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APPLICATION OF A SINGLE SAMPLING PLAN MINIMIZING THE SUM OF RISKS IN AN EOQ MODEL WITH TRADE CREDIT

Acceptance sampling by attributes is a universally used statistical tool for quality control. It is a technique that deals with the decision to accept or reject a batch of goods using defined procedures. An attribute single sampling plan designed under the assumption that the number of defects has a Poisson distribution is the optimal plan whenever the chance of a defect occurring in the manufacturing process is low. This study introduces the incorporation of an attribute single sampling plan minimizing the sum of risks with an economic order quantity (EOQ) model taking into account the possibility of trade credit. The plan ensures the effectiveness of the optimal design based on the minimization of costs including the inspection costs, stock holding costs and ordering costs.

Keywords: *trade credit, economic order quantity* (*EOQ*), *minimum sum of risks, Poisson distribution*

1. Introduction

Acceptance sampling plans are a significant field of statistical quality control initially developed by Dodge and Romig [2]. They were first established during World War II for bullet testing by the U.S. Military. It is a mid-way approach between no inspection and 100% inspection. Acceptance sampling is classified into variable acceptance sampling plans and acceptance sampling by attributes. Variable sampling plans involve a random sample of output from a process that is subjected to batch sampling by inspection.

An attribute acceptance sampling plan involves procedures firstly to determine the number of items to be inspected from a lot and then to decide whether to accept or reject the lot. The number of items to inspect is determined by a number of parameters includ-

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ing the lot size, acceptance quality level, producer risk, limiting quality level and consumer risk. Attribute acceptance sampling plans range from simple to profound and from practical to infeasible. Various attribute acceptance plans have been described in the literature. Despite this, the most commonly used, by far, attribute acceptance sampling plan is the single sampling plan.

A single sampling plan by attribute is characterized by two parameters, namely the sample size (n) and the acceptance number(c), i.e., the lot is accepted if the number of defects is at most c. A single sampling plan is described in such a way that the acceptance or rejection of a lot is determined on the basis of information resulting from a random sample of items from the lot. Along with the binomial and hypergeometric distributions, the Poisson distribution is widely used to model the number of defects (or defective items) in a lot under the normal functioning of the production process using acceptance sampling by attributes. Haight [6] stated that a single sampling plan under a Poisson distribution is ideal for count data pertaining to rare events (e.g., defects). For example, a production engineer may count the number of defects in the items randomly selected from a production process. The operating ratio and unity value for a single sampling plan under the assumption of a Poisson distribution was derived by Cameron [1].

Furthermore, Hald [5] developed extensions of this theory. Accordingly, the Poisson model is suitable for the case of nonconformities under type A and type B situations. In the case of a type A scenario, the Poisson model is ideal on any occasion when, $n/N = 0.10$, where n is the sample size and N is the lot size, under the assumption that n is large and p is small such that $np < 5$, where p is the proportion of items which are defective. The afore mentioned assumptions are also applicable for type B situations(where the total number of defects are counted). Schilling and Neubauer [13] gave detailed characteristics of plan parameters. Peach [8] considered the design of sampling inspection plans indexed by p_1, p_2, α, β associated with measures of the related producer's maximum level of risk (α, probability of accepting a bad lot) and consumer's maximum level of risk (β, probability of rejecting a good lot). The operating ratio is defined as the ratio of p_2 to $p_1$which are measures of quality associated with the production process functioning normally, which can be used as a measure of discrimination when designing a plan. The operating ratios are not generally integers, thus one has to select an operating ratio just below or above the desired operating ratio. The plan parameters are normally defined to be integers, so the underlying condition for fixing risks are usually changed to

$$P_a\left(p_1\right) \geq 1-\alpha \tag{1}$$

$$P_a\left(p_2\right) \leq \beta \tag{2}$$

where $P_a(p)$ is the probability of accepting a lot of process quality p. In general, the producer and consumer risk cannot be minimized concurrently.

