O P E R A T I O N S R E S E A R C H A N D D E C I S I O N S

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AN EOQ MODEL FOR DETERIORATING ITEMS WITH TIME-DEPENDENT EXPONENTIAL DEMAND RATE AND PENALTY COST

The present paper deals with an EOQ model for deteriorating items with time-dependent exponential demand rate and partial backlogging. Shortages are allowed and completely backlogged in this model. The backlogging rate of unsatisfied demand is assumed as a function of waiting time. The concept of penalty cost is introduced in the proposed model because there are many perishable products that do not deteriorate for some period of time and after that period they continuously deteriorate and lose their values. This loss can be incurred as penalty cost to the wholesalers/retailers. In any business organization, the penalty cost has an important role for special types of seasonal products and short life products. Therefore, the total cost of the product can be reduced by maximizing the demand rate and minimizing the penalty cost during a given period of time. The purpose of our study is to optimise the total variable inventory cost. A numerical example is also given to show the applicability of the developed model.

Keywords: *inventory, deterioration, penalty cost and time-dependent exponential demand rate*

1. Introduction

Academicians as well as industrialists have great interest in the development of inventory control and its uses. Many goods are either deteriorate or become obsolete with time. Such perishable products have different modelling. Perishable inventory forms a small portion of the total inventory and includes fashionable garments, food stuff, soft drinks, pharmaceuticals, chemicals, electronic items, digital products and periodicals. The deteriorating items/products can be classified in two categories: (1) deterioration and (2) obsolescence. Deterioration is a realistic phenomenon in any inventory system and it is defined as damage, decay or spoilage of the items that are stored for future use,

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and that always lose a part of their value with time. Obsolescence is defined as the replacement of products by the arrival of new and better products in the market.

In the existing literature, several inventory models have been developed by several researchers who consider that the demand rate may be either constant, increasing, decreasing function of time or price, and stock-dependent. In recent years, some researchers have also paid their attention to a time-dependent demand rate because the demand of newly launched products such as fashionable garments, electrical items/electronic items, motor vehicles, mobiles, etc. increases with time and later becomes constant.

Some scholars researching this area are worth mentioning. Ghare and Schrader [1] develop an inventory model for exponentially decaying items. Covert and Philip [2] propose an EOQ model for Weibull deteriorating items. Mishra [3] presents an optimum lot-sizing inventory model for perishable items. Weiss [4] considers an EOQ model for deteriorating items with non-linear holding cost. Mitra et al. [5] present a note on inventory models for perishable items with a linear trend in demand. Deb and Chaudhari [6] also present a note on heuristic inventory models for deteriorating items with finite replenishment rate and allowing shortages. Goswami and Chaudhuri [7] develop an EOQ model for perishable items with linear trend in demand and considering shortages. Fujiwara and Perera [8] propose an EOQ model for continuously deteriorating items using linear and exponential penalty cost. Hargia [9] considers a lot-sizing inventory model for perishable items with time-varying demand rate and considering shortages. Jain and Silver [10] construct a lot-sizing inventory model for deteriorating items. Wee [11] offers a deterministic lot-size inventory model for perishable items with time-dependent declining demand rate and taking shortages. Giri et al. [12] consider heuristic inventory models for deteriorating items with time-varying demand and costs. They were focused on the concept of shortages in their inventory model. Jalan and Chaudhuri [13] develop an EOQ model for perishable items with time-dependent declining demand and SFI policy. Lin et al. [14] provide an EOQ model for deteriorating items with time-varying demand and allowing shortages. Mondal et al. [15] present an inventory model for ameliorating items with price-dependent demand rate. Khana and Chaudhuri [16] give a note on an order level inventory model for perishable items with time-dependent quadratic demand rate. Teng and Chang [17] develop an EPQ model for deteriorating items with price-and stock-dependent demand rate. Ghosh and Chaudhuri [18] present an EOQ model for perishable items with time-dependent quadratic demand rate and allowing shortages. Ranjana and Meenakshi [19] propose an EOQ model for deteriorating items with time-varying demand and using penalty cost. Tripathy and Mishra [20] consider an inventory model for Weibull deteriorating items with time-dependent demand rate and allowing shortages. Dye and Hseih [21] present an optimal replenishment policy for perishable items with effective investment in preservation technology. Shah et al. [22] give an optimizing inventory and marketing policy for non-instantaneous deteriorating items with time-varying deterioration rate and holding cost. Latha and Uthayakumar

[23] develop a partially backlogging inventory model for perishable items with probabilistic deterioration rate and permissible delay in time. Palanivel and Uthayakumar [24] propose a production inventory model for deteriorating items with probabilistic deterioration rate and variable production cost.

