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CONSTRUCTION OF A FLEXIBLE SIMULATION MODEL OF A CORPORATION

The instability of the real structure of a firm is one of the fundamental problems in simulating microeconomic systems. This paper proposes a method, called ACV (abstraction – gradual concretization – verification) for constructing a flexible simulation model of a corporation. This method is based on the assumption that an effective approach to simulating a microeconomic system should take into account the structural instability of the modelled object. Practical implementation of the ACV method is illustrated using the EK_AN simulator of a firm. The purpose of the simulator as a scientific tool of operations research is to analyse the relations of given inputs (decisions) with the short- and medium-term forecasts of a firm's economic performance.

Keywords: *flexible simulation model, corporation, computer simulator, verification*

1. Introduction

A method, called ACV (abstraction – gradual concretization – verification) has been presented. It enables the construction of a flexible simulation model of a corporation. It is intended to use this model as a scientific tool of operations research to analyse the relations of given inputs (decisions) with short- and medium-term forecasts of a firm's economic performance. The model represents a microeconomic system, in particular an industrial firm.

A dominant characteristic of microeconomic systems, including those of a firm, is structural instability. This instability arises from a firm's drive to adapt its structure to the constantly changing economic environment and correct its management goals. This

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adaptation process is powered by the evolutionary forces *inside a firm primarily for survival, and secondarily for development* [11][3]. Moreover, as Georgescu-Roegen argued, *the evolutionary pace of economic species* (...) *is far more rapid than that of biological species* [9, p. 320][4]. If we keep in mind that constructing a model of a corporation in the form of an analytical simulator[5] is a series of time-consuming activities, then it is not uncommon that when a model has been finally developed, the structure of the system has changed so significantly that the mapping is only important for historical reference. The fact that the pace of construction falls behind the dynamic of structural changes in the economic system is reported as one of the main causes of failures in the practical application of models of corporations [10]. Thus an effective method for simulating a microeconomic system should take into account the structural instability of the mapped object, and, in effect, produces a flexible, in the structural sense, model of a corporation.

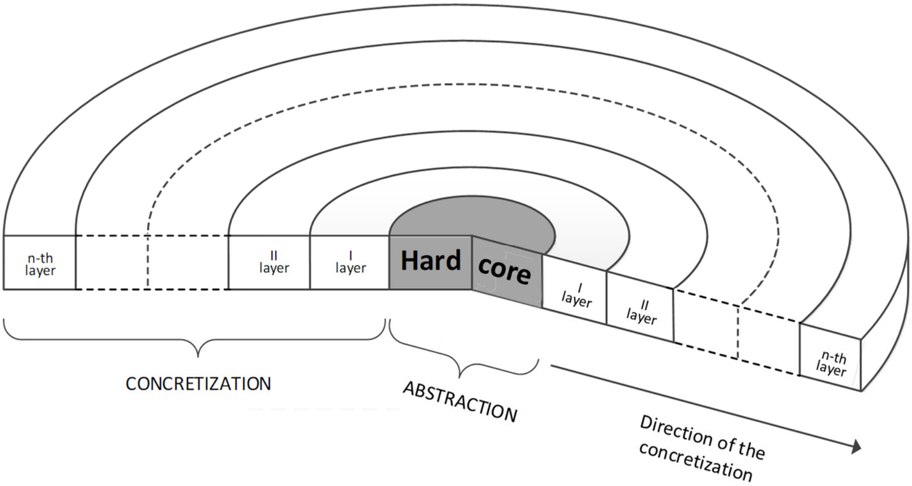

Fig. 1. The layered structure of the economic system simulator

To fulfil the requirement of flexibility, we propose to apply the ACV method as a skeleton for constructing models of microeconomic systems, industrial firms included. According to the ACV method[6], the simulator has been designed as a layered structure (Figs. 1, 2).

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[3] The instability of the subject matter under consideration is one of the fundamental problems in simulating economic systems as mentioned by Ang and Chua [1] and Ford et al. [6]. Based on surveys conducted among users of models of corporations, they found that the rigidity of a simulator's structure is the main factor limiting the practical usefulness of simulation as a scientific tool of operations research.

[4] In the opinion of Marshall, *the Mecca of the economist lies in economic biology rather than in economic dynamics. However, we have no choice but to start with economic dynamics* [13, p. XIV].

[5] Henceforth, a model in the form of a computer program will be referred to as a simulator.

[6] The initial concept of the ACV method has been presented in [17].

