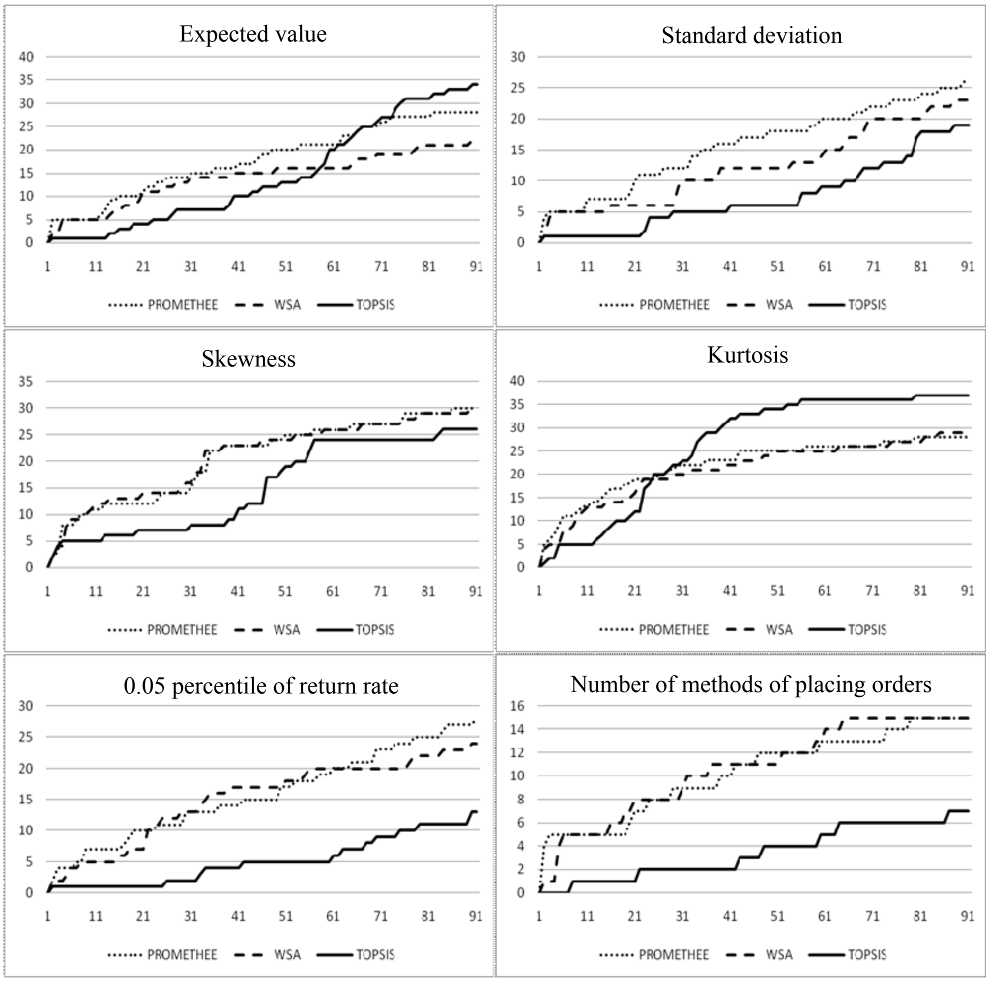

Fig. 1. Values of the *s* measure in consecutive iterations for the first group of criteria and the *number of methods of placing orders* criterion (increasing weights).
Source: author's own work

Note that for the abovementioned criteria, rankings are most sensitive to changes in the relative importance of the *kurtosis* criterion. Moreover, the values of the stability measure suggest that the range of weights were appropriately determined. We may expect that extending it would result in an even greater increase in the *s* measure and consequently in great inconsistency between the initial ranking and the new ranking.

The results obtained for the *number of methods of placing orders* criterion were very similar. However, in this case the TOPSIS ranking remained strictly stable for 6 iterations.

In table 5 we present the minimum values of the rank correlation coefficients for the criteria from the first group. Despite the high values of the *s* measure in the last iterations, the rank correlation coefficients remained high and statistically significant. On this basis we may assume that assessment based only on r_s coefficients is incomplete, because even if rankings generally remain similar, due to individual large shifts they may not be acceptable to investors.

