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INVENTORY MODEL FOR DETERIORATING ITEMS WITH NEGATIVE EXPONENTIAL DEMAND, PROBABILISTIC DETERIORATION, AND FUZZY LEAD TIME UNDER PARTIAL BACK LOGGING

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The effect of lead time plays an important role in inventory management. It is also important to study the optimal strategies when the lead time is not precisely known to the decision-makers. This paper aimed to examine the inventory model for deteriorating items with fuzzy lead time, negative exponential demand, and partially backlogged shortages. This model is unique in its nature due to probabilistic deterioration along with fuzzy lead time. The fuzzy lead time was assumed to be triangular, parabolic, trapezoidal numbers. The graded mean integration representation method was used for the defuzzification purpose. Three different types of probability distributions, namely uniform, triangular and beta were used for rate of deterioration to find optimal time and associated total inventory cost. The developed model was validated numerically, and values of optimal time and total inventory cost are given in a tabular form, corresponding to different probability distributions and fuzzy lead-time. The sensitivity analysis was performed on the variation of key parameters to observe its effect on the developed model.

Keywords: *fuzzy lead time, shortages, partially backlogged, exponentially declining demand, probabilistic deterioration*

1. Introduction

Conventional inventory models are formulated under the assumption that the time gap between placing an order and receiving goods is negligible. This indicates that suppliers of goods are quite near retailers and always ready to make the required quantity of goods available to its customers in fraction of time. But, in practice, the situation is not so fast and simple with no waste of time. The reality lies with the fact that there

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always remains a time gap between placing an order and receiving items which are produced at different production units spread over various places. Moreover, this time gap is not exactly known since some commodities whose demand is gradually decreasing but those commodities have a section of customers. Hence, non-zero lead-time in particular fuzzy lead time plays a vital role in inventory modelling. As a result, this type of problem in inventory management draws attraction of researchers and practitioners throughout the world. Over the years, the inventory problem for deteriorating item has undergone substantial improvement due to the addition of new assumptions on the nature of deterioration and other parameters. Among those, to get recent development in research in inventory problems, one may refer to the works of [19, 20, 31, 2].

2. Survey of literature

In this section, some of the major works done in the field of inventory management in respect of various parameters in the last few years, both in crisp and fuzzy environments, are discussed. Modelling any inventory problem in the absence of non-zero lead time and deterioration cannot reflect the actual scenarios. It is well known that any item, when purchased/produced, never remains the same for a long time. With time, the item changes due to spoilage, deterioration, weather condition, and other factors. So, naturally, the question arises how to maintain the stock of items for its further use, and to optimize total cost. Taking this fact into consideration, researchers developed inventory models for optimal solutions. To deal with the issue of probabilistic deterioration, a model was developed in [17]. During the formulation of the inventory model in [1], three parameters of the Weibull-distributed deterioration and selling price dependent demand are incorporated. Giving importance on time-price dependent demand [23], formulate a problem for deteriorating items in which lead time is considered as zero. Same authors [24] discuss an inventory model with price-sensitive demand for deteriorating items under full advance payment for availing discount and solve it numerically. Recently, Khurana et al. [8] intended to minimize inventory cost for deteriorating items when the demand for such type of items is variable with allowable shortages. In the literature, there is the consideration of demand as a ramp type function of time and two-parameter Weibull-distributed deterioration in formulating inventory model [6]. One can easily see that the model was developed for deteriorating items with time-dependent demand under inflation when supplier credit is linked with order quantity [30]. Also, the effect of inflation in the optimal solution of an inventory system for permissible items is studied where demand is taken as a quadratic function of time and constant rate of deterioration [18]. Considering fluctuating demand as a function of time, an inventory model for deteriorating items was developed under permissible delay in payment [22]. Apart from the aforesaid models, there are good numbers of research papers where the effect of lead time is ignored despite having a significant role in inventory decision-making policies. The consideration of other major factors, like demand, deterioration, and display cost, permissible delay in payment, etc., are frequently available in current literature. The contribution in inventory management under fuzzy environment is worth mentioning in [4] and in [28]. There are numerous papers where inventory models are developed in the fuzzy environment under various assumptions. The consideration of fuzzy parameters in mathematical modelling of inventory system is available in [15, 13, 3, 9]. Following Kazemi et al. [10], a table is made to show a comparison of contributions by different authors.