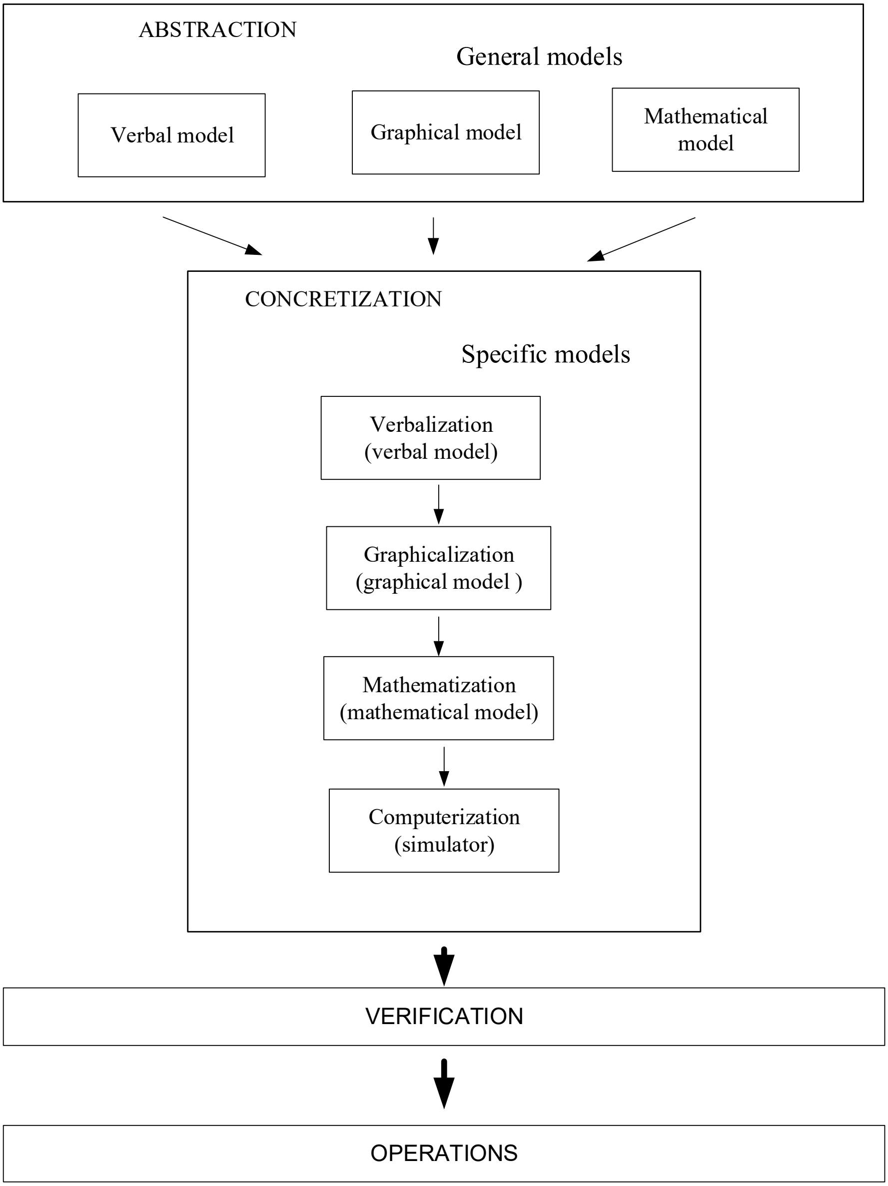

Fig. 2. The building of the computerized model of a firm

General models proposed at the stage of abstraction consist of the simulator's *hard core* (using the phrase from Lakatos [12]) which represents those properties that are immutable in terms of place and time, relevant to all things that belong to the class of economic systems. The core is enwrapped in layers (*protective belts*) defined at successive levels of gradual concretization. When the structure of the real system changes, then it might only be necessary to adapt the outer layer of the model, without disturbing the logic of its inner layers and with the hard core of the model staying intact.

2. Abstraction

At present, there is no consensus on how to perform the process of abstraction to be assured that the final products will be constructs that reflect only those qualities that are essential, universal and timeless elements of the subject matter under consideration. Because in economics as a science, there is no generally accepted interpretation of "Ockham's razor", the process for separating and filtering what is relevant from irrelevant is dependent upon arbitrary decisions of researchers. One of the decisions that must be taken by the modeller is to choose a mapping technique that will represent the informative products of abstraction. This is a very difficult decision to make, particularly if we are aware that each approach to real-life mapping in the form of a model has specific possibilities and limitations. Choosing a given mapping technique always results in unrecoverable losses of information about the system, regardless of whether we consider this information important or irrelevant. For example, mathematical mapping ignores information about a system that is immeasurable, these sorts of economic categories can be mapped using the semantic capabilities of verbal models. However, it is imprecise and ambiguous.

This one-sidedness of an abstract model is particularly troubling in how it views economic systems, which are extremely complicated conglomerations of qualitative and quantitative characteristics, static and dynamic structures, and continuous and discrete processes. Ignoring important information about a complex system only because of limitations of the mapping technique selected must be regarded as a major epistemological flaw. The contradiction between the complex nature of an economic organism and the one-sidedness of its representation in a model brings us closer toward postulating cognitive parallelism as a remedy to *the troublesome diversity*[7] of economic systems. This postulate states that the process of abstraction should result in a number of models developed by using alternative mapping techniques. In this case, accepting a variety of forms of mapping is a methodological manipulation that increases the chance of obtaining (using one-dimensional models) an adequate representation of the complexity of an economic system. These models are different, but at the same time complement one another in terms of mapping techniques. Therefore, they will describe a company from a few alternative points of view and thus – at least partially – collectively compensate

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[7] The authors owe this phrase to Boulding [4].

for their individual imperfections. This parallel method of information gathering indicates the analytical nature of general models (as products of abstraction), not in the sense of the decomposition of a system into its elements, but in breaking the raw, haywire image of an economic system into separate pictures of different forms and therefore of distinct informative content[8]. The principle of parallel modelling at the level of abstraction will be observed when the system considered is a firm, the method used – simulation, and the model's purpose is to act as a scientific tool of operations research. Considering these factors, we decided on three alternative ways of recording what we consider the essence of economic systems, namely verbal, graphic and mathematical models.

A verbal model. At the stage of abstraction, a verbal model is the product of the qualitative analysis of properties that describe in words a given group of phenomena and processes and determine its distinctiveness, and internal – despite individual differences – unity and similarity. The natural coherence between the ambiguity of verbal models and the impossibility of precisely observing an economic system implies that models of this type are irreplaceable for gaining an intuitive understanding of the essence of qualitative phenomena in complex systems. Everyday language is very good at addressing the ambiguity, uncertainty, and fuzziness of an economic system, giving one the ability to record in the form of a verbal model, *what is logical and what is sensuous, what can be seen and what cannot be seen* [20, p. 176]. Bertalanffy emphasizes: *we cannot eliminate verbal models, because we must see problems intuitively and visualize them* [3, p. 40]. Verbal description may reveal subjects that are far beyond the scope of mathematical formulation, while proving tremendously important in the determination of behaviour.

As mentioned before, the ACV procedure has been practically applied in constructing the EK_AN model, see Section 4. An excerpt from the verbal model of a firm that was formulated at the stage of abstraction is given below:

A company is managed or influenced by:

- external decision-making centres, especially government agencies, which are responsible for achieving national economic objectives,
- internal decision-making centres, especially the owners of the company, who expect the maximum return on invested capital.

If, however, because of the objectives of modelling we need something more than just an intuitive understanding, particularly when we are dealing with a problem requiring – in addition to common sense – strict deductive reasoning, we are forced to use

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[8] The general principle of creating a model of an economic system using the concept of parallel abstraction has its roots in the methodological pluralism of Feyerabend. He believes that, *pluralism is an essential feature of all knowledge which claims to be objective* [5, p. 25].

other, not so naturally compatible, forms of describing economic phenomena and processes, i.e., graphical and mathematical models.