Vijayashree and Uthayakumar develop two inventory models [25, 26]. In the first one, they consider a two-stage supply chain inventory model for perishable items with selling price-dependent demand and investment for quantity improvement, and in the second model they develop an EOQ model for time-varying deteriorating items with shortages and finite and infinite production rate. Pevekar and Nagre [27] propose an inventory model for timely deteriorating products, considering penalty cost and shortage cost.

Behera and Tripathy make up two inventory models [28, 36]. First, they propose a fuzzy EOQ model for time-varying deteriorating items and using penalty cost. In the second model they consider an inventory replenishment policy with time and reliability varying demand. Wakeel and Al-Yazidi [29] offer fuzzy constrained probabilistic inventory models depending on trapezoidal fuzzy numbers. Arora [30] presents a study of inventory models for deteriorating items with shortages. Hossen et al. [31] focus on an inventory model with price- and time-dependent demand with fuzzy valued inventory costs under inflation. Maragatham and Palani [32] propose an inventory model for deteriorating items with lead time price-dependent demand and shortages. Sekar and Uthayakumar [33] give a multi-production inventory model for deteriorating items considering penalty and environmental pollution cost with failure rework. Sahoo and Tripathi [34] considered an optimization of fuzzy inventory model with trended deterioration and salvage. Naik and Patel [35] developed an imperfect quality and repairable items inventory model with different deterioration rates under price and time-dependent demand. Haughton and Isotupa [37] give a comprehensive review of inventory system with lost sales and emergency orders. Jeyakumari et al. [38] propose a fuzzy EOQ model with penalty cost using hexagonal fuzzy numbers. UntilMay 2019 no further related work is found.

2. Assumptions and notations

We consider the following assumptions and notations corresponding to the developed model:

$R(T) = a\mathrm{e}^{b-ct}$, $a, b, c > 0$ – demand rate
$P(T) = \lambda(t-\mu)$, $t \geq \mu$ – linear penalty cost function
δ – backlogging parameter
o_C – ordering cost per order
h_C – holding cost per unit time
s_C – shortage cost per unit time
Q – the maximum inventory level at time $t = 0$

S – inventory level at time $t = \mu$
T_1 – the time of zero inventory level
T – the cycle length
$TC(\mu, T_1, T)$ – total cost per cycle

3. Mathematical formulation

Suppose an inventory system contains the maximum inventory level Q at the beginning of each cycle. During the interval $[0, \mu]$, the inventory level decreases only by demand. During the interval $[\mu, T_1]$, the inventory level decreases due to both demand and deterioration and becomes zero at $t = T_1$. The interval $[T_1, T]$ is the shortage interval during which the unsatisfied demand is backlogged at a rate of $B(t) = 1/(1+\delta(T-t))$, where δ is the backlogging parameter and t is the waiting time.

The instantaneous inventory level at any time t in $[0, T]$ is given by the following differential equations:

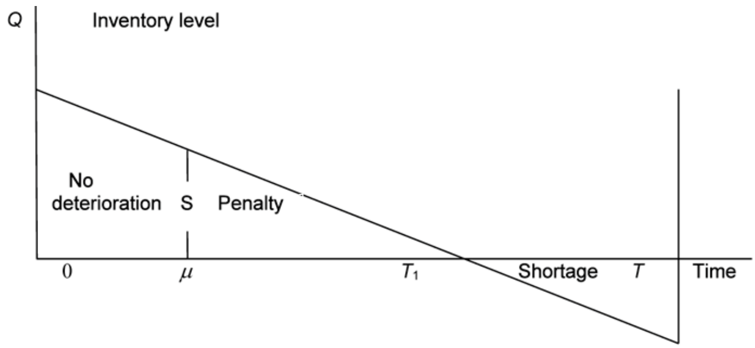

Fig. 1. Inventory model

$$\frac{dI}{dt} = -ae^{b-ct}, \quad 0 \le t \le \mu \tag{1}$$

with boundary condition $I(\mu) = S$

$$\frac{dI}{dt} = -\frac{ae^{b-ct}}{1+\delta(T-t)}, \quad T_1 \le t \le T \tag{2}$$

with boundary condition $I(T_1) = 0$.