Table 1. Classification of the literature

Reference	Environment	Demand	Deterioration	Lead time
[5]	crisp	stockdependent (non-exponential)	constant	zero
[7]	fuzzy	constant (non-exponential)	constant	constant
[11]	crisp	time dependent (non-exponential)	time dependent	zero
[12]	crisp	inflation induced (non-exponential)	Weibull distribution	zero
[14]	fuzzy	display inventory dependent (non-exponential)	no	zero
[16]	crisp	price sensitive (non-exponential)	constant	zero
[21]	fuzzy	constant (non-exponential)	constant	zero
[25]	crisp	time dependent (exponential)	time dependent	zero
[26]	fuzzy	price dependent (non-exponential)	constant	zero
[27]	fuzzy	constant	fuzzy number	zero
[29]	crisp	inventory dependent (non-exponential)	time dependent	zero
Summary	6 crisp 5 fuzzy	10 non-exponential 1 exponential	1 probabilistic 10 non-probabilistic	1 non zero 10 zero
Present study		negative exponential	probabilistic	fuzzy

To the best knowledge of the authors, and as shown in the above contributions table, there is no paper where the discussion of inventory model with probabilistic deterioration and fuzzy lead-time simultaneously took place. The present study is motivated by the work conducted in [28]. The main purpose of this study is to develop a mathematical model with a set of assumptions emphasizing probabilistic deterioration and fuzzy lead time, and simultaneously which the problems existing in the real world.

3. Notations and assumptions

Assumptions

- planning horizon is infinite,
- demand rate is exponentially decreasing,

- shortages are allowed and partially backlogged,
- holding cost is independent of time,
- deterioration is probabilistic in nature,
- there is no repair of deteriorating items,
- lead time is non-zero and fuzzy in nature.

Notations

$h = a$	– holding cost per unit time, $a > 0$, \$/unit
$\theta(t) = \theta t$	– deterioration rate $0 < \theta < 1$
$D(t) = Ae^{-\alpha t}$	– demand function
c_0	– ordering cost, \$
c_s	– shortage cost, \$/unit
c	– unit cost of the item, \$/unit
L	– lead time, months
$\tilde{L}$	– fuzzy lead time, months
T	– cycle length, month
I_0	– maximum inventory level in $(0, T)$
S	– lost sale cost per unit, \$/unit
c_d	– unit deterioration cost, \$/unit
t_1	– decision variable, time of replenishment, months
TC	objective, total cost of the inventory system, \$

4. Mathematical formulation of the models

For the crisp environment, it is assumed that the replenishment rate is finite and time gap of placing and receiving order is not fixed uncertain and that deterioration takes place randomly. So, as based on the above-mentioned assumptions, the differential equations representing the proposed inventory system as

$$\frac{dI_1(t)}{dt} + \theta t I_1(t) = -Ae^{-\alpha t} \text{ with } I_1(t) = I_0 \tag{1}$$

The rate of change of the inventory during the shortage period $[t_1, t_2]$ is governed by the differential equation

$$\frac{dI_2(t)}{dt} = -A\beta e^{-\alpha t}, \quad t_1 \le t \le t_2, \quad I_2(t_1) = 0 \tag{2}$$

The inventory level changes in $[t_2, T]$ is due to demand and deterioration

$$\frac{dI_3(t)}{dt} + \theta t I_3(t) = -A\mathrm{e}^{-\alpha t} \quad \text{with} \quad I_3(t_2) = I_0 \tag{3}$$

The solution of (1) gives

$$I_1(t) = k_1 \mathrm{e}^{-\theta t^2/2} + \frac{A\mathrm{e}^{-\alpha t}}{\theta t - \alpha} \tag{4}$$

Applying $I_1(0) = I_0$, the above expression gives

$$k_1 = I_0 + \frac{A}{\alpha} \, SoI_1(t) = \left(I_0 + \frac{A}{\alpha} \right) \mathrm{e}^{-\theta t^2/2} + \frac{A\mathrm{e}^{-\alpha t}}{\theta t - \alpha} \tag{5}$$