A graphical model. As a method of recording information about the system at the stage of abstraction, a graphical model is a natural tool for visualizing the findings of structural identification. Causal connections and hierarchical relationships are forms of interdependences between system components that can be properly and transparently mapped in the form of graphic diagrams. Figure 3 is a selected example of a graphical model of a firm that is formulated at the stage of abstraction during construction of the EK_AN.

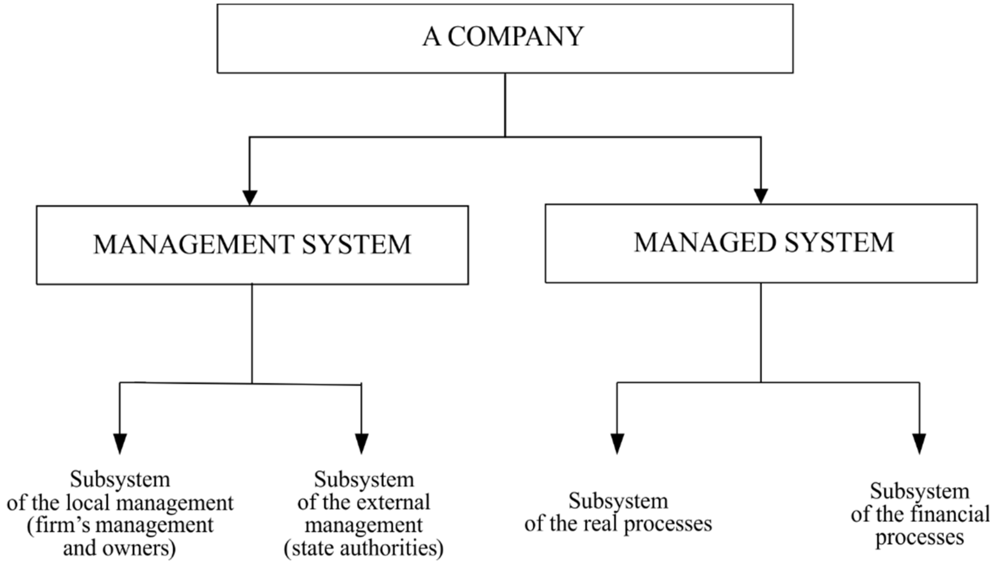

Fig. 3. Stage of abstraction, a graphical model

As the number of elements grows (which is characteristic of the identification of complex systems), graphical models become complicated and lose their explanatory power. Above all, however, the representation of a company in the form of graphics (and in verbal form as well) is useless if we are going to transform information about the system univocally, which is what simulations do. Given the limitations of verbal and graphical models, one should (according to the principle of cognitive parallelism) map an economic system anew, this time applying methods of quantitative analysis, and record information about the system under study using a suitable form, i.e., a mathematical model.

A mathematical model. A mathematical model is an abstract model that describes a system in the language of mathematics and logic. As a representation of a real-life economic system, a mathematical model is the final product of quantitative analysis. At the abstraction stage, the crucial task of quantitative analysis is to separate the measurable aspects of an economic system from those economic phenomena that cannot be correctly described by formal languages using any measure. The correctness of this separation is essential for the success of further stages of research based on simulations, because – in contrast to real experiments – simulations are not directly cognitive. In fact, a simulation is nothing more than converting into new configurations information about reality that has already been coded by a modeller in the form of the model together with input data. This information is nothing more then the petrified knowledge of a modeller about a real system, because, travestying the view of genetic empiricists, *nihil est in modo simulari, quod non prius fuerit in intellect*[9, 10].

Therefore, if, at the stage of abstraction, the principle of homomorphism between the mathematical representation and reality has been violated, then any transformation of the model at the stage of concretization does not eliminate inconsequence introduced at the stage of abstraction. Only at the stage of verification, where again we evaluate the basic relationships between the model and reality, it becomes obvious that the logic of the model's form cannot be a substitute for the logic of its informative content, and the precision of computer calculations is not equivalent to the accuracy of our view of reality.

When we start to model a firm as a formal structure, the crucial dilemma is the choice between two options: the continuity or discontinuity of functions that represent the dynamics of the variables modelled. If the company is observed and analysed from the macroscopic perspective that is characteristic for the stage of abstraction, one may accept the hypothesis that the dynamic relations classified as relevant and to be mapped in a abstract model of the system are so stable in the period under study that we can treat these characteristics as continuous. It follows that the appropriate mathematical representation of these relations are continuous functions defined in the modelling interval. Such an assumption about the continuity of these functions allows us to formulate, at the level of abstraction, a mathematical model of the firm according to the following structure[11]:

$$M^C = (T, G, Z, f) \tag{1}$$

where: T is an interval of real-time, in which t_p, t_k represent the boundary moments of the simulation, $G = \{g(t) = (g_1(t), ..., g_n(t))\}$ is a set of functions defined on (t_p, t_k), and

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[9] The authors of the paper share the views of those who argue that, the sense of reasoning supported by a model is that tending to know what is not yet known to us, in fact we examine what was already included the premises of the model. A model does not discover anything; it only orders in new ways the information that was already contained in a model's assumptions and inputs. However, the new configurations generated by simulations, may stimulate a researcher to creative reflection and, in fact, this is the only value of modelling as a way to explore the reality that surrounds us.

[10] Leontyeff wrote: *the uncritical enthusiasm for mathematical formulations makes often that ephemeral content of a paper is hidden behind a terrifying façade of algebraic characters* [14, p. 13]. In other words, econometrics too often overuses mathematics to camouflage its own cognitive helplessness.

[11] This model has been formulated based on notation proposed by Ziegler [21].

$g_j(t)$ is the jth input function. Z is a set of functions defined on the interval (t_p, t_k), $Z = \{z(t) = (z_1(t), ..., z_s(t))\}$, and $z_j(t)$ denotes the jth state function.