The solutions of equations (1) and (2) are given by the equations (3) and (4), respectively.

$$I = a\mathrm{e}^{b}\left(\mu - t - \frac{c}{2}\mu^{2} + \frac{c}{2}t^{2}\right) + S, \quad 0 \le t \le \mu \tag{3}$$

$$\begin{aligned} I = a\mathrm{e}^{b}\Bigg[& T_1 - t + \left(\frac{\delta - c}{2}\right)T_1^{2} - \left(\frac{\delta - c}{2}\right)t^{2} + \delta T t - \delta T T_1 \\ & + \frac{c\delta}{2}TT_1^{2} - \frac{c\delta}{2}Tt^{2} + \frac{c\delta}{2}t^{3} - \frac{c\delta}{2}T_1^{3}\Bigg] \end{aligned} \tag{4}$$

The maximum inventory level Q is obtained by putting $t = 0$ in equation (3), then

$$Q = a\mathrm{e}^{b}\left(\mu - \frac{c}{2}\mu^{2}\right) + S \tag{5}$$

The ordering cost per cycle is

$$O_C = o_C \tag{6}$$

The deterioration cost per cycle is

$$D_C = \int_{\mu}^{T_1} R(t)\lambda(t-\mu)\, dt$$

or

$$D_C = a\lambda\mathrm{e}^{b}\left(\frac{1}{2}T_1^{2} + \frac{1}{2}\mu^{2} - \mu T_1 - \frac{c}{3}T_1^{3} - \frac{c}{6}\mu^{3} + \frac{c}{2}\mu T_1^{2}\right) \tag{7}$$

The holding cost per cycle is

$$H_C = \frac{Q}{2}h_C T$$

or

$$H_C = \frac{a h_C \mathrm{e}^{b}}{2}\left(ST + \mu T - \frac{c}{2}\mu^{2}T\right) \tag{8}$$

The shortage cost per cycle is

$$S_C = -s_C \int_{T_1}^{T} I(t)\,dt$$

or

$$S_C = ae^b s_C \left[\frac{1}{2}T^2 + \frac{1}{2}T_1^2 - TT_1 - \left(\frac{2\delta + c}{6}\right)T^3 + \left(\frac{\delta - c}{3}\right)T_1^3 \right.$$

$$\left. -\left(\frac{2\delta - c}{2}\right)TT_1^2 + \delta T^2 T_1 + \frac{c\delta}{12}T^4 - \frac{c\delta}{4}T_1^4 - \frac{c\delta}{2}T^2T_1^2 + \frac{2c\delta}{3}TT_1^3 \right] \tag{9}$$

The total variable inventory cost per cycle is

$$TC(\mu, T_1, T) = \frac{1}{T}\left[O_C + H_C + D_C + S_C\right] \tag{10}$$

or

$$TC(\mu,\ T_1,\ T) = \frac{1}{T}\left[o_C + \frac{ae^b h_C S}{2}T + \frac{a\lambda e^b}{2}\mu^2 + \frac{a(\lambda + s_C)e^b}{2}T_1^2 + \frac{ae^b s_C}{2}T^2 \right.$$

$$+ \frac{ae^b h_C}{2}\mu T - a\lambda e^b \mu T_1 - ae^b s_C TT_1 - \frac{ac\lambda e^b}{6}\mu^3$$

$$+ \frac{ae^b}{2}\left(3(\delta - c)s_C - c\lambda\right)T_1^3 - \frac{ae^b(2\delta + c)s_C}{6}T^3 - \frac{ace^b h_C}{4}\mu^2 T$$

$$+ \frac{ac\lambda e^b}{2}\mu T_1^2 - \frac{ae^b(2\delta - c)s_C}{2}TT_1^2 + a\delta e^b s_C T^2 T_1 - 3c\delta T_1^4 + c\delta T^4$$

$$\left. -\ 6c\delta T^2 T_1^2 + 8c\delta TT_1^3 \right] \tag{11}$$

The necessary conditions for $TC(\mu,\ T_1,\ T)$ are

$$\frac{\partial TC(\mu,\ T_1,\ T)}{\partial \mu} = 0, \quad \frac{\partial TC(\mu,\ T_1,\ T)}{\partial T_1} = 0, \quad \text{and} \quad \frac{\partial TC(\mu,\ T_1,\ T)}{\partial T} = 0 \tag{12}$$