The solution of equation (2) gives

$$I_2(t) = -A\beta \left(-\frac{\mathrm{e}^{-\alpha t}}{\alpha} \right) + k_2$$

On applying $I_2(t_1) = 0$, we get $k_2 = -\frac{A\beta}{\alpha} \mathrm{e}^{-\alpha t_1}$ implying

$$I_2(t) = \frac{A\beta}{\alpha} \left(\mathrm{e}^{-\alpha t} - \mathrm{e}^{-\alpha t_1} \right) \tag{6}$$

The solution of equation (3) gives

$$I_3(t) = k_3 \mathrm{e}^{-\theta t^2/2} + \frac{A\mathrm{e}^{-\alpha t}}{\theta t - \alpha} \tag{7}$$

At $t = t_2$, $I_3(t) = I_0$, giving

$$k_3 = \mathrm{e}^{\theta t_2^2/2} \left(I_0 - \frac{A\mathrm{e}^{-\alpha t_2}}{\theta t_2 - \alpha} \right) \tag{8}$$

using k_3 in equation (7),

$$I_3(t)=\left(I_0-\frac{Ae^{-\alpha t_2}}{\theta t_2-\alpha}\right)\mathrm{e}^{\theta\left(t_2^2-t^2\right)}+\frac{A\mathrm{e}^{-\alpha t}}{\theta t-\alpha} \tag{9}$$

But $t_2 - t_1 = L \Rightarrow t_2 = t_1 + L$. So, equation (9) becomes

$$I_3(t)=\left(I_0-\frac{A\mathrm{e}^{t_1+L}}{\theta\left(t_1+L\right)-\alpha}\right)\mathrm{e}^{\theta\left(\left(t_1+L\right)^2-t^2\right)/2}+\frac{A\mathrm{e}^{-\alpha t}}{\theta t-\alpha} \tag{10}$$

The remaining quantities are

- backlogged quantity

$$BQ=\int_{t_1}^{t_2} I_2(t)dt=\frac{A\beta\mathrm{e}^{-\alpha t_1}}{\alpha^2}\left(\mathrm{e}^{-\alpha L}-1+L\alpha\right)$$

- lost sale

$$\int_{t_1}^{t_2} D(t)dt-\int_{t_1}^{t_2} I_2(t)dt=\frac{A\mathrm{e}^{-\alpha t_1}}{\alpha^2}\left((\beta-\alpha)(\mathrm{e}^{-\alpha L}-1)+\beta\alpha L\right)$$

- deteriorated quantity

$$Q_d=2I_0+\frac{A}{\alpha}\left(\mathrm{e}^{-\alpha T}-1\right)+\frac{A}{\alpha}\mathrm{e}^{-\alpha t_1}\left(1-\mathrm{e}^{-\alpha L}\right)$$

- holding cost $HC = HC$ in $[0, t_1]$ + HC in $[t_1, T]$

$$HC=h\int_0^{t_1} I_1(t)dt+h\int_{t_1}^{T} I_3(t)dt=h\left(t_1-\frac{A}{\alpha}\left(\frac{\theta}{\alpha}-\alpha\right)\frac{t_1^2}{2}-\frac{1}{6}\left(I_0-\frac{A\theta}{\alpha}\right)t_1^3\right)$$
$$+h\left(\left(X_1-\frac{A}{\alpha}\right)\left(T-t_1-L\right)+\frac{A}{2}\left(\frac{\theta}{\alpha^2}+1\right)\left(T^2-t_1^2-L^2-2t_1L\right)\right.$$
$$\left.-\frac{\theta}{3}\left(\frac{X_1}{2}+\frac{A}{\alpha}\right)\left(T^3-t_1^3-3t_1^2L-3t_1L^2-T^3\right)\right)$$

where

$$X_1 = \left(I_0 - \frac{A\mathrm{e}^{t_1+L}}{\theta\left(t_1+L\right)-\alpha} \right)\mathrm{e}^{-\theta\left(t_1+L\right)^2}$$