On the interval (t_p, t_k), $z(t)$ is the solution of the following differential equation:

$$\frac{dz(t)}{dt} = f\left(z(t),\ g(t)\right) \tag{2}$$

with initial condition $z(t_p) = z_p$, where f is the state transition function f: $Z \times G \to Z$.

3. Concretization

The stage of abstraction produces general (verbal, graphical, and mathematical) models, which are hypotheses, expressed in alternative forms, about the main regularities that govern the processes and structure of a firm. However, any form of abstract model is unsuitable for performing computer simulation. Only through the gradual concretization from higher to lower levels of generalization, in effect adding details about the system under study and attaching specialized software modules, will a model achieve the degree of specificity and level of technical efficiency that enables performing computer simulation.

Concretization not only refers to increasing the specificity of a model, but also, and perhaps more importantly, is involved with a different view of reality. If the basis of abstract observation is Plato's standpoint that *universalia sunt in rebus*, i.e., that there is a stable structure of elements and relationships independent of time and space, then at the stage of concretization, one tries to see what are the signs of the times and place in which an economic system is situated. This change of perspective used for observing a system results in the need for reassessment of the hypothesis about the stability of the elements and relationships mapped by the model. This hypothesis, which is reasonable at the stage of abstraction, in which we record what is permanent and unchanging, becomes questionable as we approach concretization. From a concrete perspective that spans no more than a few years, it becomes obvious that structural instability is a dominant characteristic of micro-economic systems, including those of a firm.

As far as a model of the firm is concerned, the approach to concretization is determined by our earlier decision on how to carry out the stage of abstraction. The postulate of cognitive parallelism, which was accepted as a paradigm of abstraction, implies that the starting point for concretization is not a single unified general model, but a number of them, with each one having been built using alternative mapping techniques. Thus, concretization will rely not only on attaching an increasing number of specific elements and relationships, but also on the synthesis of parallel general models into one integrated specific model. Because the set of general models consists of three different types of mappings (verbal, graphical and mathematical), the concretization stage is conducted as a linear process that involves three consecutive phases: verbalization, graphicalization and mathematization.

The verbalization phase. In the verbalization phase, the economic system defined at the stage of abstraction is concretized in time and space. Concerning the cognitive goals of the simulator, for verbal concretization, we selected an industrial firm (concretization in space) that operates in an economic system typical of the beginning of the 21st century, the so-called 2000+ system (concretization in time). Below, an initial description (a verbal model) of the 2000+ system is given:

A firm which is the object of modelling is:

- an industrial company because of the type of business,
- a joint-stock company because of the form of ownership,
- active in the Polish economy due to the area of its operations,
- running under the legal system defined by the Polish Accounting Act of 29 September 1994.

The graphicalization phase. In the graphicalization phase, a map of the static and dynamic interdependencies for the 2000+ system are identified in the form of graphs and diagrams. These figures were constructed on the basis of information defined at the stage of abstraction, as well as in the verbal description of the 2000+ system. Figure 4 is an example of the graphs constituting a part of the graphical model of the 2000+ system. As was said, the 2000+ system makes legal and economic environment for the EK_AN model.

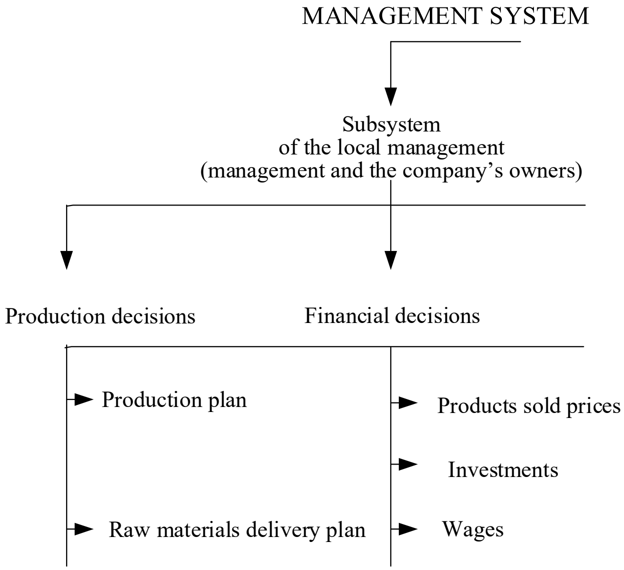

Fig. 4. Stage of concretization, a graphical model

The mathematization phase. During the mathematization phase, mathematical and logic formulas are assigned to each interdependent relationship illustrated by diagrams defined as a graphical model, the concretization stage. The transition from an abstract mathematical model to a concrete one involves a radical change of the modelling perspective. Characteristically, at the stage of abstraction, the macroscopic perspective allows us to assume that the crucial processes in the economic system observed can be treated as continuous ones. But at the stage of concretization, we focus our observation on microscopic details, when the duration of observations is limited to a couple of years, the same processes appear to us as rapidly changing phenomena. As a result, we must accept that the continuous sequence of functions, acceptable at the level of abstraction, breaks down at instantaneous events, such as: completion of an investment, liquidation of a cash deposit or loan payment. For these reasons in the mathematical model, the stage of concretization, the assumption about the continuity of dynamic characteristics must be repealed.

In consequence, the final product of this phase is a mathematical model of the system in the form of a set of dynamic non-continuous differential equations. As a mathematical representation of the EK_AN system, we propose the following structure:

$$M^{c-d} = \left(T, G, Z, f, \beta\right) \tag{3}$$

where: T is an interval of real-time, in which t_p, t_k represent the boundary moments of the simulation, $G = \{g(t) = (g_1(t), ..., g_n(t))\}$ is a set of functions defined on (t_p, t_k), and $g_j(t)$ is the jth input function. The set of the points of discontinuity of the function $g_j(t)$ is denoted by T_j^d, thus

$$T_j^d = \left\{t_{1,j}, t_{2,j}, ..., t_{l(j),j}\right\}$$

Furthermore, let

$$T^d = \bigcup_{j=1}^{n} T_j^d = \left\{t_1, ..., t_i, t_{i+1}, ..., t_m\right\}$$

where m is the total number of points of discontinuity of $g(t) \in G$.