Golub [3] proposed a model for minimizing the sum of the risks of the producer and consumer when using a single sampling plan by attributes under the condition that the sample size is fixed. With the sample size *n* fixed, the study derived the acceptance number minimizing the sum of risks based on a binomial model using the mathematical expression

$$c = -\frac{1}{2} + \frac{n}{\dfrac{\log\left(\dfrac{p_2}{p_1}\right)}{\log\left(\dfrac{q_1}{q_2}\right)}} \tag{3}$$

This model was extended by Soundararajan [15] based on a Poisson's model for the operating characteristic (OC) curve. The tables for the selection of a single sampling plan when the sample size is fixed are based on this Poisson's model. These tables were only defined for $n = 10, 15, 25, 40$. Later on, many authors such as Soundararajan [14], Soundararajan and Govindaraju [16], Soundararajan and Govindaraju [17], Subramani [18], Subramani and Haridoss [19], Suresh and Kavithamani [20] greatly extended the research on minimizing the sum of risks.

In order to strive and thrive in a competitive market, managing quality, as well as ones inventory, is essential. Inventory control involves being well-informed about the whereabouts of stocks and ensuring that there is an appropriate amount of each item in stock. This is particularly crucial when items are perishable. Defining the Economic Order Quantity (EOQ) is the classical approach to optimizing the costs involved in the inventory. It gives a good indication of whether or not the current order quantities are reasonable. One shortcoming of the conventional EOQ model is the assumption of immediate payment. In practice, the vendor accepts a mutually agreed delayed payment. An EOQ model with the provision of an admissible fixed delay in payments, i.e. trade credit, was pioneered by Goyal [4], who assumed that the purchase price and selling price are identical. This model implicitly assumes that the buyer initiates paying a higher interest rate on items in stock and pays back the remainder when the deadline approaches. Published three decades ago, this study initiated an enormous number of studies dealing with diverse situations regarding trade credit. Teng [21] relaxed Goyal's assumption that the purchase and selling price are equal by allowing them to differ. Tsao [23] further enhanced the model by incorporating acceptance sampling and trade credit. Sana [10] studied an EOQ model where it is assumed that the demand is deterministic and the retailer is offered a price discount and permissible delay in payment. Sana [9] developed an EOQ model where after the screening of products, non-conforming items are sold at reduced price. Sarkar [11] proposed an inventory model with different types of time dependent demand, trade credit and price discounts. Pal [7] studied two echelon competitive supply chains involving two rival retailers and one common supplier. Saxena [12] proposed a green supply chain model for the integrated production of new items and remanufacturing of redeemable returned items with trade credit. Tiwari [22] developed an economic lot sizing model based on stochastic demand and a controllable lead time. The intent of this paper is to design an EOQ model integrating a single sampling plan under the assumption that the number of defects has a Poisson distribution with the method of minimizing the sum of risks. Such an optimal strategy provides better quality at minimal risk and cost.

Notation

N – lot size
n – sample size
c – acceptance number
d – number of defectives
P – unit retail price
W – unit wholesale price
C_i – inspection cost
C_r – rejection cost
C_o – ordering cost
C_h – unit inventory holding cost per unit time
M – credit period
I_p – interest paid
I_e – interest earned
T – length of replenishment cycle
D – total annual demand
Q – order quantity, $Q = DT$

2. Selection of optimal single sampling plans for given p_1, p_2, α, β minimizing the sum of risks

A new method for designing single sampling plans (SSPs) under the assumption that the number of defects has a Poisson distribution by minimizing the sum of risks is considered. Unlike Golub's approach, it is assumed that the sample size n is unknown. This method does not require explicit knowledge of n, but requires the specification of the operating ratio and the quality level p_1. Theoretically, the sum of the producer's and consumer's risk can be expressed as

$$\alpha + \beta = 1 - P_a(p_1) + P_a(p_2) \tag{4}$$

The parameters for the sampling plan can be found using the following procedure

1. The operating ratio $p_2/p_1$is computed.

2. Enter a value into the table headed by p_2/p_1which is equal to or just smaller than the computed ratio.

3. The sample size is obtained using the unity value approach $n = np_1/p_1$ and the corresponding acceptance and rejection numbers for the acceptable levels of risk are obtained from the table.

