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FLEXIBLE MEASURE IN THE PRESENCE OF PARTIAL INPUT TO OUTPUT IMPACTS PROCESS

Precise recognition of the nonparametric measurement approach in the production process and proper application of accurate techniques to categorise the variables play a key role in the process of improving performance of decision-making units (DMUs). The classical data envelopment analysis (DEA) models require that the status of all inputs and outputs measures be precisely specified in advance. However, there are situations where a performance measure can play input role for some DMUs and output role for the others. This paper introduces an approach to determine the situation of such flexibility where in the presence of resource sharing among subunits, the partial input will impact output in DEA. As a result, DMUs have a fair evaluation when compared to each other. Likewise, the maximum improvement is obtained in aggregate efficiency due to partial input to output impacts. The proposed approach is applied to a set of real data collected from 30 branches of an Iranian bank.

Keywords: *data envelopment analysis, flexible measure, efficiency, integer programming, partial impact, bank industry*

1. Introduction

Since the development of data envelopment analysis (DEA) by Charnes et al. [10], it has been widely applied to evaluate the relative efficiencies of a set of decision-making units (DMUs). The main work of Banker et al. [3] extends the constant returns to

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the scale (CRS) model of Charnes et al. [10] to permit variable returns to scale (VRS). Both models are a radial projection model which assumes that all inputs/outputs (in the case of input/output-oriented method) are decreased or increased commensurately in projecting to the efficient frontier. The conventional applications of DEA reveal that the status of performance measures from the perspective of input or output is specified in advance.

However, there are some cases in which the statuses of some measures are unknown in advance as inputs or outputs. Beasley [5] first addresses the factors that could be treated as both inputs and outputs. For instance, in the evaluation of university performance as in Beasley [5, 6], research income is considered as a flexible measure. As another example, Cook et al. [12] and Cook and Hababou [13] consider the number of high-value customers as either input or output for evaluating bank branches performance. Cook and Bala [14] suggest an improved measure for evaluating the performance in the bank sector with flexible measures. In another study, Cook et al. [15] propose moving the input role from the denominator to the numerator, the output side, but with the opposite sign in its weight. Particularly, the authors consider the dual-role factor as an exogenously fixed or non-discretionary variable (see, e.g., [4]), which is not controlled but can affect the DEA evaluation. Also, Cook and Zhu [16] propose an approach to classify them in DEA rather than considering these factors as inputs and outputs simultaneously. Further, Bi et al. [7] tried to address this theme from the view of a production process, not multi-criteria performance aggregation.

Saen [25] presents a method for searching the proper suppliers in conditions where flexible measures and decision maker's preferences are altogether included in the analysis. Amirteimoori et al. [1] suggest a flexible slack-based measure of efficiency to maximise performance. Chen [11] develops a simple three-dimensional case for visualisation, and also generalise it to multi-dimensional cases with considering the multiple factors. Moreover, the author determines the input/output behaviour (tendency) of a dual-role factor from multi-criteria performance aggregation, geometry, and economics perspectives, and then concludes that it is a property on the projected boundary, not the data point itself. Kumar and Jain [22] improve Saen's [25] method to present a Green DEA framework for the aim of formulating supplier selection problems with carbon footprints of suppliers as a flexible measure. Kordrostami and Jahani [21] propose a model that examines the situation of flexible measures in the presence of interval data. Ding et al. [18]) develop a two-step model to pinpoint the proper suppliers in the presence of flexible measures. In a recent study, Toloo et al. [26] present a new model which integrates both pessimistic and optimistic models to identify a unique status of each imprecise dual-role factor. The authors employ a fuzzy decision-making approach where the interval efficiency score is used to define the fuzzy goal for each DMU.

Taking a look at the vast literature in DEA reveals that the original DEA approach presumes that in multiple inputs and multiple outputs environments, all inputs impact all outputs. Nonetheless, there are situations where this hypothesis cannot be held (see, e.g., [17, 19, 20, 23] for further information). For example, in a bank setting with multiple types of staff and activities, all activities are not influenced by all staff. The service personnel members, for instance, do not impact some operational outputs. Also, in a hospital setting, clinical staff members do not impact many functional outputs. Further, there are many situations in which shared resources or resources on the multi-stage processes are influenced by some/all shared products or outputs (see, e.g., [8, 28, 29, 30] and many others). Therefore, assessing such outputs versus such special inputs yields invalid efficiency measures. Further, there are some factors in such environments that are referred to as dual-role factors/flexible measure, for instances, the number of customers in the evaluation of bank branch performance and research income, the number of researchers and graduate students in the efficiency assessment of university departments or the number of nurse trainees on staff in a hospital setting can be considered as a dual-role factor with input and output roles at the same time (see, e.g., [15]).

Due to the importance of banks in the economy of each country, their performance evaluation has been a matter of main interest for financiers, customers and market correctors (see, e.g., [24]). Wang et al. [28] study the efficiency of the Chinese commercial banking system applying an additive two-stage DEA model. The authors utilised the DEA network method to disaggregate, evaluate and test the efficiencies of 16 major Chinese commercial banks during the third round of the Chinese banking reform period (2003–2011) with the consideration of undesirable output. Also, An et al. [2] develop a new two-stage data envelopment analysis approach for measuring the slacks-based efficiency of Chinese commercial banks during years 2008–2012, where the banking operation process of each bank is divided into a deposit-generation division and a deposit-utilisation division. In their approach, the increase of desirable outputs and the decrease of undesirable outputs are simultaneously considered in order to identify the inefficiency of a bank. In another study, Wanke et al. [27] use a dynamic slack-based method to assess the efficiency of the Malaysian dual banking system.

Because the banking system is a multiple-input and multiple-output structure, a suitable multiple criteria evaluation technique is essential to extensively and impartially measure its efficiency. In this paper, we aim to present a method that assesses the performance of DMUs in the presence of flexibility in a bank setting where the partial input-output impacts exist. Therefore, we incorporate the partial impacts model (PIM) with flexible measures so that each of them is either input or output in their related subunits in order to maximise the efficiency of the DMU under evaluation. Here, DMUs (i.e., bank branches) are considered business units, containing several discrete subunits based on Imanirad et al. model [19], where the activity in terms of the products created and resources used, varies from one subunit to the others. Accordingly, the efficiency of the bank branches can be specified as a weighted average of the efficiencies of the subunits. Also, a factor such as "the number of customers" can be assumed that both of as an input and as an output. On the one hand, such a measure plays the role of representative for future enterprise, therefore can be categorised as an output. On the other hand, it can be assumed as an input that helps the branch in producing its existing enterprise investments. Thus, in order to maximise the efficiency of branches, we intend to offer an approach for evaluating the performance of branches in the presence of flexibility and PIM in DEA. It should be emphasized that evaluating the level of performance through all sub-units by the proposed model gives more precise results of the efficiency for each DMU. Also, the lack of shared case here is another advantage. This study makes two main contributions to the literature. First, to the best of our knowledge, it is the first to develop a PIM method to investigate the partial efficiency effects with flexibility in the banking sector. Second, the paper not only provides reliable information on the efficiency of the Iranian bank industry but also assists us in understanding how to improve banks overall efficiency. Hence, it offers important guidance for policy design and implementation in the future development of the bank industry.