Z is a set of functions defined on the interval (t_p, t_k); $Z = \{z(t) = (z_1(t), ..., z_s(t))\}$, and $z_j(t)$ denotes the jth state function. On the interval (t_i, t_{i+1}), with t_i and t_{i+1} being two consecutive elements of T^d, $z(t)$ is the solution of the following differential equation:

$$\frac{dz(t)}{dt} = f(z(t), g(t)) \tag{4}$$

with initial condition $z(t_i) = z_i$, where f is the state transition function $f: Z \times G \to Z$.

For each $t_i \in T^d$, there exists a static function β_i: $R_z \to R_z$; also for t_k β_k: $R_z \to R_z$ where R_z represents the range of the function Z.

Concretization of the mathematical model for the EK_AN system will be carried out in this way, such that each state function that belongs to Z constitutes a partial hypothesis about the correspondence between a differential equation and the dynamics of this function in a real system.

The computerization phase. At present, there is no method of analytically solving the set of Eqs. (4). Consequently, an exact solution is beyond our reach. In practice, the only effective means of solving a set of such equations is to employ computerized numerical methods. For this reason, the specific mathematical model of a firm must be rewritten, in a separate phase of concretization, called computerization, in a form that complies with the requirements of information processing.

Converting a mathematical model into a computer program consists not only of a chain of trivial IT operations but is a process that involves setting up further assumptions, simplifications and arbitrary choices concerning the method of numerical integration to be employed, characteristics of the software utilized, and the technical parameters of simulation. Therefore, we consider this part of modelling as a distinct phase of concretization, called computerization, which completes the linear sequence of concretization.

In the phase of computerization, we can utilize numerical methods to find approximate solutions of Eq. (4). If we apply, for example, the forward Euler method, then we should redefine some sets in the structure given by Eq. (1):

$$M_E^{c-d} = \langle T, G, Z, f, \beta, \Delta t \rangle \tag{5}$$

Let $(t_k - t_p)/\Delta t = b_k$; then b_k is replaced by n'_k, which is the nearest integer to b_k, such that $|t_k - t'_k| < \Delta t/2$. It follows that $t_k \cong t'_k = t_p + n'_k \Delta t$. Then $t_i \to t'_i$, where: $t_i \cong t'_i + n'_i \Delta t$, $|t_i - t'_i| < \Delta t/2$, $T^{d'} = \{t'_1, ..., t'_i, t'_{i+1}, ..., t'_m\}$.

Basing on the above formulas, we are allowed that in each time interval (t'_i, t'_{i+1}) we have r_i points on t where $r_i = (t'_{i+1} - t'_i)/\Delta t$. Thus for $h \in \{0, 1, ..., r_i\}$

$$z\left(t'_{i,h+1}\right) = z\left(t'_{i,h}\right) + \Delta t f\left(z\left(t'_{i,h}\right), g\left(t'_{i,h}\right)\right), \text{ and } z\left(t'_i\right) = z_i \tag{6}$$

A similar situation applies to the interval (t_p, t_1'). For each $t_i' \in T^{d'}$, $\beta_i : R_z \to R_z$, also for t_k', $\beta_k : R_z \to R_z$.

We should be aware that the proposed numerical solution (6) of Eq. (4) is inaccurate, because of, e.g., floating-point, round off and truncation errors. Moreover, for a certain unknown-in-advance set of input data, there is no guarantee that the imprecision resulting from a global error in approximation is damped rather than accelerated. Such acceleration could cause the global deviation from the precise form of the antiderivative to grow exponentially. In consequence, the iteration process would not be stable numerically. This weakness of numerical methods substantially reduces the confidence of a researcher in a continuous-discrete simulator as a scientific tool of operations research. This provides another reason that, according to the principle of ceaseless criticism [18], each cognitive experiment using the simulator should also be a verification experiment.

The product of the computerization phase and the final outcome of concretization is a scientific tool of operations research in the form of a flexible simulator of a firm. During verification, the simulator will be subject to tests to assess the validity of the construction process, i.e., the stages of abstraction and concretization.

4. Verification procedure. The case of the EK_AN simulator

The process of constructing a model of the firm involves a series of numerous assumptions, simplifications and arbitrary choices. Therefore, verification is a crucial stage of the ACV procedure. At this stage, it is assessed whether the flexible simulator can be accepted as a scientific tool of operations research. This evaluation will be carried out based on the methodological recommendations formulated by the authors of [18] as the RAD–VER procedure. In this procedure, it is assumed that verification (in fact, attempting to refute) is a ceaseless process of the evaluation of the model's scientificness from the standpoints of deductive reasoning, coherency and empiricism. The practical implementation of the proposed procedure is illustrated by the verification of the EK_AN simulator of a firm, see Section 4.2.

4.1. The verification procedure

In the RAD–VER procedure, testing is divided into two steps: the verification of the simulator and the verification of the assumptions underlying the model of an economic system, in our case, a firm[12]. In the case of the EK_AN simulator, this is an industrial firm that operate in a concrete economic and legal environment.

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[12] This part of the text is based on a previous paper by the authors, see [18].

Verification of the simulator. From the point of view of sentential calculus, a computer program, i.e., the simulator, is a set of analytical sentences. The verification of the simulator must indicate whether the transformations – carried out in the course of a simulation – of the synthetic sentences in the form of the model's underlying assumptions, together with the input data (synthetic sentences) via the simulator (analytical sentences), into output data (synthetic sentences) may be regarded as a flawless, tautological chain of deductive implications. Therefore, the deductive criteria derived from mathematical and formal logic can be accepted as appropriate foundations for verifying the internal consistency of the simulator. Considering that the system under study is an economic one, deductive accounting rules are also applied, in the case of the EK_AN, this takes the form of the BSV module, see Section 4.3.

Verification of the model's assumptions. Positive verification of the simulator as a deductive machine opens the way to verifying the model's assumptions (Fig. 6). The verification of the model's assumptions can be conducted from the standpoints of deductive reasoning, coherency and empiricism.

Firstly, we *a priori* accept a set of sentences that consist of the assumptions of the model. Thereafter, simulation tests are performed. The outcomes of these runs are subjected to analysis that leads to implications in the form of basic, simulational sentences. This set constitutes the simulational basis of verification.