• deteriorated cost

$$DC = C_d Q_d = C_d \left(2I_0 + \frac{A}{\alpha}\left(\mathrm{e}^{-\alpha T}-1\right) + \frac{A}{\alpha}\mathrm{e}^{-\alpha t_1}\left(1-\mathrm{e}^{\alpha L}\right) \right)$$

• total shortages during $[t_1, t_2]$

$$\int_{t_1}^{t_2} D(t)\,dt = -\frac{A}{\alpha}\mathrm{e}^{-\alpha t_1}\left(\mathrm{e}^{-\alpha L}-1\right)$$

• shortage cost

$$SC = -c_s \frac{A}{\alpha}\mathrm{e}^{-\alpha t_1}\left(\mathrm{e}^{-\alpha L}-1\right)$$

• purchase cost

$$PC = c(2I_0 - BQ) = c\left(2I_0 - \frac{A\beta}{\alpha^2}\mathrm{e}^{-\alpha t_1}\left(\mathrm{e}^{-\alpha L}+L\alpha-1\right) \right)$$

• lost cost sale

$$LSC = S\,\frac{A\mathrm{e}^{-\alpha t_1}}{\alpha^2}\left((e^{-\alpha L}-1)(\beta-\alpha)+\alpha\beta L\right)$$

Total cost of the inventory system is as follows

$$TC = OC + PC + HC + SC + LSC + DC$$

$$\begin{aligned} TC = {} & k_0 + k_1 t_1 + k_2 t_1^2 + k_3 t_1^3 + k_4 t_1^4 + M_1 L + M_2 L^2 + M_3 L^3 + M_4 L^4 \\ & + N_1 L t_1 + N_2 L t_1^2 + N_3 L t_1^3 + N_4 L^2 t_1 + N_5 L^2 t_1^2 + N_6 L^3 t_1 - A c_d T \end{aligned} \qquad (11)$$

where

$$k_0 = 2A + 2CI_0 + hI_0 + \frac{hA}{\alpha} + 2c_d I_0$$

$$k_1 = h\left(1 + \frac{A}{\alpha}\left(\frac{\theta}{\alpha} - \alpha\right)\right), \quad k_2 = h\left(\frac{-h}{2\alpha}\left(\frac{\theta}{\alpha} - \alpha\right) + \frac{I_0\theta}{2} + \frac{A}{\alpha} - \frac{A\theta}{\alpha}\right)$$

$$k_3 = h\left(\frac{-h}{6}\left(I_0 - \frac{A\theta}{\alpha}\right) + \frac{A\theta}{2\alpha}\left(\frac{\theta}{\alpha} - \alpha\right)\right), \quad k_4 = -h\frac{A\theta^2}{2\alpha}$$

$$M_1 = A\left(\frac{h}{\alpha}\left(\frac{\theta}{\alpha} - \alpha\right) - c_s + S + c_d\right), \quad M_2 = h\left(\frac{I_0\theta}{2} + \frac{A}{\alpha} - \frac{A\theta}{\alpha}\right)$$

$$M_3 = \frac{hA\theta}{2\alpha}\left(\frac{\theta}{\alpha} - \alpha\right), \quad M_4 = -\frac{hA\theta^2}{2\alpha}$$

$$N_1 = 2h\left(\frac{I_0\theta}{2} + \frac{A}{\alpha} - \frac{A\theta}{\alpha}\right), \quad N_2 = \frac{3hA\theta}{2\alpha}, \quad N_3 = -\frac{2hA\theta^2}{\alpha}$$

$$N_4 = \frac{3hA}{\alpha}\left(\frac{\theta}{\alpha} - \alpha\right), \quad N_5 = -\frac{3hA\theta^2}{\alpha}, \quad N_6 = -\frac{2hA\theta^2}{\alpha}$$

5. Numerical results of the model in various environments

The numerical values of different parameters are given below in appropriate units: $h = 2$, $A = 100$, $\beta = 0.3$, $\alpha = 0.001$, $I_0 = 1000$, $c_0 = 50$, $c = 50$, $c_d = 0.75$, $L = 0.34$, $\theta = 0.12$.