The below part of this paper is organised as follows. Section 2 gives a brief explanation of the PIM model. Section 3 introduces the new model with flexible measures in the presence of partial input to output manufacturing process. Section 4 illustrates the applicability of the proposed model in assessing branches of an Iranian bank in Guilan province. Conclusions and further remarks are given in Section 5.

2. A PIM model

In the original DEA model, a set of $n\ DMU_j\ (j = 1, ..., n)$ is to be evaluated as based on a set of I inputs x_{ij} and R outputs y_{rj}. In model (1) below, we see CCR input-oriented DEA model for evaluating the technical efficiency of DMUs under the assumption of CRS that is proposed by Charnes et al. [10]. In this model, u_r, v_i are the decision variables and are multipliers assigned to outputs y_{rj} and inputs x_{ij}, respectively:

$$
\begin{aligned}
& e = \max \frac{\sum_r u_r y_{rj_o}}{\sum_i v_i x_{ij_o}} \\
& \text{s.t.} \qquad\qquad\qquad\qquad\qquad\qquad (1) \\
& \frac{\sum_r u_r y_{rj}}{\sum_i v_i x_{ij}} \le 1, \quad j = 1, ..., n
\end{aligned}
$$

Model (1) is proposed for efficiency measurement in such environments where all outputs are influenced by all inputs. Imanirad and Cook [19] propose a PIM model as a model (2) through the following three steps for evaluating the measurement of efficiency in situations where only partial input-output impacts exist:

1. Derive a proper split α_{ik} of each input *i* to each bundle *k* of which it is a member.

2. Using the split of inputs from step 1, apply the original CCR-DEA model to each of the *k* subunits.

3. Derive an overall efficiency score by combining the subunit scores from step 2.

Let us present a brief description of the PIM model here; suppose a DMU as a business unit including a set of *K* separate subunits, each of which can be treated as in the original DEA. Each input subunit is represented against its own input-output subunit (I_k, R_k), where each input in I_k affects each output in R_k. Since the goal is to derive the aggregate efficiency of each DMU, let us first define it.

Definition 2.1. The aggregate efficiency of each DMU is the weighted combination of its *K* subunits efficiencies.

Now consider PIM model for any DMU_{j_o} as follows:

$$e_{j_o} = \max \sum_{k=1}^{K} w_{kj_o} \frac{\sum_{r \in R_k} u_r y_{rj_o}}{\sum_{i \in I_k} v_i \alpha_{ik} x_{ij_o} + \sum_{i \in I_{ns}} v_i^k x_{ij_o}} \tag{2.1}$$

s.t.

$$\sum_{k=1}^{K} w_{kj} \frac{\sum_{r \in R_k} u_r y_{rj}}{\sum_{i \in I_k} v_i \alpha_{ik} x_{ij} + \sum_{i \in I_{ns}} v_i^k x_{ij}} \le 1, \qquad \forall j \tag{2.2}$$

$$\frac{\sum_{r \in R_k} u_r y_{rj}}{\sum_{i \in I_k} v_i \alpha_{ik} x_{ij} + \sum_{i \in I_{ns}} v_i^k x_{ij}} \le 1, \qquad \forall j, k \tag{2.3}$$

$$\sum_{k \in L_i} \alpha_{ik} = 1, \qquad \forall i \tag{2.4}$$

$$a_{ik} \le \alpha_{ik} \le b_{ik}, \qquad \forall i, k \tag{2.5}$$

$$u_r, v_i, \alpha_{ik} \geq \varepsilon, \quad \forall r, i, k \tag{2.6}$$

where the weights w_{kj_o} represent the importance of each subunit for DMU_{j_o} under consideration (see, e.g., [17, 19]) that is defined as follows:

$$w_{kj_o} = \frac{\sum_{i \in I_k} v_i \alpha_{ik} x_{ij_o} + \sum_{i \in I_{ns}} v_i^k x_{ij_o}}{\sum_{k=1}^{K} \left[\sum_{i \in I_k} v_i \alpha_{ik} x_{ij_o} + \sum_{i \in I_{ns}} v_i^k x_{ij_o} \right]} \tag{3}$$

Note that for any DMU_{j_o}, the w_{kj_o} assigned to each subunit should represent the importance of that subunit to the DMU_{j_o}. Also, one could define importance (or, terms of outputs in) w_{kj_o} generated, however, since its input reduction, not output expansion, that is an issue here, it appears proper to define the weights in terms of inputs. From a computational perspective, it is reasonable to let the contribution of inputs assigned to a subunit dictate the importance of that subunit. For instance, 30% of the inputs are dedicated to a special subunit and 70% to another, allocating a weight of 30% to the efficiency ratio of the first subunit and 70% to the other is reasonable. The α_{ik} provide a suitable assigning of inputs as $\alpha_{ik} x_{ij_o}$ to their respective subunits. Thus, from constraints (2.3), such α_{ik} values should be chosen so that the efficiency score relevant to any subunit k of DMU_{j_o} does not exceed unity for some amounts of the multipliers u_r, v_i. Note that, in this model, there are two types of inputs so that one type includes separable inputs and the other type non-separable inputs. Here, v_i^k is the weight assigned to the non-separable input x_{ij_o}, $i \in I_{ns}$, and represents the impact of that input on the outputs in subunit k. This weight can be different from one subgroup to another, while v_i is the weight assigned to the separable input x_{ij_o}, $i \in I_k$. Constraints (2.4) enforce the usual convexity restriction on the α_{ik} amounts in every subunit k, for any input i relatives to that subunit. The set L_i in constraints (2.4) shows those subunits k that have i as a member. Finally, constraints (2.5) limit the size of the α_{ik} variables. The following lemma shows that in the presence of constraints (2.3), constraints (2.2) are redundant.

Lemma 1. Constraints (2.2) are redundant in the presence of constraints (2.3).