Table 5. Minimum values of the r_s coefficients for the first group of criteria and the *number of methods of placing orders* criterion (increasing weights)

	Expected value	Standard deviation	Skewness	Kurtosis	Minimum required value of the first input	Minimum required value of following inputs
PROMETHEE	0.8005***	0.8292***	0.7858***	0.7636***	0.7684***	0.9199***
WSA	0.8363***	0.8388***	0.7872***	0.7617***	0.8451***	0.9170***
TOPSIS	0.7987***	0.8920***	0.9457***	0.4521***	0.9689***	0.9899***

Source: author's own work.

Stability analysis for the second set of criteria leads to different conclusions. In this group the similarity between the WSA and PROMETHEE methods is not noticeable, except in the case of the *minimum required value of following inputs* criterion. The stability of order measure *s* is significantly higher than in the previous case. It is also easier to determine the boundaries within which the maximum change of ranks remained constant.

The rank correlation coefficients in this case are statistically significant and (in most cases) very high.

Table 6. Minimum value of r_s for the second group of criteria (increasing weights)

	Minimum required value of the first input	Minimum required value of following inputs	Management fee
PROMETHEE	0.9983***	0.9608***	0.8126***
WSA	0.9868***	0.9341***	0.8430***
TOPSIS	0.9987***	0.8607***	0.5103***

Source: author's own work.

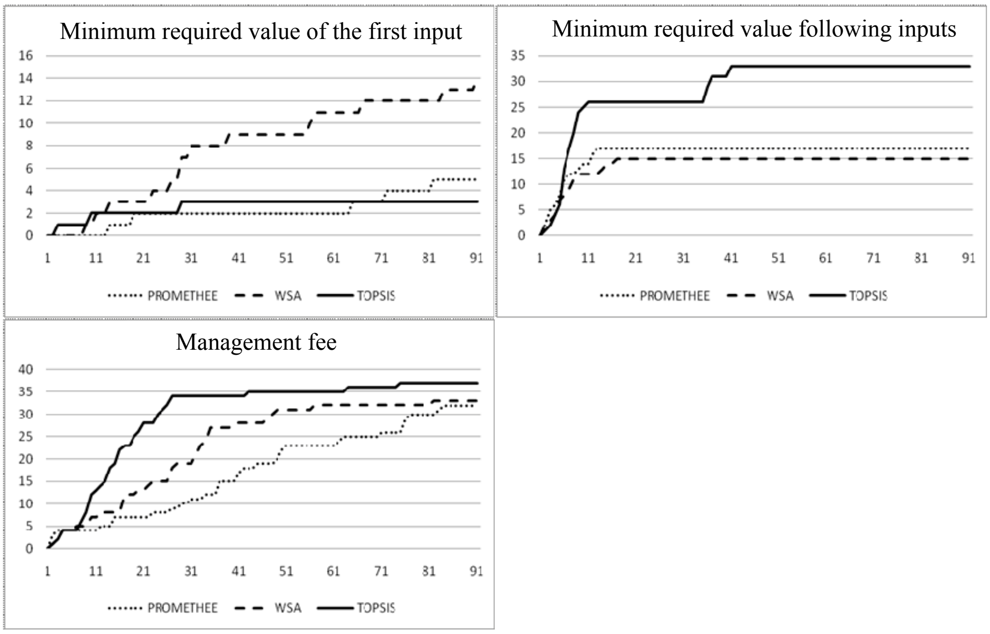

Fig. 2. Values of the *s* measure in consecutive iterations for the second group of criteria (increasing weights). Source: author's own work

4.2. Results for decreasing weights

In this section we present the results obtained by modifying the weights as described by formula (5).

It should be noted, that for the first group of criteria and the *number of methods of placing orders* criterion in most of the cases the rankings constructed using the TOPSIS method were strictly stable. Values of the measure *s* were lower than in the case of increasing weights. The only exception is the *skewness* criterion, for which all the methods give rankings of a similar stability of order.

The rank correlation coefficients remained high and significant. A statistically significant similarity to the initial ranking was preserved, except in the case of the *skewness* criterion using the TOPSIS method.

In the case of the second group of criteria, a similarity can be seen between WSA and PROMETHEE. For the changes in the weights of the *minimum required value of the first input* criterion, the values of the stability measure for rankings are lower than in the case of increasing weights. The only exception is the TOPSIS method for which the stability of order measure values are higher.