On solving the equations in equation (12), we find the optimum values of μ, T_1 and T for which the total cost is minimum. The sufficient conditions for $TC(\mu, T_1, T)$ to be minimum are that the principal minors of the Hessian matrix or **H** matrix are positive definite. The **H** matrix is defined as

$$\mathbf{H} = \begin{bmatrix} \dfrac{\partial^2 TC(\mu, T_1, T)}{\partial \mu^2} & \dfrac{\partial^2 TC(\mu, T_1, T)}{\partial \mu\, \partial T_1} & \dfrac{\partial^2 TC(\mu, T_1, T)}{\partial \mu\, \partial T} \\ \dfrac{\partial^2 TC(\mu, T_1, T)}{\partial T_1 \partial \mu} & \dfrac{\partial^2 TC(\mu, T_1, T)}{\partial T_1^2} & \dfrac{\partial^2 TC(\mu, T_1, T)}{\partial T_1 \partial T} \\ \dfrac{\partial^2 TC(\mu, T_1, T)}{\partial T \partial \mu} & \dfrac{\partial^2 TC(\mu, T_1, T)}{\partial T \partial T_1} & \dfrac{\partial^2 TC(\mu, T_1, T)}{\partial T^2} \end{bmatrix}$$

Differentiating equation (11), we obtain

$$\frac{\partial TC(\mu, T_1, T)}{\partial \mu} = \frac{a\mathrm{e}^b}{T}\left[\lambda\mu + \frac{h_C}{2}T - \lambda T_1 - \frac{c\lambda}{2}\mu^2 - \frac{ch_C}{2}\mu T + \frac{c\lambda}{2}T_1^2 \right] \tag{13}$$

$$\begin{aligned} \frac{\partial TC(\mu, T_1, T)}{\partial T_1} = \frac{1}{T}\Big[& a\mathrm{e}^b(\lambda + s_C)\,T_1 - a\lambda\mathrm{e}^b\mu - a\mathrm{e}^b s_C T + a\mathrm{e}^b \\ & + \{3(\delta - c)s_C - c\lambda\}\,T_1^2 + ac\mathrm{e}^b\lambda\mu T_1 - a\mathrm{e}^b(2\delta - c)s_C T T_1 \\ & + a\delta\mathrm{e}^b s_C T^2 - 12c\delta T_1^3 - 12c\delta T^2 T_1 + 24c\delta T T_1^2 \Big] \end{aligned} \tag{14}$$

$$\begin{aligned} \frac{\partial TC(\mu, T_1, T)}{\partial T} = \frac{1}{T}\Bigg[& \frac{a\mathrm{e}^b h_C S}{2} + a\mathrm{e}^b s_C T + \frac{a\mathrm{e}^b h_C}{2}\mu - a\mathrm{e}^b s_C T_1 - \frac{a\mathrm{e}^b(2\delta + c)s_C}{2}T^2 \\ & - \frac{ac\mathrm{e}^b h_C}{4}\mu^2 - \frac{a\mathrm{e}^b(2\delta - c)s_C}{2}T_1^2 + 2a\delta\mathrm{e}^b s_C T T_1 + 4c\delta T^3 \\ & - 12c\delta T T_1^2 + 8c\delta T_1^3 \Bigg] - \frac{1}{T^2}\Bigg[o_C + \frac{a\mathrm{e}^b h_C S}{2}T + \frac{a\mathrm{e}^b\lambda}{2}\mu^2 \\ & + \frac{a\mathrm{e}^b(\lambda + s_C)}{2}T_1^2 + \frac{a\mathrm{e}^b s_C}{2}T^2 + \frac{a\mathrm{e}^b h_C}{2}\mu T - a\mathrm{e}^b\lambda\mu T_1 - a\mathrm{e}^b s_C T T_1 \\ & - \frac{ac\lambda\mathrm{e}^b}{6}\mu^3 + \frac{a\mathrm{e}^b}{3}\{3(\delta - c)s_C - c\lambda\}\,T_1^3 - \frac{a\mathrm{e}^b(2\delta + c)s_C}{6}T^3 \end{aligned}$$

$$-\frac{ace^b h_C}{4}\mu^2 T+\frac{ac\lambda e^b}{2}\mu T_1^2-\frac{ae^b(2\delta-c)s_C}{2}TT_1^2+a\delta e^b s_C T^2 T_1$$