Proof. Since $a_{ik} \leq \alpha_{ik} \leq b_{ik}$ and $\sum_{k \in L_i} \alpha_{ik} = 1$, the ratio of the weighted outputs to the weighted inputs at every subgroup becomes less than or equal to one. Hence, constraints

(2.3) determine that the α_{ik} values can be chosen in a way that ensures the efficiency ratio corresponding with any DMU in each of its subunits is less than one for some amounts of the multipliers u_r, v_i. It is clear that, the *k*th subunit efficiency score for DMU_{j_o} is given by

$$e_{kj_o} = \frac{\sum_{r\in R_k} u_r y_{rj_o}}{\sum_{i\in I_k} v_i \alpha_{ik} x_{ij_o} + \sum_{i\in I_{ns}} v_i^k x_{ij_o}} \le 1, \quad \forall k, j$$

Further, from constraints (2.2) and

$$\sum_{k\in L_i} \alpha_{ik} = 1 \rightarrow \sum_{k\in L_i} v_i \alpha_{ik} = v_i$$

we have

$$\sum_{k=1}^{K} w_{kj_o} e_{kj_o} = \sum_{k=1}^{K} \frac{\sum_{i\in I_k} v_i \alpha_{ik} x_{ij_o} + \sum_{i\in I_{ns}} v_i^k x_{ij_o}}{\sum_{k=1}^{K}\left[\sum_{i\in I_k} v_i \alpha_{ik} x_{ij_o} + \sum_{i\in I_{ns}} v_i^k x_{ij_o}\right]} \times \frac{\sum_{r\in R_k} u_r y_{rj_o}}{\sum_{i\in I_k} v_i \alpha_{ik} x_{ij_o} + \sum_{i\in I_{ns}} v_i^k x_{ij_o}}$$

$$= \sum_{k=1}^{K} \frac{\sum_{r\in R_k} u_r y_{rj_o}}{\sum_{k=1}^{K}\left[\sum_{i\in I_k} v_i \alpha_{ik} x_{ij_o} + \sum_{i\in I_{ns}} v_i^k x_{ij_o}\right]} = \frac{\sum_{k=1}^{K}\sum_{r\in R_k} u_r y_{rj_o}}{\sum_{k=1}^{K}\left[\sum_{i\in I_k} v_i \alpha_{ik} x_{ij_o} + \sum_{i\in I_{ns}} v_i^k x_{ij_o}\right]}$$

$$= \frac{\sum_{r\in R_k} u_r y_{rj_o}}{\sum_{i\in I_k} v_i x_{ij_o} + \sum_{i\in I_{ns}}\left(\sum_{k} v_i^k\right) x_{ij_o}} \le \frac{\sum_{r\in R_k} u_r y_{rj_o}}{\sum_{i\in I_k} v_i x_{ij_o}} \le 1$$

Thus, constraints (2.2) hold and it can be dropped from model (2). Model (2) is a nonlinear model which delivers assessments for a set of *k* subunits with an output maximisation orientation. Now make the change of variables $z_{ik} = v_i \alpha_{ik}$ and observe that $\sum_{k\in L_i} \alpha_{ik} = 1 \rightarrow v_i \sum_{k\in L_i} \alpha_{ik} = v_i \rightarrow \sum_{k\in L_i} z_{ik} = v_i$. Then, by using the definition of w_{kj_o} and simplification, its objective function model (2) becomes as the following:

$$e_{j_o} = \max \frac{\sum_{r\in R} u_r y_{rj_o}}{\sum_{i\in I} v_i x_{ij_o} + \sum_{i\in I_{ns}} \left(\sum_k v_i^k \right) x_{ij_o}} \tag{4}$$

Using the Charnes and Cooper [9] transformation

$$t = \frac{1}{\sum_{i\in I} v_i x_{ij_o} + \sum_{i\in I_{ns}} \left(\sum_k v_i^k \right) x_{ij_o}},$$

defining $\mu_r = tu_r, \upsilon_i = tv_i, \gamma_{ik} = tz_{ik}$ and $\vartheta_i^k = tv_i^k$, the linear form of model (2) becomes

$$\max \sum_{r\in R} \mu_r y_{rj_o}$$

s.t.

$$\begin{aligned}
&\sum_{i\in I} \vartheta_i x_{ij_o} + \sum_{i\in I_{ns}} \left(\sum_k \vartheta_i^k \right) x_{ij_o} = 1 \\
&\sum_{r\in R_k} \mu_r y_{rj} - \sum_{i\in I_k} \gamma_{ik} x_{ij} - \sum_{i\in I_{ns}} \vartheta_i^k x_{ij} \le 0, \qquad \forall j,k \\
&\sum_{k\in L_i} \gamma_{ik} = \vartheta_i, \qquad \forall i \\
&\vartheta_i a_{ik} \le \gamma_{ik} \le \vartheta_i b_{ik}, \qquad \forall i,\ k \\
&\mu_r, \vartheta_i, \gamma_{ik} \ge \varepsilon, \qquad \forall r,i,k
\end{aligned} \tag{5}$$

The γ_{ik} and ϑ_i obtained from model (5) are required to calculate α_{ik}, i.e., $\alpha_{ik} = \gamma_{ik} / \vartheta_i$. Next, α_{ik} inputs i is used to generate the input data needed for the efficiency evaluation of the kth subunit. As a result, a set of K subunits efficiency scores are obtained.

In many situations, there is a measure that does not lend itself to a subdivision in the procedure described above. Additionally, this measure can also act as a flexible measure. Therefore, if such a measure plays the role of an input factor, it affects all outputs in each subunit k; otherwise, if it plays the role of an output factor, then it will be added to output bundles. In the sequel, we present a PIM model in the presence of such a flexible measure.

3. A PIM model in the presence of flexible measures

Consider the input-oriented CRS PIM model (2), and L flexible measures $\partial_{lj}\ (l=1,...,L)$ in which their input and output status are unknown. As based on the model of Cook and Zhu [16], we introduce γ_l^k as a binary variable for each l, where $d_l^k=0$ means l is an input factor into subunit k, and $d_l^k=1$ means it is an output factor into subunit k. Let γ_l^k be the weight of flexible measure pertaining to each subunit k. Therefore, model (2) can be reformulated in the following mixed integer nonlinear programming form:

$$e_{j_o}=\max\sum_{k=1}^{K} w_{kj_o}\left[\frac{\sum_{r\in R_k}u_r y_{rj_o}+\sum_{l}\sum_{k}d_l^k\gamma_l^k\partial_{lj_o}}{\sum_{i\in I_k}v_i\alpha_{ik}x_{ij_o}+\sum_{i\in I_{ns}}v_i^k x_{ij_o}+\sum_{l}\sum_{k}\left(1-d_l^k\right)\gamma_l^k\partial_{lj_o}}\right] \tag{6.1}$$

s.t.