Table 2. Values of θ for various distributions

	Triangular	Uniform	Beta
θ	(0.002, 0.007, 0.3)	(0.002, 0.2)	(0.002, 0.02)

Table 3. Values of lead time for various fuzzy numbers

	Triangular	Parabolic	Trapezoidal
$\tilde{L}$	(0.33, 0.40, 0.47)	(0.20, 0.3, 0.47)	(0.20, 0.27, 0.33, 0.47)

Due to the complex nature of the cost function, the optimality is checked graphically in crisp and fuzzy environment using MATLAB. The optimal results are listed for three different cases for rate of deterioration when it follows triangular, uniform, and beta distribution.

5.1. Numerical results

Numerical results in the crisp environment are as follows:

Table 4. Optimal values of t_1 and TC corresponding to various probability distributions

θ follows triangular distribution		θ follows uniform distribution		θ follows beta distribution	
t_1 [months]	TC [$]	t_1 [months]	TC [$]	t_1 [months]	TC [$]
5	7 468 000	5	6 541 000	6	460 000

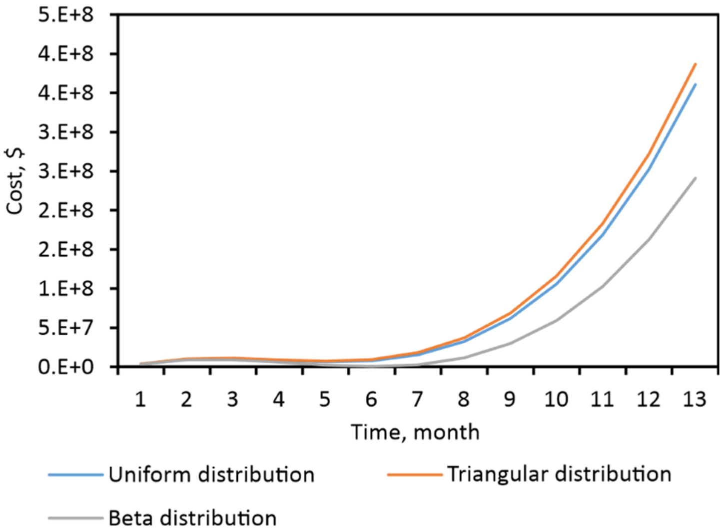

Fig. 1. Graphical representation of the results presented in Table 4

Numerical result in fuzzy environment are following when the rate of deterioration follows various distributions and lead time is fuzzy numbers:

Table 5. Optimal values corresponding to various fuzzy lead times when the deterioration follows beta distribution

Triangular fuzzy number		Parabolic fuzzy number		Trapezoidal number	
t_1 [months]	TC [$]	t_1 [months]	TC [$]	t_1 [months]	TC [$]
6	3 194 000	6	129 200 000	6	–2 300 000

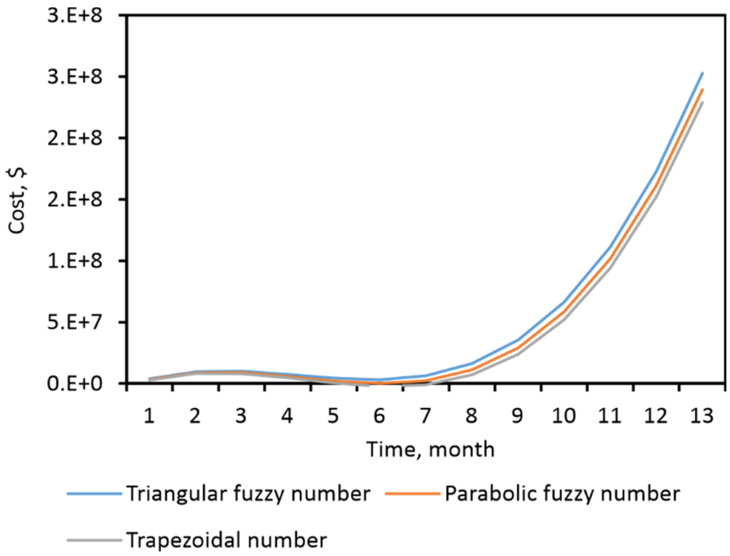

Fig. 2. Graphical representation of the results presented in Table 5

Table 6. Optimal values corresponding to various fuzzy lead times when the deterioration follows triangular distribution