$$-3c\delta T_1^4+c\delta T^4-6c\delta T^2T_1^2+8c\delta TT_1^3\Big] \tag{15}$$

$$\frac{\partial^2 TC(\mu,\,T_1,\,T}{\partial\mu^2}=\frac{ae^b}{T}\left[\lambda-c\lambda\mu-\frac{ch_C}{2}T\right] \tag{16}$$

$$\frac{\partial^2 TC(\mu,\,T_1,\,T)}{\partial\mu\partial T_1}=\frac{1}{T}\left[-a\lambda e^b+ac\lambda e^b T_1\right] \tag{17}$$

$$\frac{\partial^2 TC(\mu,\,T_1,\,T)}{\partial\mu\partial T}=\frac{1}{T}\left[\frac{ae^b h_C}{2}-\frac{ace^b h_C}{2}\mu\right]-\frac{1}{T^2}\left[ae^b\lambda\mu+\frac{ae^b h_C}{2}T\right.$$

$$\left.-ae^b\lambda T_1-\frac{ae^b c\lambda}{2}\mu^2-\frac{ace^b h_C}{2}\mu T+\frac{ac\lambda e^b}{2}T_1^2\right] \tag{18}$$

$$\frac{\partial^2 TC(\mu,\,T_1,\,T)}{\partial T_1\partial\mu}=\frac{ae^b}{T}\left[-\lambda+c\lambda T_1\right] \tag{19}$$

$$\frac{\partial^2 TC(\mu,\,T_1,\,T)}{\partial T_1^2}=\frac{1}{T}\Big[ae^b(\lambda+s_C)+2ae^b\left\{3(\delta-c)-c\lambda\right\}T_1+ace^b\lambda\mu$$

$$-ae^b(2\delta-c)s_C T-36c\delta T_1^2-12c\delta T^2+24c\delta TT_1\Big] \tag{20}$$

$$\frac{\partial^2 TC(\mu,\,T_1,\,T)}{\partial T_1\partial T}=\frac{1}{T}\Big[-ae^b s_C-ae^b(2\delta-c)s_C T_1+2a\delta e^b s_C T-24c\delta TT_1$$

$$+\,24c\delta T_1^2\Big]-\frac{1}{T^2}\Big[ae^b(\lambda+s_C)T_1-a\lambda e^b\mu-ae^b s_C T$$

$$+\,ae^b\left\{3(\delta-c)s_C-c\lambda\right\}T_1^2+ac\lambda e^b\mu T_1-\frac{ae^b(2\delta-c)s_C}{2}TT_1$$

$$+\,ae^b\delta s_C T^2-12c\delta T_1^3-12c\delta T^2T_1+24c\delta TT_1^2\Big] \tag{21}$$

$$\frac{\partial^2 TC(\mu,\,T_1,\,T)}{\partial T\partial\mu}=\frac{ae^b}{T}\left[\frac{h_C}{2}-\frac{ch_C}{2}\mu\right] \tag{22}$$

$$\frac{\partial^2 TC(\mu,\, T_1,\, T)}{\partial T \partial T_1} = \frac{1}{T}\Big[-ae^b s_C + ae^b(2\delta - c)s_C T_1 + 2ae^b \delta s_C T - 24c\delta T T_1$$

$$+ 24c\delta T_1^2\Big] - \frac{1}{T^2}\Big[ae^b(\lambda + s_C)T_1 - a\lambda e^b \mu - ae^b s_C T$$

$$+ ae^b\left\{3(\delta - c) - c\lambda\right\}T_1^2 + ac\lambda e^b \mu T_1 - ae^b(2\delta - c)s_C T T_1$$

$$+ a\delta e^b s_C T^2 - 12c\delta T_1^3 - 12c\delta T^2 T_1 + 24c\delta T T_1^2\Big] \qquad (23)$$

$$\frac{\partial^2 TC(\mu,\, T_1,\, T)}{\partial T^2} \geq \frac{1}{T}\Big[ae^b s_C - ae^b(2\delta + c)s_C T + 2ae^b \delta s_C T_1 + 12c\delta T^2$$

$$-12c\delta T_1^2\Big] - \frac{1}{T^2}\Bigg[\frac{ae^b h_C S}{2} + ae^b h_C S + 2ae^b s_C T + ae^b h_C \mu$$