$$\frac{\sum_{r\in R_k}u_r y_{rj}+\sum_{l}\sum_{k}d_l^k\gamma_l^k\partial_{lj}}{\sum_{i\in I_k}v_i\alpha_{ik}x_{ij}+\sum_{i\in I_{ns}}v_i^k x_{ij}+\sum_{l}\sum_{k}\left(1-d_l^k\right)\gamma_l^k\partial_{lj}}\le 1,\quad \forall j,k \tag{6.2}$$

$$\sum_{k\in L_i}\alpha_{ik}=1,\quad \forall i \tag{6.3}$$

$$a_{ik}\le\alpha_{ik}\le b_{ik},\quad \forall i,k \tag{6.4}$$

$$u_r, v_i, \alpha_{ik}\ \gamma_l^k\ge\varepsilon,\quad \forall r,i,k,l \tag{6.5}$$

$$d_l^k\in\{0,1\},\quad \forall l,k \tag{6.6}$$

where

$$w_{kj_o} = \frac{\sum_{i \in I_k} v_i \alpha_{ik} x_{ij_o} + \sum_{i \in I_{ns}} v_i^k x_{ij_o} + \sum_{l=1}^{L} \sum_{k=1}^{K} d_l^k \gamma_l^k \partial_{lj_o}}{\sum_{k=1}^{K} \left[\sum_{i \in I_k} v_i \alpha_{ik} x_{ij_o} + \sum_{i \in I_{ns}} v_i^k x_{ij_o} + \sum_{l=1}^{L} \sum_{k=1}^{K} \left(1 - d_l^k\right) \gamma_l^k \partial_{lj_o} \right]} \tag{7}$$

represents the importance of each subunit in the presence of flexible measures for DMU_{j_o} under consideration. First note that, according to (7), the objective function (6.1a) is

$$e_{j_o} = \max \frac{\sum_{k=1}^{K} \sum_{r \in R_k} u_r y_{rj_o} + \sum_{l=1}^{L} \sum_{k=1}^{K} d_l^k \gamma_l^k \partial_{lj_o}}{\sum_{k=1}^{K} \left[\sum_{i \in I_k} v_i \alpha_{ik} x_{ij_o} + \sum_{i \in I_{ns}} v_i^k x_{ij_o} + \sum_{l=1}^{L} \sum_{k=1}^{K} \left(1 - d_l^k\right) \gamma_l^k \partial_{lj_o} \right]}$$

Now, by making the change of variables $z_{ik} = v_i \alpha_{ik}$, it is noted that

$$\sum_{k \in L_i} \alpha_{ik} = 1 \to v_i \sum_{k \in L_i} \alpha_{ik} = v_i \to \sum_{k \in L_i} v_i \alpha_{ik} = v_i \to \sum_{k \in L_i} z_{ik} = v_i$$

Then, as seen before in Lemma 1, it is clear that

$$\sum_{k=1}^{K} \sum_{r \in R_k} u_r y_{rj_o} = \sum_{r \in R} u_r y_{rj_o}$$

and

$$\sum_{k=1}^{K} \left[\sum_{i \in I_k} v_i \alpha_{ik} x_{ij_o} + \sum_{i \in I_{ns}} v_i^k x_{ij_o} \right] = \sum_{i \in I} v_i x_{ij_o} + \sum_{i \in I_{ns}} \left(\sum_{k=1}^{K} v_i^k \right) x_{ij_o}$$

Therefore, e_{j_o} becomes

$$e_{j_o} = \max \frac{\sum_{r \in R} u_r y_{rj_o} + \sum_{l=1}^{L} \sum_{k=1}^{K} d_l^k \gamma_l^k \partial_{lj_o}}{\sum_{i \in I} v_i x_{ij_o} + \sum_{i \in I_{ns}} \left(\sum_{k=1}^{K} v_i^k \right) x_{ij_o} + \sum_{l=1}^{L} \sum_{k=1}^{K} \left(1 - d_l^k\right) \gamma_l^k \partial_{lj_o}}$$

Now, using the Charnes and Cooper [9] transformation,

$$t' = \frac{1}{\sum_{i\in I} v_i x_{ij_o} + \sum_{i\in I_{ns}} \left(\sum_{k=1}^{K} v_i^k \right) x_{ij_o} + \sum_{l=1}^{L} \sum_{k=1}^{K} \left(1 - d_l^k\right) \gamma_l^k \partial_{lj_o}}$$

defining $\mu_r = t'u_r, \vartheta_i = t'v_i,\ \gamma_{ik} = t'z_{ik},\ \vartheta_i^k = t'v_i^k,\ d_l^k \gamma_l^k = \delta_l^k$, and by imposing constraints $o \le \delta_l^k \le Md_l^k$ and $\delta_l^k \le \gamma_l^k \le \delta_l^k + M\left(1 - d_l^k\right)$ when M is a large positive number, model (6) becomes the following linear model:

$$e_{j_o} = \max \sum_{r\in R} \mu_r y_{rj_o} + \sum_{l} \left(\sum_{k} \delta_l^k \right) \partial_{lj_o} \tag{8.1}$$

s.t.

$$\sum_{i\in I} \vartheta_i x_{ij_o} + \sum_{i\in I_{ns}} \left(\sum_{k} \vartheta_i^k \right) x_{ij_o} + \sum_{l} \left(\sum_{k} \gamma_l^k \right) \partial_{lj_o} - \sum_{l} \left(\sum_{k} \delta_l^k \right) \partial_{lj_o} = 1 \tag{8.2}$$

$$\sum_{r\in R_k} \mu_r y_{rj} + 2\sum_{l} \left(\sum_{k} \delta_l^k \right) \partial_{lj} - \sum_{i\in I_k} \vartheta_i x_{ij} + \sum_{i\in I_{ns}} \left(\sum_{k} \vartheta_i^k \right) x_{ij} - \sum_{l} \left(\sum_{k} \gamma_l^k \right) \partial_{lj} \le 0,\ \forall j, k \tag{8.3}$$

$$\sum_{k\in L_i} \beta_{ik} = \vartheta_i,\quad \forall i, k \tag{8.4}$$

$$\vartheta_i a_{ik} \le \beta_{ik} \le \vartheta_i b_{ik},\quad \forall i, k \tag{8.5}$$

$$0 \le \delta_l^k \le Md_l^k,\quad \forall k, l \tag{8.6}$$

$$\delta_l^k \le \gamma_l^k \le \delta_l^k + M\left(1 - d_l^k\right),\quad \forall k, l \tag{8.7}$$

$$\mu_r, \vartheta_i, \beta_{ik}, \gamma_l^k \ge \varepsilon,\quad \forall r, i, k, l \tag{8.8}$$

$$d_l^k \in \{0, 1\},\quad \forall k, l\ (8.1) \tag{8.9}$$

The β_{ik} and ϑ_i obtained from the solution of model (8) are required to compute the α_{ik}, i.e., $\alpha_{ik} = \beta_{ik} / \vartheta_i$. Then, α_{ik} are used to the respective inputs i to generate the input data needed for the efficiency evaluation of the kth subunit. As a result, a set of K subunits efficiency scores and optimised values d_l^k are derived. Next, K subunits scores are combined and their weighted average, i.e., w_{kj_o} are computed.