For triangular fuzzy number		For parabolic fuzzy number		For trapezoidal number	
t_1 [months]	TC [$]	t_1 [months]	TC [$]	t_1 [months]	TC [$]
5	9 946 000	5	7 168 000	5	4 973 000

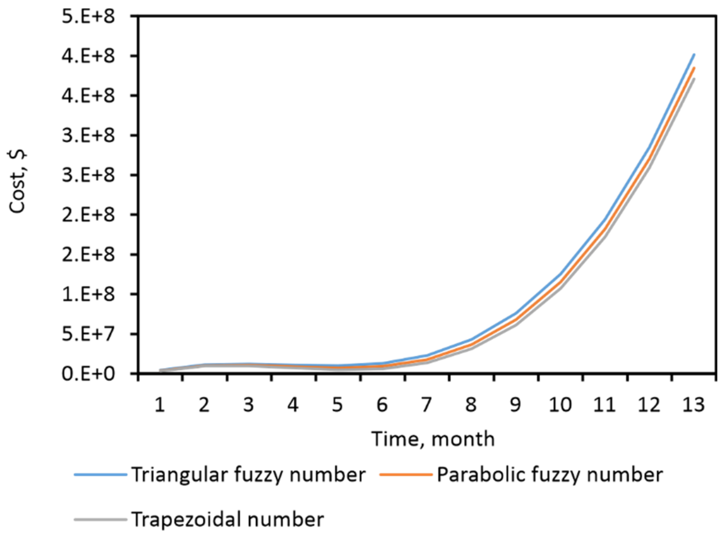

Fig. 3. Graphical representation of the results obtained in Table 6

Table 7. Optimal values corresponding to different fuzzy lead times
when the deterioration follows uniform distribution

For triangular fuzzy number		For parabolic fuzzy number		For trapezoidal number	
t_1 [months]	TC [\$]	t_1 [months]	TC [\$]	t_1 [months]	TC [\$]
5	8 937 000	5	6 251 000	5	4 128 000

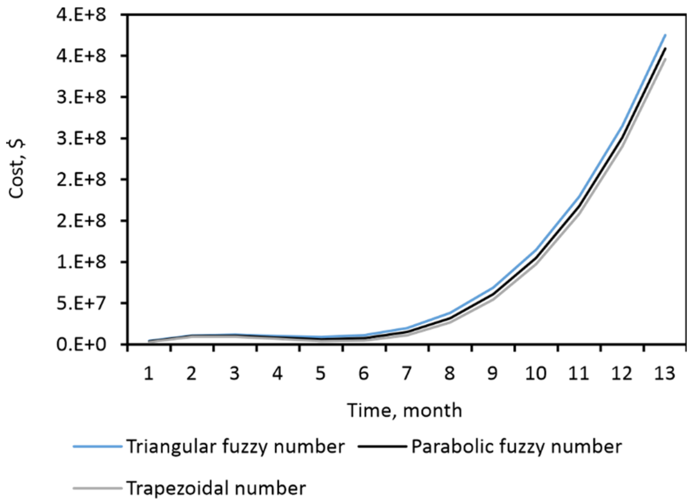

Fig. 4. Graphical representation of the results presented in Table 7

6. Sensitivity analysis

Table 8. Sensitivity analysis of the proposed model; (t_1, TC) values for various θ distributions and L being triangular, parabolic, and trapezoidal fuzzy numbers

θ follows beta distribution					
Parameter	Change		L		
			Triangular fuzzy number	Parabolic fuzzy number	Trapezoidal fuzzy number
1	2	3	4	5	6
	50%	3			
	25%	2.5			
h	0%	2	(6, 3 194 000)	(6, 129 200)	
	–25%	1.5			
	–50%	1			TC negative
	50%	150	(6, 4 749 000)	(6, 152 200)	
	25%	125	(6, 3 971 000)	(6, 140 700)	
A	0%	100	(6, 3 194 000)	(6, 129 200)	
	–25%	75	(6, 2 416 000)	(6, 117 600)	
	–50%	50	(6, 1 638 000)	(6, 106 100)	