$$-2ae^b s_C T_1 - a(2\delta + c)e^b s_C T^2 - a(2\delta - c)e^b s_C T_1^2$$

$$+4a\delta e^b s_C T T_1 + 8c\delta T^3 - 24c\delta T T_1^2 + 16c\delta T_1^3 - \frac{ace^b h_C}{2}\mu^2\Bigg]$$

$$+\frac{2}{T^3}\Bigg[o_C + \frac{ae^b h_C S}{2}T + \frac{ae^b}{3}\left\{3(\delta - c)s_C - c\lambda\right\}T_1^3$$

$$-\frac{ae^b(2\delta + c)s_C}{6}T^3 - \frac{ach_C e^b}{4}\mu^2 T + \frac{ac\lambda e^b}{2}\mu T_1^2$$

$$-\frac{a(2\delta - c)s_C e^b}{2}T T_1^2 + a\delta e^b s_C T^2 T_1 - 3c\delta T_1^4 + c\delta T^4$$

$$-6c\delta T^2 T_1^2 + 8c\delta T T_1^3\Bigg] \qquad (24)$$

Numerically, the Hessian matrix or **H** matrix is given by

$$\mathbf{H} = \begin{bmatrix} 7.5402 & 6.2546 & 0.2350 \\ 6.2546 & 166.0273 & 165.3946 \\ 0.2350 & 1.3144 & 4.6255 \end{bmatrix}$$

4. Numerical example

Let us consider the following data for parameters of the model in appropriate units: $a = 20$, $b = 0.05$, $c = 1$, $\lambda = 10$, $\delta = 0.5$, $o_C = 15$, $h_C = 0.4$, $s_C = 0.6$, $S = 50$.

Table 1. Variation in total inventory cost with respect to a

a	μ	T_1	T	$TC(\mu, T_1, T)$
20	0.7632	0.8739	4.2390	210.3253
30	0.5729	1.2577	6.7899	281.4225
40	0.2672	1.5106	9.4270	299.5380
50	2.5287	2.7496	11.8449	277.3333
60	3.0962	3.3668	14.4054	160.6548

From Table 1 we observe that as we increase the values of the demand parameter a, then the values of μ, T_1 and T increase but the values of $TC(\mu, T_1, T)$ first increase and then decrease.

Table 2. Variation in total inventory cost with respect to b

b	μ	T_1	T	$TC(\mu, T_1, T)$
0.05	0.7632	0.8739	4.2390	210.3253
0.10	0.8006	0.9367	4.4827	219.2178
0.15	0.8181	0.9971	4.7459	228.2676
0.20	0.7875	1.0489	5.0302	237.4076
0.25	0.7513	1.0937	5.3349	246.5482

From Table 2, we observe that as we increase the demand parameter b, then the values of T_1, T and $TC(\mu, T_1, T)$ increase as well.

Table 3. Variation in total inventory cost with respect to c

c	μ	T_1	T	$TC(\mu, T_1, T)$
1	0.7632	0.8739	4.2390	210.3253
2	0.3389	0.4459	3.5728	205.0544
3	0.1951	0.3074	3.3342	199.7278
4	0.1201	0.2362	3.2099	194.3231
5	0.0742	0.1923	3.1334	166.8760

From Table 3, we observe that as we increase the demand parameter c, then the values of μ, T_1 and T and $TC(\mu, T_1, T)$ decrease. From Table 4 we observe that as we increase the penalty parameter λ, then the values of μ, T_1 and $TC(\mu, T_1, T)$ increase but the values of T_1 decrease.

Table 4. Variation in total inventory cost with respect to λ

λ	μ	T_1	T	$TC(\mu, T_1, T)$
10	0.7632	0.8739	4.2390	210.3253
20	1.3242	0.6440	4.4536	212.8275
30	1.3529	0.6179	4.5564	215.0922
40	1.3699	0.6076	4.6393	217.2965
50	1.3795	0.6021	4.7119	219.4584

5. Conclusion

From the results of the developed model we see that the parameters a, b and c are more sensitive than the parameter λ. This is due to the reason that the total cost is affected by the penalty cost. If the penalty cost on a product is minimum, the total cost will also be minimum. Therefore, the total cost of the wholesaler/retailer can be reduced by the maximising the demand rate of a product and minimising the penalty cost on that product. Finally, in particular, our study provides an ample scope for further research and exploration. For instance, we have proposed an EOQ model for deteriorating items with time-dependent exponential demand rate and penalty cost. This study can be further developed by considering a full range of different assumptions and conditions on demands and costs.

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