Definition 3.1. DMU_{j_o} in model (8) is efficient if its aggregate score, e_{j_o} is equal to one.

Definition 3.2. DMU_{j_o} is efficient in the kth subunit if $e_{kj_o} = 1$ in the kth subunit.

Note. The kth subunit efficiency score for DMU_{j_o} is given by e_{kj_o}. It is noteworthy that, we obtain e_{kj_o} through the following two steps:

1. At first, we derive a proper split α_{ik} of each input i to each bundle k of which it is a member.
2. Next, by using the split of inputs from step 1, we apply the original CCR-DEA model to each of the k subunits.

It is easy to see that in the model (8), a DMU is efficient if and only if it is efficient in each of its subunits.

Note. The objective value gained in the model (8), for each DMU_{j_o} is always greater than or equal to those objective values gained in models with assuming all flexible measures as input variables or output variables.

4. Case study

Bank industry has been playing a significant role in financial support for the economic development in each country. The banks gaining high efficiency, commonly function with lower costs and do not merge to do perilous market revenues. By contrast, inefficient banks have a tendency to risky periods on the market, and this can be serious for the whole financial system of a country. Therefore, banks should be appraised using the most careful and modern evaluation method until warranting a healthy fiscal system. The proposed approach in the previous section is used to evaluate the efficiency of an Iranian commercial bank. Based on the banking law of Iran, the main task of the commercial bank is to attract customers in that, when receiving deposits from them and distribute them among other customers, they improve their own performance. In addition, customer management and level of staff education are two factors that play significant roles to attract customers. Accordingly, the branches attracting more customers will function more efficiently, and thus compete with each other to gain high efficiency.

In a bank environment, there are multiple types of staff and activities, but not all of them affect all output activities. For instance, administrative staff as an input variable does not entirely and directly impact the receivables as an output variable, thus, evaluating such outputs in terms of such inputs results in distorted efficiency scores. Such partial impacts mean that in some environments like a bank, a DMU (a branch of a bank) has to be considered as a business unit, consisting of several sub-branches, where the activity in terms of outputs made and inputs consumed, differs from one sub-branch to another. Moreover, factors such as poor work performance, lack of skills and training may affect the daily profit of a branch, regardless of the cost of staff available for a bank sub-branch. Also, in a bank setting, to reduce the staff costs, bank managers may be interested in setting up a branch with less numerous staff, while a branch manager may prefer more staff to serve more customers. As a result, the management of the workload would be increased significantly and therefore, the branch manager would overcome the decline in the profits caused by the inefficiency of some existing staff. In other words, the number of staff and hence the number of customers may be an unwanted feature for the bank manager, but for the branch manager, it may be desirable. Therefore, on the one hand, the number of customers plays the role of representative for future enterprise, so it can be categorised as an output. On the other hand, it can be assumed as an environmental input that supports the branch in producing its existing investment funds. Also, the number of customers cannot be an input factor that lends itself to the subunits in the PIM model as other factors. In addition, sufficient discrimination among the efficiency scores in any DEA analysis is often an issue. In this case, and for the setting herein, such discrimination is generally achieved through the PIM model. For these reasons, we classify them as based on their input-output impact and activities. Hence, according to the performance of the bank branches, we have three different subunits (i.e., sub-branches) which are service department, administrative part, and the financial sector. We call these sub-branches k_1, k_2, k_3, respectively. In line with this idea, a bank branch operates like a business unit comprising a set of discrete sub-branches, thus, the efficiency of the branch can be defined as a weighted average of the efficiencies of the sub-branches.

Figure 1 shows the resource splitting versus sub-branches based on which resources /inputs have a membership, and also a flexible measure as an input variable and Figure 2 displays the role of a flexible measure as an output variable. Also, here, there is a non-separable variable as customer-orientation that does not give itself to subdivision as mentioned above. This type of variable is supposed to affect the outputs in each sub-branch when that treats as an input variable. The monetary variables are measured in million rials.

Model (8) is applied to data of a set of 30 branches of an Iranian bank to evaluate their financial performance during 2017 for which the summary of information is reported in Table 1.

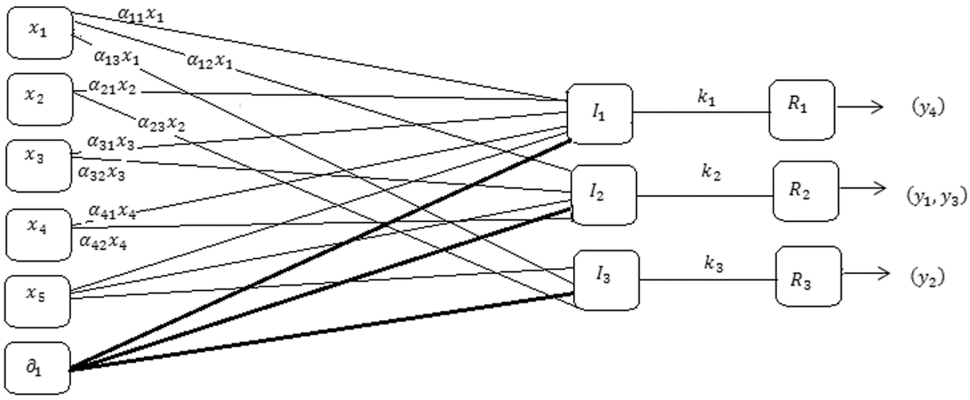

Fig. 1. Resource splitting versus subunits from one DMU and flexible measure (input)

Table 1. Summary of statistics for the inputs and outputs data in bank branches

Input and output	Minimum	Maximum	Mean	STD
Service staff	3	7	5	0.915
Administrative staff	5	12	8	2
Level of staff education	12	20	16	2.024
Customer management	328.53	987	613.1	190.43
Receivables	0	7260	2626	2.173
Documents and services	1494	4741	2939	791
Facilities	21.031	71 888	41 312	12.941
Deposits	44 026	125 229	81 315	19.026
Customer orientation	12	20	16	1.77
The number of customers	892	4348	1746	784.92

It is worth noting that each output variable includes several indicators that can be classified as below:

- documents and services (total average transaction turnover, average payment amount by ATM, total average transaction pertaining to other branches, average balance of accounts associated with POS, etc.),
- facilities (average amount of donated facilities, total number of current facilities, total number of capital facilities, etc.),
- receivables (average cost of the facilities, average amount of receipts of doubtful and delayed receivables, etc.), and
- deposits (daily average of a non-interest current account, daily average of interest-free saving account, daily average of short term deposits, daily average of long term deposits, etc.).