Table 8 continued

1	2	3	4	5	6
c_s	50%	7.5	(6, 3 193 000)	(6, 129 100)	*TC* negative
	25%	6.25	(6, 3 194 000)		
	0%	5		(6, 129 200)	
	–25%	3.75			
	–50%	2.5			
C	50%	75	(6, 3 244 000)	(6, 179 100)	
	25%	62.5	(6, 3 219 000)	(6, 154 100)	
	0%	50	(6, 3 194 000)	(6, 129 200)	
	–25%	37.5	(6, 3 169 000)	(6, 104 200)	
	–50%	25	(6, 3 144 000)	(6, 79 200)	
c_d	50%	1.125	(6, 3 194 000)	(6, 129 500)	
	25%	0.9375		(6, 129 300)	
	0%	0.75		(6, 129 200)	
	–25%	0.5625	(6, 3 193 000)	(6, 129 000)	
	–50%	0.375		(6, 128 800)	

θ follows triangular distribution

Parameter	Change		*L* Triangular fuzzy number	*L* Parabolic fuzzy number	*L* Trapezoidal fuzzy number
h	50%	3	(5, 9 946 000)	(5, 7 168 000)	(5, 4973 000)
	25%	2.5			
	0%	2			
	–25%	1.5			
	–50%	1			
A	50%	150	(5, 14 870 000)	(5, 10 710 000)	(5, 4 173 000)
	25%	125	(5, 12 410 000)	(5, 8 937 000)	(5,6 193 000)
	0%	100	(5, 9 946 000)	(5, 7 168 000)	(5, 4 973 000)
	–25%	75	(5, 7 482 000)	(5, 5 399 000)	(5, 3 75 0000)
	–50%	50	(5, 5 019 000)	(5, 3 631 000)	(5, 2 533 000)
c_s	50%	7.5	(5, 9 945 000)	(5, 7 168 000)	(5, 4973000)
	25%	6.25	(5, 9 946 000)		
	0%	5			
	–25%	3.75			
	–50%	2.5			
C	50%	75			
	25%	62.5			
	0%	50			
	–25%	37.5			
	–50%	25			
c_d	50%	1.125			
	25%	0.9375			
	0%	0.75			
	–25%	0.5625	(5, 9 945 000)		
	–50%	0.375			

Table 8 continued

<table>
<tr><td colspan="6">θ follows uniform distribution</td></tr>
<tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td rowspan="5">h</td><td>50%</td><td>3</td><td rowspan="5">(5, 8 937 000)</td><td rowspan="5">(5, 6 251 000)</td><td rowspan="5">(5, 4128 000)</td></tr>
<tr><td>25%</td><td>2.5</td></tr>
<tr><td>0%</td><td>2</td></tr>
<tr><td>–25%</td><td>1.5</td></tr>
<tr><td>–50%</td><td>1</td></tr>
<tr><td rowspan="5">A</td><td>50%</td><td>150</td><td>(5, 13 360 000)</td><td>(5, 9 331 000)</td><td>(5, 6 146 000)</td></tr>
<tr><td>25%</td><td>125</td><td>(5, 11 150 000)</td><td>(5, 7 791 000)</td><td>(5, 5 137 000)</td></tr>
<tr><td>0%</td><td>100</td><td>(5, 8 937 000)</td><td>(5, 6 251 000)</td><td>(5, 4 128 000)</td></tr>
<tr><td>–25%</td><td>75</td><td>(5, 6 726 000)</td><td>(5, 4 712 000)</td><td>(5, 3 119 000)</td></tr>
<tr><td>–50%</td><td>50</td><td>(5, 4 515 000)</td><td rowspan="6">(5, 3 172 000)</td><td>(5, 2 111 000)</td></tr>
<tr><td rowspan="5">c_s</td><td>50%</td><td>7.5</td><td rowspan="6">(5, 8 937 000)</td><td rowspan="5">(5, 4 128 000)</td></tr>
<tr><td>25%</td><td>6.25</td></tr>
<tr><td>0%</td><td>5</td></tr>
<tr><td>–25%</td><td>3.75</td></tr>
<tr><td>–50%</td><td>2.5</td></tr>
<tr><td rowspan="5">C</td><td>50%</td><td>75</td><td>(5, 6 301 000)</td><td>(5, 4 178 000)</td></tr>
<tr><td>25%</td><td>62.5</td><td>(5, 8 962 000)</td><td>(5, 6 276 000)</td><td>(5, 4 153 000)</td></tr>
<tr><td>0%</td><td>50</td><td>(5, 8 937 000)</td><td>(5, 6 251 000)</td><td>(5, 4 128 000)</td></tr>
<tr><td>–25%</td><td>37.5</td><td>(5, 8 912 000)</td><td>(5, 6 226 000)</td><td>(5, 4 103 000)</td></tr>
<tr><td>–50%</td><td>25</td><td>(5, 8 887 000)</td><td>(5, 6 201 000)</td><td>(5, 4 078 000)</td></tr>
<tr><td rowspan="5">c_d</td><td>50%</td><td>1.125</td><td>(5, 8 938 000)</td><td>(5, 6 252 000)</td><td>(5, 4 129 000)</td></tr>
<tr><td>25%</td><td>0.9375</td><td rowspan="4">(5, 8 937 000)</td><td>(5, 6 525 000)</td><td rowspan="4">(5, 4 128 000)</td></tr>
<tr><td>0%</td><td>0.75</td><td rowspan="3">(5, 6 251 000)</td></tr>
<tr><td>–25%</td><td>0.5625</td></tr>
<tr><td>–50%</td><td>0.375</td></tr>
</table>