As a parallel stage of the verification procedure, statements about the behaviour of a real firm based on observation are derived inductively by generalizing experience. The set of these sentences constitutes the empirical basis for verification.

The syntactic structure of the simulational sentences should be as close to the syntactic structure of the observational statements as possible. This enables us to directly compare (so called sentential confrontation) a particular simulational statement with its counterpart in the empirical base. In the case of the EK_AN simulator, the procedure of sentential confrontation is described in Section 4.3.

Positive verification of the simulator, as a deductive machine, and the positive results of sentential confrontation, allow us to state the correctness of the model's assumptions on the basis of the *modus tollens* rule of inference. In propositional logic, *modus tollens* is an application of the general truth that if a statement is true, then so is its contra-positive. In our case, this rule of inference is as follows: model's assumptions → sentential confrontation and ¬ sentential confrontation, then ¬ model's assumptions. In other words: if the model's assumptions are accepted *a priori* as TRUE, it implies that sentential confrontation must be TRUE but if sentential confrontation is FALSE, it implies that the model's assumptions must be FALSE.

If sentential confrontation fails, then the statement about the validity of the model's assumptions must be rejected. A negative appraisal from the verification procedures requires performing a critical analysis of the detailed assumptions made during concretization, and in some cases it might be necessary to rebuild the general hypotheses formulated at the stage of abstraction. In turn, positive confrontation of the outcomes of simulations with reality allows us to state that, so far as the RAD-VER procedure is accepted, the statement about the validity of the model's assumptions has not been falsified.

At the operational stage, the validity of each cognitive experiment is also systematically assessed by the verification sub-model, according to the previously accepted postulate that "each simulation experiment is also a verification process".

It is expected that, due to the ACV method applied, we are able to make the next step toward developing a concept of flexible models. This means that the simulator of a firm has been adapted a priori to ongoing modifications that will be made in the software to maintain the required compatibility between the unstable structure of a real system and the structure mapped in the model.

4.2. A flexible simulator of the 2000+ firm. The EK_AN simulator

The final product of the concretization stage is a flexible, continuous – discrete simulator called EK_AN with a model of the 2000+ firm embedded. The simulator has been built using the ACV method. In mathematical terms, the simulator is based on a set of dynamic, discontinuous differential equations (Eq. 5) using the principle of system dynamics [2, 7, 15]. In computational terms, the simulator is a stack of sequentially processed numerical procedures which execute a simulation, in particular, numerical integration (Eq. (6)) and discrete event processing (Eq. (7)), including verification trials. The purpose of the simulator as a scientific tool of operations research is to perform controlled, *ceteris paribus* simulations in a computer laboratory. It is expected that by actively participating in the modelling and simulation, an experimenter can enrich his knowledge about the internal causes of the dynamic phenomena observed in a firm[13].

As was said an industrial firm (concretization in space) simulated by the EK_AN system operates under set of legal and economic rules typical of the Polish economy (concretization in a place) at the beginning of the 21st century, the so-called 2000+ system (concretization in time). This simulator has been employed practically for carrying out structural tests of the 2000+ system. The framework of the 2000+ system is based on the Law on Accounting passed by the Polish Parliament on 29th September, 1994. In particular, the aim of simulation experiments has been to evaluate the coefficients of observability and controllability so far as they apply to the leading indicators (eg., net profit) applied in the 2000+ system. It is well known that the main task of leading indicators is to provide a synthetic assessment of the overall economic activity of a company. For this reason, these indicators should react to any internal or external event that may have an impact on the company's performance (e.g., volatility in the

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[13] A detailed description of the EK_AN system is available at www.eprotal.pwr.edu.pl, the files: ekan.zip – a source code for the EK_AN system with a model of the 2000+ system included, ekan.exe – the system EK_AN ready to use.

prices of raw materials). In other words, these indicators should be related to capacity, due to the changes taking place within a enterprise or its immediate vicinity. The reactions of leading indicators should indicate not only the occurrence of an event, but also inform about the conformance of its effects with the criteria for rational management. Based on the results of simulation tests carried out in *ceteris paribus* conditions, the coefficient of observability is calculated for a given leading indicator. Large values of this coefficient suggest that the information function of the indicator is appropriately satisfied.

Another important property of the leading indicators is controllability. The coefficient of controllability informs us how accurately internal states can be inferred from the factors under the control of state institutions that affect overall economic performance. Using the EK_AN system as a simulation platform, the vulnerability of the 2000+ system to changes in such factors (e.g., rates of taxation) was examined.

4.3. The practical implementation of the RAD VER procedure to verify the EK_AN simulator

The proposed procedure for verifying the EK_AN simulator entails three stages : construction, testing and operations (Fig. 5).

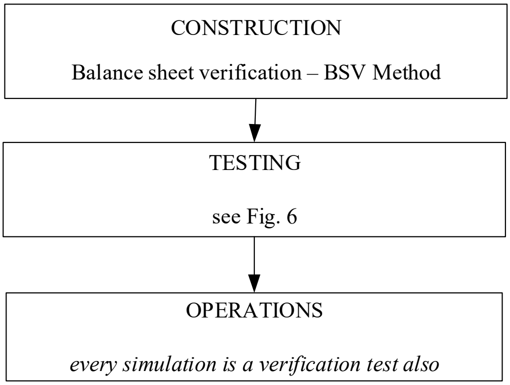

Fig. 5. Verification procedure for the EK_AN model

Construction. The balance sheet verification (BSV) method. The aim of the construction phase of the verification process is to create a software environment that enables fulfilling in practice the following two postulates of the ACV method, to the extent that they apply to the EK_AN simulator:

- a postulate addressing the concept of verification, namely the postulate of ceaseless criticism,
- a postulate addressing the concept of model building, namely the idea of a flexible simulator of a firm.