These four product groupings organised the outputs for aims analysis. Also, some input variables include several indicators as follows such as service staff, administrative staff, level of staff education, customer management (organisation behaviour, etc.).

Here, we use 10 variables from the data set as inputs and outputs, and a flexible measure. The inputs include service staff (x_1), administrative staff (x_2), level of staff education (x_3), customer management(x_4), and customer-orientation (x_5) as a non-separable input. It is obvious that customer-orientation is assumed to affect the outputs in each subunit. Also, this variable is a non-proportional input that cannot be separated into each subunit as other inputs in Figs. 1 and 2 while documents and services (y_1), facilities (y_2), receivables (y_3), deposits (y_4), comprise outputs, the number of customers (∂_1), is considered as a flexible measure. The status of this measure for each branch and subunits is determined by the model. Further, (a_{ik}, b_{ik}) ranges for the α_{ik} as needed in the model are set to $(0.1, 0.6)$ that is the largest contribution of any input dedicated to any bundle of which it is a member is 60%, and the smallest is 10%. It is noteworthy that to limit the contribution of any input i dedicated to any bundle (I_k, R_k) of which it is a member, two (a_{ik}, b_{ik}) ranges can be used for α_{ik}, i.e., $(0.1, 0.6)$ and $(0.3, 0.7)$. We use both of them in the study.

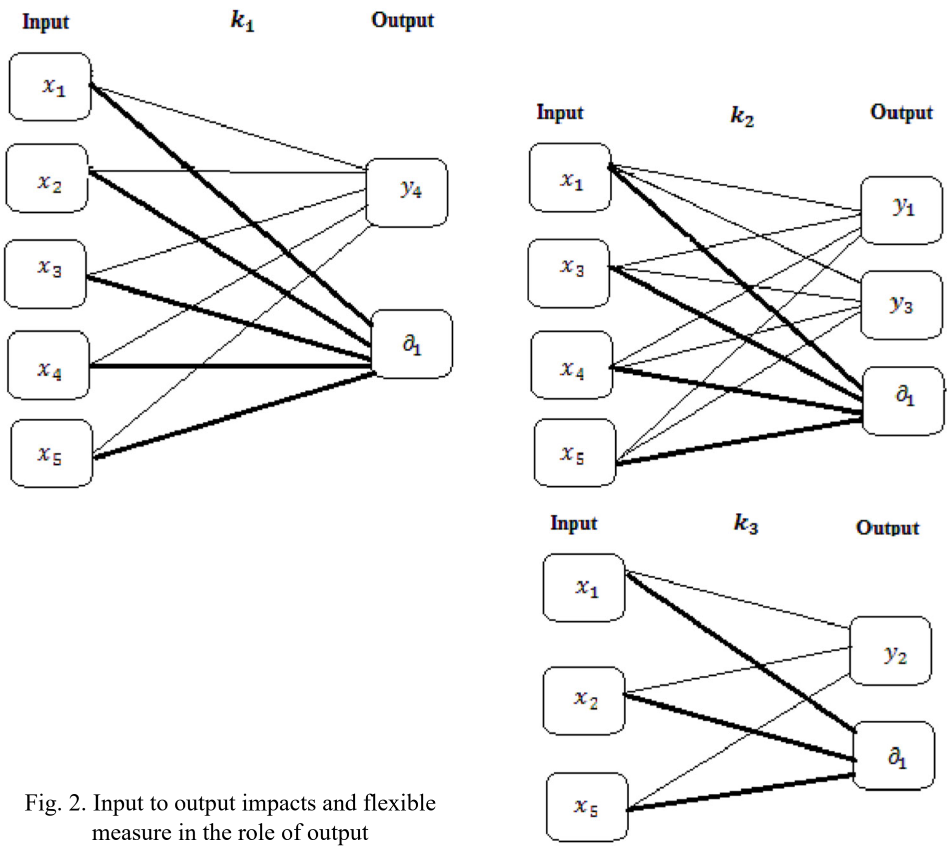

Fig. 2. Input to output impacts and flexible measure in the role of output

For example, the situation of the number of customers as a flexible measure is unknown for each branch. In total, it can play different roles from one subunit to another.

Figure 1 demonstrates resource splitting versus subunits from one DMU and the situation of flexible measure as an input variable. It is worth noting that this factor, as the other variables in this study, cannot be inputs that contribute themselves to a subdivision in terms of partial input to output impacts, since it can only take integer values. Therefore, if the number of customer is treated as an input factor, then it affects the outputs in each subunit *k*. The dash lines in Fig. 1 show this case. Otherwise, if the number of customers is treated as an output factor, then it is added to output variables, and it may be affected by any bundle of inputs that are entirely displayed as three different structures (or, different subunits). Figure 2 displays three different subunits of input-output impacts, and the dash lines on it depict flexible measure connections. In such a case, obviously, the number of output bundles do not make any change. Therefore, each branch of the bank is evaluated as a unit (i.e., DMU), which includes three distinct subunits named k_1, k_2, k_3 (see, e.g., [17, 19] for further information).

Table 2. Efficiency of Cook's model [16][a]

DMU	Flexible	Input	Output	*d*	DMU	Flexible	Input	Output	*d*
1	1.00	1.00	1.00	0 or 1	16	0.84	0.87	0.88	1
2	1.00	1.00	1.00	0 or 1	17	1.00	1.00	1.00	0 or 1
3	1.00	1.00	1.00	0 or 1	18	1.00	1.00	1.00	0 or 1
4	1.00	1.00	1.00	0 or 1	19	1.00	1.00	1.00	0 or 1
5	1.00	1.00	1.00	0 or 1	20	0.26	0.96	0.82	0
6	1.00	1.00	1.00	0 or 1	21	0.98	0.98	0.91	0
7	0.97	0.97	0.97	0 or 1	22	0.81	0.98	1.00	1
8	0.98	1.00	0.98	1	23	1.00	1.00	1.00	0 or 1
9	1.00	1.00	1.00	0 or 1	24	0.66	0.61	0.66	1
10	1.00	1.00	0.89	0	25	1.00	0.89	1.00	1
11	0.92	0.96	0.92	1	26	0.64	0.79	0.78	0
12	0.74	0.74	0.89	0	27	0.97	0.99	1.00	1
13	1.00	1.00	1.00	0 or 1	28	0.31	0.86	0.74	0
14	1.00	1.00	1.00	0 or 1	29	1.00	1.00	1.00	0 or 1
15	0.85	0.85	0.85	0 or 1	30	0.76	0.76	0.76	0 or 1

[a]The number of customers is as input/output/ flexible measure.