Table 7. Minimum values of r_s coefficients for the first group of criteria and the *number of methods of placing orders* criterion (decreasing weights)

	Expected value	Standard deviation	Skewness	Kurtosis	0.05 percentile of the return rate	Number of methods of placing orders
PROMETHEE	0.9927^{***}	0.9970^{***}	0.7246^{***}	0.9594^{***}	0.9978^{***}	0.9960^{***}
WSA	0.9935^{***}	0.9966^{***}	0.7300^{***}	0.9757^{***}	0.9968^{***}	0.9951^{***}
TOPSIS	1^{***}	1^{***}	0.2373	0.9979^{***}	1^{***}	0.9999^{***}

Source: author's own calculations

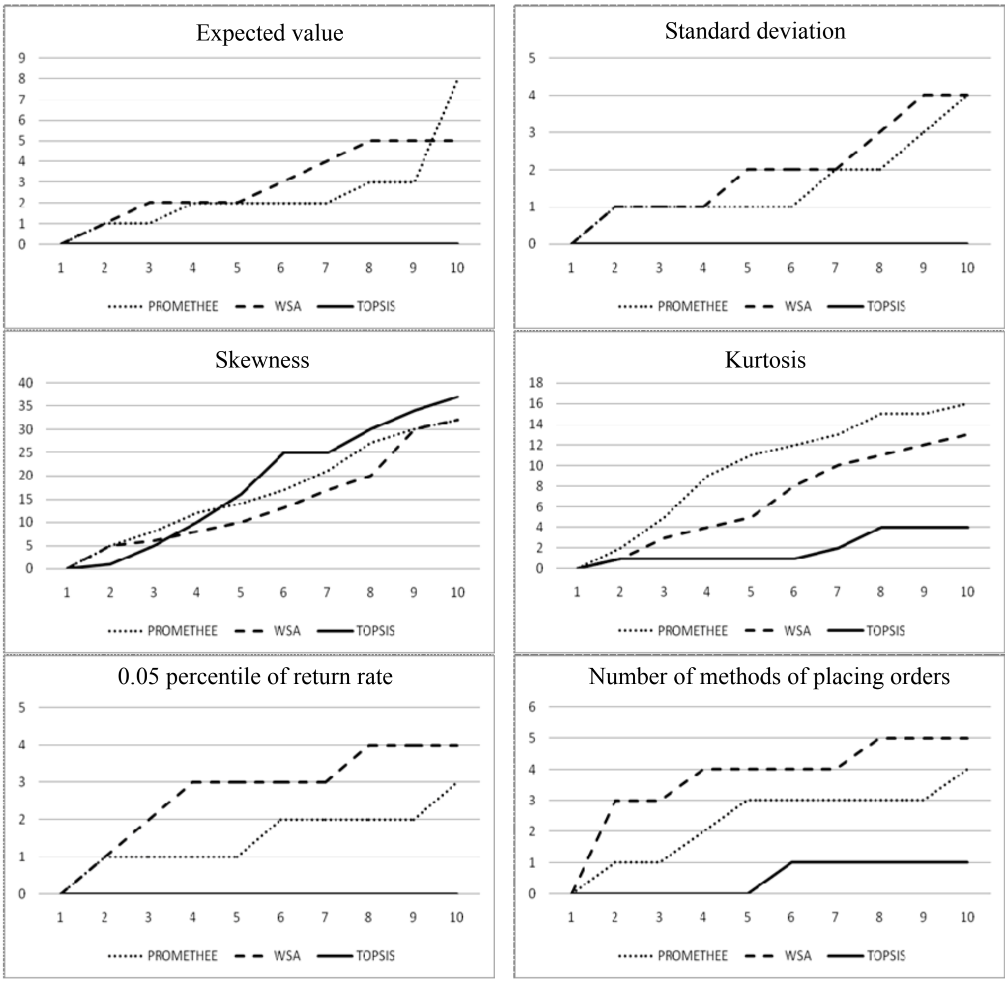

Fig. 3. Values of the *s* measure in consecutive iterations for the first group of criteria and the *number of methods of placing orders* criterion (decreasing weights). Source: author's own calculations

The situation is the reverse in the case of the *minimum required value of following inputs* criterion. Note that whereas the TOPSIS method has the lowest stability of order in the case of the modifications discussed in section 4.1, in this case it is the least stable method up to the 5th iteration. For this criterion, in the early iterations the differences in the stability of order values are insignificant, whereas in later iterations the TOPSIS method remains stable while in the case of WSA and PROMETHEE the stability of order measure increases rapidly.

In the case of the *minimum required value of the first input* criterion, all the rankings were strictly stable in the first iterations and this feature was longest preserved by the PROMETHEE method.

Again, the values of the rank correlation coefficients were large and statistically significant.