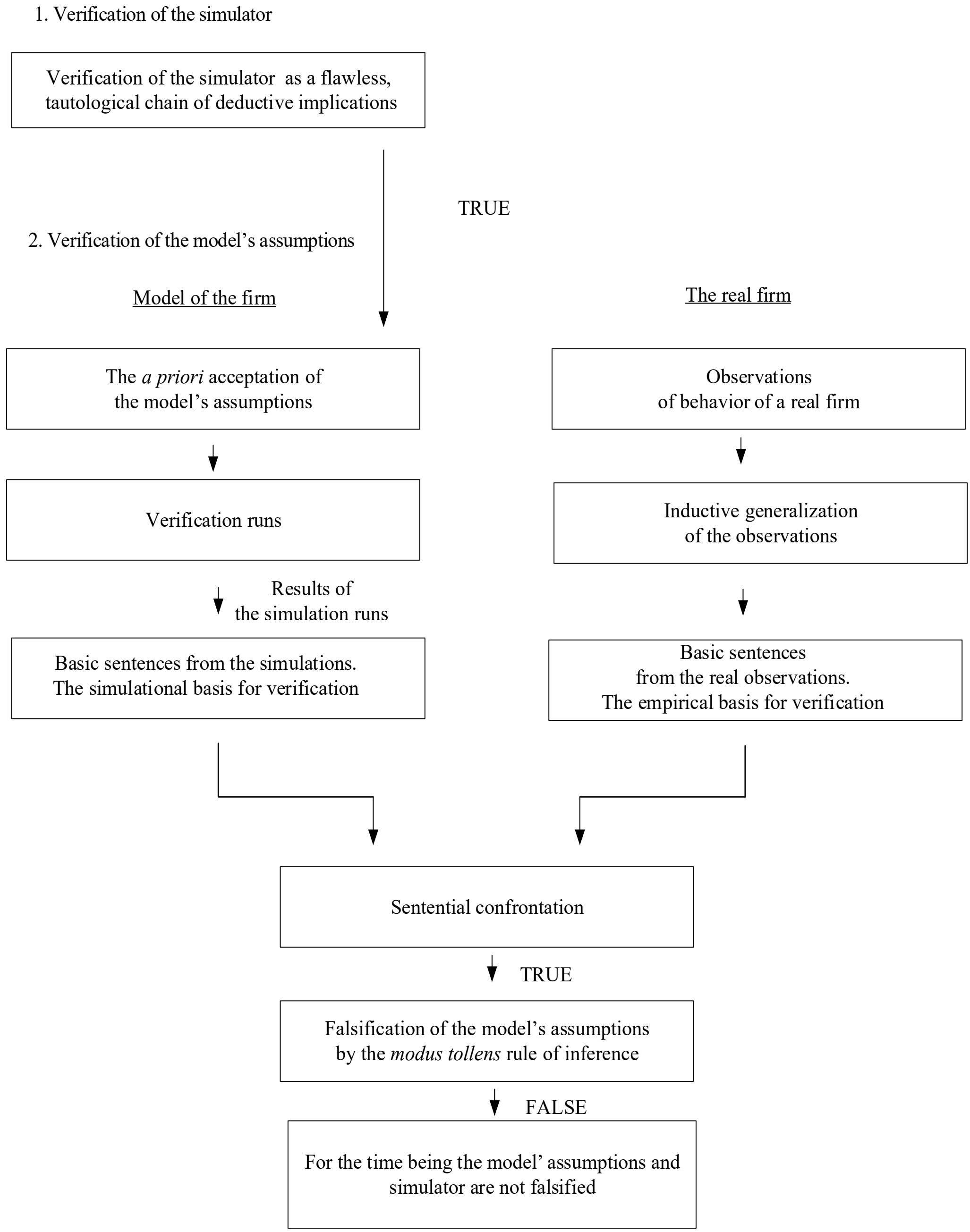

Fig. 6. Verification of the computerized model of a firm.
The RAD-VER procedure, testing

It should be pointed out that the RAD-VER procedure implies that each cognitive experiment using the simulator should also be a verification experiment. To fulfil this requirement during the construction process, the verification modules must be developed as an integral part of the simulator software. These modules are used to automatically test each simulation run.

The principle of ceaseless criticism underlying verification is inseparably tied to the concept of a flexible simulator to be understood as a computer model which is structurally prearranged for numerous modifications. Any modification of the structure of the model forces the modeller to repeat all of the – time consuming – verification tests. Therefore, taking into account the effectiveness of simulation as a scientific method, the aim of computer programming in the construction phase is to rationalize the verification process by automizing it to the greatest extent possible.

The proposed approach of ongoing verification is named the balance sheet verification (BSV) method. Using concepts borrowed from accounting, certain identities, set up on the basis of the model's state variables, are constantly checked by two, completely independent, calculation systems. When the deductive reasoning executed by the simulator is flawless, then these identities must hold, regardless of what values are plugged in for the input variables and no matter what modifications of the model's structure have been made.

Completion of the construction stage opens the way to the testing (experimental) stage of verification (Fig. 6), when the newly constructed simulator is assessed via verification runs. A positive result from these tests allows us to accept the model as a scientific tool of operations research.

Testing. After completing the construction stage, verification experiments are performed with the final form of the model of the firm (the EK_AN simulator). Positive outcomes from these experiments give us authorization to utilize the simulator as a scientific tool of operations research. When setting up the scope of verification tests, we should take into account that the EK_AN simulator is based on assumptions and hypotheses that are gradually set up in the successive phases of model building (see the section on Concretization). Therefore, it appears appropriate that in the first step of the verification tests, numerical errors that might be made in the computerization phase are traced and counteracted; then, mathematical and logical (mathematization) errors, structural (graphicalization) errors and, finally, functional (verbalization) errors are eliminated (Fig. 7).

In the computerization and mathematization phases of the experimental stage, the EK_AN simulator is verified from the standpoint of deductive reasoning. In other words, statements derived from the output of simulations are compared with statements inferred from the laws of logic and mathematics. Positive results from these tests imply that the EK_AN simulator can be used as a computational device for transforming the input data into simulation outcomes.