The results of flexible CCR model by Cook and Zhu [16] and PIM model (8) are reported in Tables 2 and 3, respectively. The values of *d* in Table 2 indicate that the number of customers should either be considered as an input measure or as an output measure. According to the data from Table 2, in 17 branches, which is 56.6% of the whole branches, that measure is considered as both input and output. Clearly, these numbers of assessment units in classifying flexibility measure cannot be accounted for as input or output. However, among the remaining branches, 7 branches consider that measure as an output measure, and 6 branches as an input one. Therefore, in this case, it can be concluded that the number of customers acts as an output measure. Clearly, since the situation of most branches is unknown, as Table 2 shows, most branches are efficient, thus we cannot derive fair assessment from this measure. On the other hand, in this model one cannot easily specify the situation of the number of customers in branches.

Table 3. Efficiency of model (8)[a]

DMU	Flexible	d_l^1	d_l^2	d_l^3	Input	Output	d
1	0.94	1	0	1	0.91	0.94	1
2	0.88	1	0	0	0.87	0.88	1
3	0.73	1	1	0	0.73	0.69	0
4	0.77	0	1	0	0.71	0.77	1
5	0.75	0	0	1	0.75	0.73	0
6	0.92	1	1	0	0.87	0.92	1
7	0.70	1	1	1	0.64	0.70	1
8	0.80	0	0	0	0.80	0.73	0
9	0.80	0	0	1	0.80	0.74	0
10	0.71	0	0	0	0.85	0.71	1
11	0.65	1	0	0	0.73	0.65	1
12	0.49	1	1	1	0.47	0.49	1
13	0.82	1	0	1	0.81	0.82	1
14	0.90	1	1	1	0.68	0.90	1
15	0.54	1	0	0	0.61	0.54	0
16	0.61	0	0	1	0.61	0.60	0
17	0.93	0	0	0	0.93	0.87	0
18	0.83	1	1	1	0.74	0.83	1
19	0.86	1	1	1	0.81	0.86	1
20	0.50	1	0	0	0.50	0.45	0
21	0.61	0	0	1	0.60	0.61	1
22	0.66	1	1	1	0.57	0.66	1
23	0.76	0	0	0	0.76	0.75	0
24	0.47	1	1	1	0.32	0.47	1
25	0.83	1	1	1	0.63	0.83	1
26	0.48	0	0	0	0.48	0.44	0
27	0.68	1	1	1	0.67	0.68	1
28	0.41	1	0	0	0.41	0.34	0
29	0.76	1	0	1	0.76	0.78	0
30	0.56	1	0	1	0.55	0.56	1

[a]The number of customers as a flexible/ input/output measure.

The results of the model (8) are shown in Table 3. In this table, the efficiency scores of assessment units (after calculating the $\alpha_{ik} = \gamma_{ik} / \vartheta_i$ values) and optimised values of d_l^k are presented under the conditions that the flexible measure has been considered once as input, once as output and once as flexible. As can be seen, the status of the number of customers can be accurately specified, and also there is no common status here. In model (8), the efficiency scores show that assigning a share of each input to each subunit of which it is a member can avoid resources waste. There are different values of model (8) that determine the status of the number of customers as the role of input or output at sub-branches. Now, for comparison, we specify the role of flexible measure through subunits by Cook's model [16].

The second column, Flexible, in Table 3 shows efficiency scores for each of the evaluating unit, and the next three columns display the optimal values of d_l^k in all subunits. Likewise, the eighth column illustrates the optimal values of flexible measure for each DMU that here is shown with the notation *d* that is due to the comparison of the second, sixth and seventh columns. So, in such a condition, the status of a flexible measure can be concluded via the optimal d_l^k values from subunits of each unit. In such cases, if for all subunits of the evaluating unit, all values are $d_l^k = 1$, this measure plays the role of output for the related unit and if for each subunit $d_l^k = 0$, it acts as an input role.

Table 4. Results (k_1) of Cook's model [16][a]

DMU	Flexible	Input	Output	d_l^1	DMU	Flexible	Input	Output	d_l^1
1	1.00	0.00	1.00	1	16	0.84	0.84	0.81	0
2	1.00	1.00	1.00	0 or 1	17	1.00	1.00	0.91	0
3	0.58	0.38	0.58	1	18	1.00	0.96	1.00	1
4	1.00	0.67	1.00	1	19	1.00	1.00	1.00	0 or1
5	1.00	1.00	1.00	0 or 1	20	0.26	0.26	0.24	0
6	1.00	0.79	1.00	1	21	0.98	0.91	0.98	1
7	0.74	0.57	0.74	1	22	0.81	0.45	0.81	1
8	1.00	1.00	0.92	0	23	1.00	1.00	1.00	0 or 1
9	1.00	1.00	1.00	0 or 1	24	0.66	0.24	0.66	1
10	1.00	1.00	0.89	0	25	1.00	0.89	1.00	1
11	0.67	0.67	0.62	0	26	0.64	0.64	0.56	0
12	0.68	0.50	0.68	1	27	0.97	0.97	0.96	0
13	0.98	0.98	0.97	0	28	0.31	0.31	0.29	0
14	1.00	0.55	1.00	1	29	1.00	1.00	1.00	0 or 1
15	0.68	0.68	0.65	0	30	0.76	0.75	0.76	1

[a]The number of customers as a flexible/input/output measure.

A glimpse at Table 4 reveals that 6 branches treat the number of customers either as an input or as an output, i.e., equal to 23.3% of the whole branches which must not be taken into account in classifying inputs and outputs, while 11 out of 24 of the remaining branches consider it as an input. We conclude that 13 branches treat it as an output. Also, according to the majority choice, the flexible measure is identified as an output. The share case in Tables 5 and 6 is equal 5, and in 5 units which are equal to 16.6%. Thus, 11, 10 and 13 branches at the subunits consider this measure as an input, respectively, and 13, 15, and 12 branches consider an output role for this measure. In sum, as based on the majority choice, the flexible measure is identified as an output in subunit k_2, and as an in input in subunit k_3. As can be observed, the status of flexible measure is different in these tables. Therefore, it is not unexpected that one branch considers the measure of the number of customers as an input and in another one as an output.