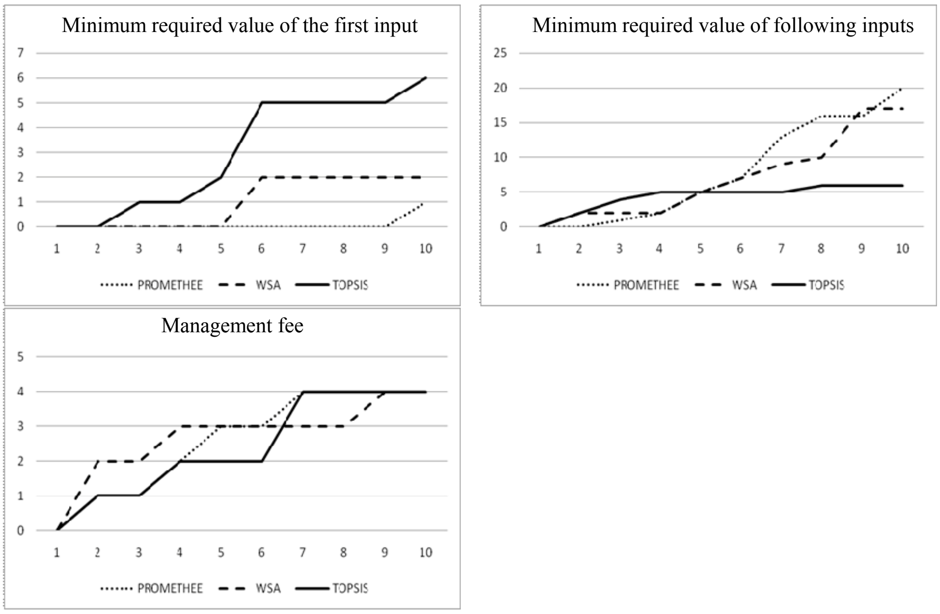

Fig. 4. Values of the *s* measure in consecutive iterations for the second group of criteria (decreasing weights)
Source: author's own calculations

Table 8. Minimum value of r_s for the second group of criteria (decreasing weights)

	Minimum required value of the first input	Minimum required value of the next input	Management fee
PROMETHEE	0.9999***	0.9637***	0.9958***
WSA	0.9997***	0.9677***	0.9979***
TOPSIS	0.9907***	0.9964***	0.9979***

Source: author's own calculations.

5. Conclusions

On the basis of the presented results, it can be concluded that rankings constructed using the TOPSIS method by a decision-maker whose preferences are similar to the preferences of a decision-maker who considers only differentiation between the alternatives according to the criteria are similar to the initial ranking. For small modifications in the weights, the measure of stability *s* rises more slowly for the TOPSIS method than in the case of the WSA and PROMETHEE methods.

It is important that an assessment of the similarity of rankings only on the basis of Spearman's rank correlation coefficient may lead to incomplete conclusions. With a tenfold increase in the weight of a given criterion, the value of the stability of order measure rises to 37, which indicates big shifts in the structure of a ranking. It should be noted that if we rank 47 investment funds, such a big change must involve a change between a very high and a very low position. However, in all cases the values of Spearman's rank correlation coefficients suggested that the rankings are similar.

On the basis of the assumptions about an investor's preferences, we can conclude that the outcomes of PROMETHEE and WSA are most prone to change due to a change in preferences. Moreover, the rankings constructed by the TOPSIS method can be regarded as the most universal. In the case of this method, the rankings obtained by different types of investors are similar and in the case of strictly stable rankings – identical. On the other hand, if the stability of order measure *s* for the TOPSIS method rises, the increase is rapid (decreasing weight of *minimum required value of the first input* and increasing weight of *kurtosis*, *minimum required value of following inputs, management fee*). The similarity of the initial rankings constructed using the WSA and PROMETHEE methods is reflected by the stability of modified rankings. The analysis of stability according to modified weighting vectors allows determining the boundaries within which changes in the relative importance of criteria do not lead to big shifts from the initial rankings. We may also select the method which is most stable for a decision-maker whose preferences are described by a modified weighting vector.

The reason for the differing stability of the methods compared needs further examination. On the basis of the results obtained, we may suppose that these differences result from the mutual relationships between the weights and between the values according to the individual criteria. Note that due to the assumption concerning decision-makers' preferences, the results from other methods of determining relative importance may turn out to be different from those presented here. In this study we focused only on the case of preferences related to differentiation according to the criteria. More precise analysis of the stability of methods requires numerical experiments and more detailed examination.

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