4. Then the operating procedure for the single sampling plan under the assumptions of the Poisson distribution is followed to make a decision on the lot.

The single sampling plan is designated by two parameters, namely n and c. The operating procedure involves the following steps:

1. From a lot of N units, a random sample of n units are drawn.

2. By counting the number of non-conforming items, d, the lot is accepted if $d \leq c$ or else the lot is rejected.

The OC function of the SSP is given as

$$P_a(p) = P(X \leq c) \tag{5}$$

The probability of rejection is

$$P_r\left(P\right) = 1 - P_a\left(P\right) \tag{6}$$

3. Assumptions of the model

The model is formulated in such a way that the cost is optimized under the condition that the sum of the consumer's and producer's risk is minimized. The following assumptions are made:

1. The replenishment order is delivered instantaneously, i.e., zero lead time.

2. A single product inventory is studied.

3. Demand is known.

4. Unbounded time horizon.

5. No shortage is granted.

6. The acceptance of a lot is based on single sampling under the assumptions of a Poisson distribution.

7. The revenue generated from each item sold during the credit period m is invested in an interest-bearing account at interest rate I_e. The buyer pays at the conclusion of the credit period at interest rate I_p on the items in stock.

8. There are no discounts on defective products or goodwill cost for selling defective items.

9. $W < P$, $I_e < I_P$.

4. Evaluation of the total cost

The total costs involved are

$$\text{Annual ordering cost} = \frac{C_o}{T} \tag{7}$$

$$\text{Annual inventory holding cost} = \frac{DTC_h}{2} \tag{8}$$

Annual cost of inspection: the sample size n is calculated using $n = np_1/p_1$ or np_2/p_2 whichever is just higher or equal to the operating ratio and the acceptance number c is obtained directly from the table.

$$\text{Cost of inspection} = \frac{C_i n}{T} \tag{9}$$

The annual interest earned can come from two sources

- when $T \geq M$

$$\text{Annual interest earned} = \frac{PI_e DM^2}{2T} \tag{10}$$

- when $T \leq M$

$$\text{Annual interest earned} = \frac{PI_e DT}{2} + PI_e D(M - T) \tag{11}$$

Annual interest paid is

- when $T \geq M$

$$\text{Annual interest paid} = \frac{WDI_p (T - M)^2}{T} \tag{12}$$

- when $T \leq M$, annual interest paid equals 0. In this case, no interest is paid for the items.

Total cost:

• when $T \geq M$

$$Tc_1(T) = \frac{C_o}{T} + \frac{DTC_h}{2} + \frac{C_i n}{T} - \frac{PI_e DM^2}{2T} + \frac{WDI_p (T-M)^2}{2T} \tag{13}$$

• when $T \leq M$

$$Tc_2(T) = \frac{C_o}{T} + \frac{DTC_h}{2} + \frac{C_i n}{T} - \frac{PI_e DT}{2} - PI_e D(M-T) \tag{14}$$

The total cost is defined as long as $T > 0$.

Theorem

1. If $\lambda - DM^2(H + PI_e) > 0$, then the optimal replenishment interval is T_1^*, where $T_1^* > M$.

2. If $\lambda - DM^2(H + PI_e) < 0$, then the optimal replenishment interval is T_2^*, where $T_2^* < M$.

3. If $\lambda - DM^2(H + PI_e) = 0$, then the optimal replenishment interval is $T_1^* = T_2^* = M$, where $\lambda = 2(C_o + C_i n)$. The formulas for T_1^* and T_2^* are given below by Eqs. (16), (21).