Table 5. Results (k_2) of Cook's model [16][a]

DMU	Flexible	Input	Output	d_l^2	DMU	Flexible	Input	Output	d_l^2
1	1.00	0.96	1.00	1	16	0.70	0.69	0.70	1
2	0.96	0.94	0.96	1	17	1.00	1.00	1.00	0 or 1
3	0.62	0.60	0.62	1	18	1.00	1.00	1.00	0 or 1
4	1.00	1.00	1.00	0 or 1	19	0.96	0.64	0.96	1
5	0.76	0.76	0.67	0	20	0.68	0.68	0.50	0
6	1.00	1.00	1.00	0 or 1	21	0.66	0.61	0.66	1
7	0.71	0.66	0.71	1	22	0.81	0.66	0.81	1
8	0.69	0.69	0.59	0	23	0.64	0.64	0.58	0
9	1.00	1.00	0.84	0	24	0.63	0.46	0.63	1
10	1.00	1.00	0.62	0	25	1.00	0.68	1.00	1
11	0.64	0.63	0.64	1	26	0.68	0.68	0.58	0
12	0.45	0.45	0.44	0	27	0.94	0.92	0.94	1
13	1.00	1.00	1.00	0 or 1	28	0.66	0.66	0.40	0
14	1.00	0.81	1.00	1	29	1.00	1.00	0.93	0
15	0.65	0.64	0.65	1	30	0.56	0.48	0.56	1

[a]The number of customers as a flexible/input/output measure.

When we apply Cook's model [16] for each subunit separately, it can be seen that there is the share case of the d_l^k in some subunits. For instance, consider branch #17 in Tables 4, 5, and 6. The values of d_l^k are $k_1 = 0$, $k_2 = 0.1$, and $k_3 = 0.1$, while in Table 3 there is not any share case, and the aggregate efficiency score of this branch is 93%, and the optimal values of d_l^k in its subunits are $d_l^1 = 0$, $d_l^2 = 0$ and $d_l^3 = 0$. Therefore, the status of a flexible measure is perfectly known in each subunit from input or output perspective in PIM. It is obvious that after solving model (8), we obtain the unique optimal values of d_l^k for all subunits unlike in Cook's model [16]. Thus, the role of flexible measure can be easily guessed in each subunit in PIM. As can be seen, branches that are efficient in Cook's model [16], in the PIM model may not be efficient, and also there is not any share case in it. Further, we can observe in subunit k_1 from Table 3 that the number of customers plays the role of output in 20 subunits and acts in 10 subunits as input, whereas, in subunit k_2 acts in 12 subunits as output and for 18 subunits as an input. Therefore, according to the majority choice, the flexible measure is identified as output for subunit k_1 and as an input for subunit k_2. Also, as based on the majority choice, for subunit k_3 the number of customers is specified as an output since for 17 subunits it takes the role of output and for the rest of subunits, the role of input. Moreover, the eighth column from Table 3 shows that 15 out of the 30 DMUs treat the number of customers, measure as an input, i.e., the majority of 18 treat it as an output. Then, as based on the majority choice, the flexible measure is considered as an output for all DMUs.

Table 6. Results (k_3) of Cook's model [16][a]

DMU	Flexible	Input	Output	d_l^3	DMU	Flexible	Input	Output	d_l^3
1	1.00	0.98	1.00	1	16	0.66	0.66	0.65	0
2	0.89	0.87	0.89	1	17	1.00	1.00	1.00	0 or 1
3	1.00	1.00	1.00	0 or 1	18	0.96	0.88	0.96	1
4	0.95	0.95	0.85	0	19	0.84	0.79	0.84	1
5	0.61	0.61	0.57	0	20	0.96	0.96	0.80	0
6	1.00	1.00	1.00	0 or 1	21	0.82	0.75	0.82	1
7	0.95	0.89	0.95	1	22	1.00	0.97	1.00	1
8	0.87	0.87	0.77	0	23	1.00	1.00	1.00	0 or 1
9	1.00	1.00	1.00	0 or 1	24	0.62	0.59	0.62	1
10	1.00	1.00	0.78	0	25	0.83	0.56	0.83	1
11	0.95	0.95	0.86	0	26	0.70	0.70	0.59	0
12	0.75	0.74	0.75	1	27	0.51	0.51	0.42	0
13	0.64	0.64	0.63	0	28	0.82	0.82	0.60	0
14	1.00	0.92	1.00	1	29	0.64	0.64	0.56	0
15	0.77	0.77	0.62	0	30	0.58	0.57	0.58	1

[a]The number of customers as a flexible/input/output measure.

In summary, in some sub-branches, the number of customers is regarded as an output, in that it is a source of revenue for the sub-branch. At the same time, arguments are made that staff time expended in processing customers who make deposits or open deposit accounts could be used to better the advantage to sell more profitable products to the customer.

5. Conclusions

In the classical DEA models given a set of accessible measures, it is presumed that the nature of each measure is known as an input or as an output in the production process. However, in some applications, there are measures which can be treated either as an input or as an output. This study extends the literature on banking efficiency evaluation by developing the PIM model. In a bank setting with multiple types of staff and activities, not all activities are influenced by all staff. In other words, some activities can differ from one sub-branch to another hence flexible measures can play a different role in each sub-branch. Therefore, we combine the flexibility with the PIM model of Imanirad and Cook [19] to calculate the aggregate efficiency of DMUs. Unlike the previous studies, in our approach, performance assessment is done by the interior structure of units. We can determine the role of the flexible measure and also its subunits. Evaluating the level of the performance through all sub-branches by PIM model provides more accurate results, and decreases the discriminatory power resulting from Cook's model [16] with respect to the number of efficient DMUs generated. Further, the lack of share case in it is the major advantage of it. As a result, DMUs have a fair evaluation as compared to other. Therefore, PIM model is suggested as the methodology for deriving the most appropriate designations for flexible measures.

The application of the presented approach in 30 branches of an Iranian bank shows that grouping input/output variables and classifying the flexible measures leads to increased aggregate efficiency of branches. Furthermore, this method shows that attention to flexibility where the partial input to output impact exists in branches somewhat led to common status which reaches the minimum and, occasionally, the status of measure is determined through the status of its sub-branches.

The concept of congestion and return to scale has widespread applicability and many valuable expansions. Thus, studying them in the presence of flexible measure could be an interesting future research direction.

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