Changes in the values of system parameters may take place due to uncertainties in any decision-making situation. So, to study the implications of these changes, it is essential to perform sensitivity analysis. Here we study the effects of changes in the values of system parameters on the optimal values of time of replenishment and total cost when the deterioration rate follows different probability distributions. As based on our numerical calculation presented in Table 8, we have the following observations.

• There is no effect on the time of replenishment when lead time is taken as triangular, parabolic, and trapezoidal numbers under the assumptions of different probability distributions for the deterioration rate.

• The optimal total cost (*TC*) is not acceptable due to changes in system parameters when lead time is taken as the trapezoidal fuzzy number and *TC* remains unchanged under beta distribution for deterioration rate as *h* is increased or decreased when lead time is taken triangular, parabolic fuzzy number. 50% changes in the value of *h*, *TC* changes with lead time as triangular, parabolic, and trapezoidal fuzzy number in the case of triangular distribution for deterioration rate. For any change in the value of *h*, *TC* remains unaltered with lead time as triangular, parabolic, and trapezoidal fuzzy number, in the case of uniform distribution for the deterioration rate.

• It is clear from the sensitivity analysis that there is a direct relation of optimal *TC* with the parameters *A*, *C*. That is, the maximum value of *A* and *C* will maximize *TC*. To avoid such type of situation, the decision manager should purchase the goods at a reasonable price to minimize *TC*.

• 50% decrease in unit deterioration cost minimizes *TC* when lead time is a parabolic fuzzy number under beta distribution, whereas the same percentage of decrease in unit deterioration cost does not minimize *TC* when lead time is triangular, parabolic, and trapezoidal fuzzy number under triangular and uniform distributions.

• Our next observation is on optimal *TC* due to changes in the value of unit shortage cost c_s. *TC* is minimal due to 50% positive changes on the said parameter when lead time is taken as a triangular fuzzy number under beta and triangular distributions.

7. Conclusion

For the first time, the inventory model for deteriorating items with negative exponential demand is developed in the presence of probabilistic deterioration and fuzzy lead time. In this study, the model is formulated and solved with non-zero lead time and different probabilistic deterioration to obtain optimal values. Following this, the model is considered in a fuzzy environment. Corresponding to different probabilistic distributions, lead time was considered as different fuzzy numbers under three different cases of probabilistic deterioration, and optimal values may be compared. The developed model might be helpful for retailers in dealing with the problems of items like fashionable goods, a certain brand of medicine, etc., due to their demand pattern. The present investigation may be further extended with new assumptions like salvage cost, preservation technology, and variable holding cost.

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