Proof

In order to minimize TC_1, find $\frac{dTC_1}{dT}$, then equate it to zero:

$$\frac{dTC_1}{dT} = \frac{-2(C_o + C_i n) - DM^2(WI_p - PI_e) + DT^2(C_h + WI_p)}{2T^2} \tag{15}$$

Thus the value of T_1^* is

$$T_1^* = \sqrt{\frac{2(C_o + C_i n) + DM^2(WI_p - PI_e)}{D(C_h + WI_p)}} \tag{16}$$

The second derivative of

$$TC_1 = \frac{2(C_o + C_i n) + DM^2(WI_p - PI_e)}{T^3} \tag{17}$$

To prove $T_1^* > M$, substitute the value of T_1^* into this inequality, thus obtaining

$$\sqrt{\frac{\lambda + DM^2(WI_p - PI_e)}{D(C_h + WI_p)}} > M \tag{18}$$

Taking squares on both sides we obtain

$$\frac{\lambda + DM^2(WI_p - PI_e)}{D(C_h + WI_P)} > M^2 \tag{19}$$

$$\lambda + DM^2 WI_p - DM^2 PI_e > M^2 D(C_h + WI_P) \tag{20}$$

Hence, $\lambda - DM^2(C_h + PI_e) > 0$ if and only if $T_1^* > M$.

The value of T_2^* is obtained in an analogous way,

$$T_2^* = \sqrt{\frac{2(C_o + C_i n)}{D(C_h + PI_e)}} \tag{21}$$

By substituting the value of T_2^* into the inequality $T_2^* < M$, we obtain

$$\lambda - DM^2(C_h + PI_e) < 0$$

Similarly, Theorem 3 can also be proven by equating $T_1^* = T_2^* = M$.

5. Numerical illustrations

Numerical examples illustrate the optimal strategies and costs based on this model. The optimum replenishment cycle length and total cost are derived by applying the theorem to various combinations of α, β, p_1, p_2, $C_o = 60$ \$/unit, $P = 5$ \$/unit, $W = 4$ \$/unit, $C_h = 0.02$ \$/unit per year, $C_i = 0.1$ \$/unit, $C_r = 0.2$ \$/unit, $I_p = 0.15$ \$/dollar, $I_e = 0.10$ 1/dollar, $M = 0.3$.

Table 1. Various parametric and cost values of the model

Operating ratio	np_1	α [%]	β [%]	c	n	T_1^*	TC^*
6.0	2.5	1	1	6	50	0.3943	440.98
2.2	10	5	5	15	100	0.7715	540.49
4.75	2	1	5	6	40	0.5521	418.60
4.0	6	5	10	4	14	0.4231	355.89
3.3	4	5	5	7	20	0.4561	370.87

By applying trade credit in the model, the cost is lower compared to the conventional EOQ model. This is due to the optimal strategy involving less sampling than 100% screening, thus it requires less labour in inspection activities, but maintains low rates of inspection error. By applying the minimum sum of risks to the EOQ model, the sum of the probability of accepting a bad lot and the probability of rejecting a good lot, i.e. the producer risk and consumer risk, is minimized. It should be remembered that the methodology not only minimizes the costs, but risks are also minimized by employing this model.

6. Conclusion

By applying a single sampling plan under the assumption that the number of defects has a Poisson distribution, minimizing the sum of risks and using trade credit, a buyer achieves better quality, minimized risk and flexible payment, while the vendor minimizes the risk of a lot being rejected. Future research will focus on the skiplot sampling plan under the condition that the number of defects has a Poisson distribution and trade credit is available, based on minimizing the sum of risks. The demand may be modelled by, e.g., a Markovian process, exponential smoothing or a moving average process. One drawback of the proposed study is that it requires planning and documentation of the entire procedure. It may generate little information about the product and its production process.

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