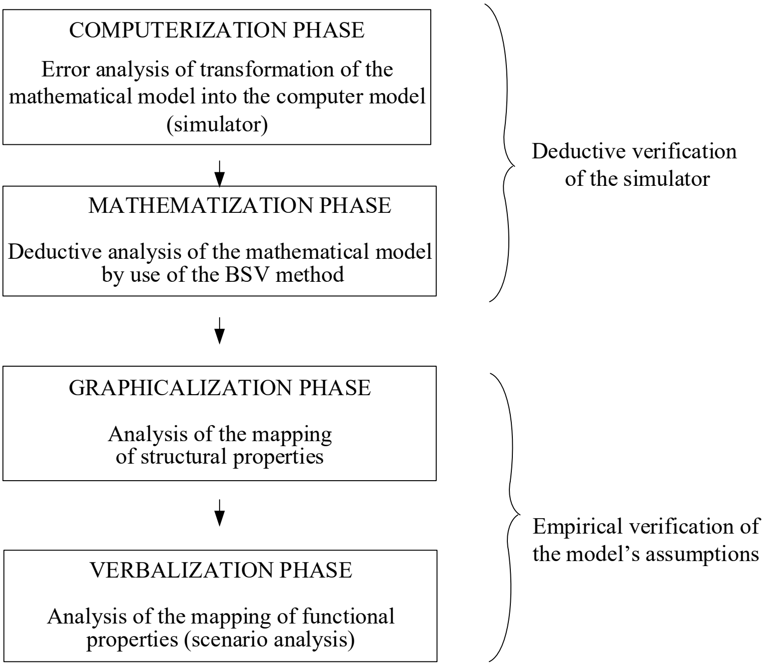

Fig. 7. Verification procedure for the simulator of a firm. Experimental stage

In the computerization phase, the aim of the verification tests is to determine whether the transformation of the mathematical model into a computer program (simulator) is correct to the greatest extent possible. The task of the verification tests is to provide a rational background for selecting the parameters of the iteration method applied (single or multiple step, extrapolation or non-extrapolation) that lead to a reasonable compromise between the scale of the global approximation error and the efficiency of the computation process. In particular, we must demonstrate that the integration method selected for solving the system of differential equations (Eq. 6) possesses all of the desirable properties of a numerical algorithm, i.e., consistency, stability, and convergence. For example, tests were performed to show that the iteration step ensures that the value of the global truncation error is less than a given error bound. In the experiments controlled by the BSV subprogram, the step length used in conjunction with the selected method of integration was reduced until the compared values of a state variable calculated by alternative verification algorithms did not differ from each other by more than 0.0001% of the initial value. In consequence, in this phase of verification, the one-step forward Euler method was selected as the method of integration for the EK_AN simulator.

In the mathematization phase, we should demonstrate that the set of differential equations (Eq. 6) is free of mathematical and logical errors. Such errors are suspected when, despite the appropriate choice of the step length in the computerization phase, the BSV program indicates non-compliance of the simulator with balance-sheet totals. Such a failure to conform means that the EK_AN simulator does not satisfy the criterion of deductive verification. Consequently, a detailed analysis of the simulator is performed to find the answer to the question of why the modified simulator no longer meets the criterion of balancing the books. This concept of verification is in conformance with the conclusion about the informative value of falsification and the principle of ceaseless criticism [18].

In the graphicalization and verbalization phases of the verification, the EK_AN simulator is verified from an empirical standpoint by use of the RAD-VER procedure presented in [18]. In other words, the outcomes of the tests in the form of simulational basic sentences are compared with basic sentences derived from empirical observation (so-called sentential confrontation). In the graphicalization phase, the purpose of verification is to assess the validity of how the EK_AN model maps the structural properties of the firm considered. Verification is based on the principles of structural verification proposed by Shannon [19], see also Forrester and Senge [8]. Positive outcomes from these tests testify that, at least in relation to the selected criteria of structural verification – continuity, absurdity and extremity – the EK_AN simulator appropriately maps the structural characteristics of the economic system studied[14].

In the verbalization phase of the concretization stage, the simulator is concretized as a representation of a specific type of firm, in this case a 2000+ firm. Verification examines whether the simulator correctly maps the functional features of this class of economic systems. In our case, *ceteris paribus* tests were applied to see whether, in the policy scenarios considered, the performance of the firm as simulated by the EK_AN model is similar to the real behaviour of a 2000+ firm. For example, the following scenarios were studied: the effect of the prices of raw materials on the dynamics of production costs, the influence of the length of the production cycle on the dynamics of financial profit, and the relationship between company liquidity and the demand for a product[15].

Tests to verify the verbalization phase close the experimental stage of the verification. Positive results from these tests provide at least a temporary basis for assuming that the EK_AN model is a scientific tool of operations research for studying the economic system considered[16]. As mentioned previously, verification is continued during the operational phase, according to the principle that every simulation is also a verification test.

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[14] An example of a basic verification statement (both of the simulational and empirical bases) is the following: An extreme value (close to 0 or ∞) of an economic variable results in extreme behaviour by the system.

[15] An example of a basic verification statement (both of the simulational and empirical bases) is the following: Shortening the production cycle results in increasing financial profit.

[16] In this paper, only positive results of verification tests are reported. However, positive verification was accompanied by hundreds of tests that resulted in falsification of a model.

Operations. As was said, verification is continued during the operational phase, according to the principle that every simulation is also a verification test. In the case of the EK_AN simulator, each time the simulator is used, the BSV procedure is put into operation to detect and discard those runs for which the verification criteria are not fulfilled.

5. Final conclusions

The verification procedure described above does not pretend to be a method that allows one to formulate an unambiguous judgement concerning the scientificity of the EK_AN simulator. It was noted by Hume (*An Enquiry Concerning Human Understanding*), that the use of inductive inference is inevitably associated with drawing uncertain conclusions from relatively limited empirical experiences. As mentioned in [18], a problem occurs concerning what can and what cannot be accepted as a basic statement used for sentential confrontation. However, as Popper states: *Every test of a theory, whether resulting in its collaboration or falsification, must stop at some basic statement or other which we decide to accept* (...) *This procedure has no natural end. Thus, if the test is to lead us anywhere, nothing remains but to stop at some point or other and say that we are satisfied, for the time being*" [16, p. 86].

Summing up, it is expected that, thanks to the ACV method applied, we are able to make the next step toward developing a concept of flexible models. This means that the simulator of a firm has been adapted a priori to ongoing modifications that will be made in the software to maintain the required compatibility between the unstable structure of a real system and the structure mapped by the model.

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