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VERIFICATION OF A MODEL AS A SCIENTIFIC TOOL OF OPERATIONS RESEARCH. A METHODOLOGICAL APPROACH

The aim of the research presented in this paper was to solve one of the fundamental problems of modelling and simulation, i.e., verification of a model as a scientific tool of operations research. To attack this problem, certain crucial issues in the philosophy of science (the demarcation problem, the principle of verifiability) must be redefined. In discussing the question of verification, a procedure (the so called RAD-VER procedure) for verifying a model of a microeconomic system, in our case – a firm, is formulated. It is assumed that verification is a ceaseless process of evaluating a model's scientificity from the standpoints of deductive reasoning, coherency and empiricism. Verification has been divided into two stages: the verification of the assumptions underlying the model of a firm and the verification of the simulator.

Keywords: *validation of OR computations, scientific tools of operations research, verification*

1. Questions of verification

If we wish to use modelling and simulation as a scientific method of operations research, it is necessary to define the range of problems to which the model of an organization, in our case – a firm, can be successfully applied. It should be stated how scientific experiments should be carried out to ensure that the model mirrors an organization's behaviour as a substitute for economic reality as well as possible. We must also, to the greatest extent possible, estimate the errors that can occur due to the actual in the

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field experiment being replaced by a simulation in a computer laboratory. In other words, one relevant issue of concern is how statements inferred from simulations correspond to those derived from direct observation and/or personal experience of a real life organization[3].

The above-formulated topics constitute one of the most elusive of all still-unresolved methodological problems, namely verification[4], associated with the application of models as a tool of operations research. Despite the utmost importance of this issue for operations research, there have only been a few projects, and a very limited number of publications, devoted to discussing the question of verification. To date no satisfactory solution to the problem of how to verify a model in operations research has been offered.

Georgescu-Roegen [13, p. 332] tells us explicitly, *there is no acid test for the validity of an economic model*. Landry and Oral [25, p. 166] express similar concerns, stating: *There is no one universal scientific method, and therefore there cannot be a universal set of criteria for model validation* (...) *different schools of thought in epistemology have selected different criteria for coping with this issue and have come up with different suggestions. Similarly, OR scholars and practitioners do not seem to share a common set of criteria or a method to evaluate the validity of a model*. Dery, Landry and Banville [6, p. 169] insist that: *There is no agreement either on what is a valid model or on what is the way to validate models*. Sargent [40, p. 129] concludes that verification and validation *are critical in the development of the simulation model. Unfortunately, there is no set of specific tests that can easily be applied to determine the "correctness" of a model. Furthermore, no algorithm exists to determine what techniques or procedures to use*.

1.1. Basis of verification

Following the recommendations of Reichenbach [34], we start out our analysis of the problem (verification of models) from an analysis of language. The etymological meaning of the term verification comes from the root word *verificare* (medieval Latin) – make true. It follows that verification of a model can be regarded as a procedure intended to confirm or to reject the sentence: this model is true. If we accept classical correspondence theory, truth is the conformity of a thing (a real-life system) to a thought

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[3]The authors propose very broad definitions of a system, a model, simulation and a simulator as follows: a system – a complex entity treated in isolation from its environment, a model – a set of information about a real-life system, simulation – any realization of a model, a simulator – a computerized calculation device with a model embedded.

[4]Some authors divide the process of model evaluation into two stages: validation and verification (see [26, 18, 7, 30, 46]). But as stated by Klejninen [21, p. 146], in practice, *verification and validation are often mixed*. Extensive terminological discussion of verification versus validation was performed by Dybkaer [7].

(a model of the system)[5]. However, we must agree with those, see Plato's *Timaeus*, who are convinced that in the case of any sentence about reality, including sentences about economic systems formulated with the aid of modelling and simulation, truth, understood as a complete, ideal correspondence between a real-life economic system and its model, as such remains unattainable. In fact, a sentence about an economic system is only unquestionably true if we derive it from an axiomatically independent set of analytical sentences that represent a virtual economic system that is solely the product of our imagination (e.g., corporate games)[6]. But if we are going to use such a virtual system to study a real-life economic system, we are forced to set up relationships (in the form of synthetic sentences) between elements of our model and perceived real-life objects. However, perception of these relationships is not flawless but depends upon, and can be deformed by the individual characteristics of the physical, mental and social profiles of the modeller. Thus, a lack of certainty should not be viewed as an incidental failure, due to the imperfect model that we have, but as an inherent feature of all possible sentences about a real economic system, including synthetic sentences derived via simulations. Reichenbach [36, p. 304) writes: *scientific philosophy, in contrast, refuses to accept any knowledge of the physical world as absolutely certain. Neither individual occurrences, nor laws controlling the universe, can be stated with certainty. The principles of logic and mathematics represent the only domain in which certainty is attainable; but these principles are analytic and empty. Certainty is inseparable from emptiness*[7, 8].

The conviction that we are unable to construct the true model of a real economic system should not, nevertheless, push us into "epistemological anarchism" or nihilism, arguing that "nothing is true" and "anything goes"[9], and thus verification is senseless[10]. On the contrary, we ought to accept the opinion that, although all possible synthetic sentences (based on direct observation, personal experience or simulation) about a real economic system are admittedly subject to doubt and cannot provide us with absolute,

[5]For example Tarski [45, p. 14] holds that *the truth of a sentence consists in its agreement with (or correspondence to) reality*.

[6]In this paper, we adopt the terminology of sentential calculus.

[7]A classic phrase from a lecture given by Einstein in Berlin, 1921, is: *As far as the laws of mathematics refer to reality, they are not certain; and to the extent that they are certain, they do not refer to reality*. But, much earlier, Tertullianis taught in De Carne Christi that, *certum est, quia impossibile est.*

[8]The authors of the paper agree with the opinion that the aspiration to gain absolute certainty in the form of the total correspondence of a thing to a thought appears to be an aim of admirable grandeur, but is usually associated with the demonstration of the unhealthy lust to achieve super-scientific knowledge. Magee [28, p. 28] said *So it is a profound mistake to try to do what scientists and philosophers have almost always tried to do, namely prove the truth of a theory, or justify our belief in a theory, since this is to attempt the logically impossible*.

[9]These phrases are borrowed from Feyerabend [10].

[10]This problem is taken up in the discussion between the logical empiricists from the Wiener Circle – *all or nothing* and Carnap – *requirement of gradual corroboration*.

certain knowledge, the extent of this uncertainty and its structure can be substantially transformed and reduced thanks to the creative capabilities of the human mind.

Our civilization regards scientific models, tools and methods as much more "valid" (truer) than other approaches to real-life cognition (for example, based on mystical, astrological or theological paradigms)[11]. As Magee [28, p. 22, 23] states, *so if not certain, they (scientific laws) are probable to the highest degree which is possible to conceive, and in practice, if not in theory, this is indistinguishable from certainty*. By accepting the point of view represented by scientism, for example in the form of neo-positivism, the more a tool of cognition (a model of an economic system) is scientifically substantiated, the greater is the chance that the conclusions derived in the form of synthetic sentences explain the real phenomenon under study *with a degree of accuracy which belongs to the subject matter* (Aristotle, *Ethica Nicomachea*)[12, 13]. In this context, verifying the model of a firm is not testing its veracity, but is rather an attempt to determine which parts of the model fulfil the criteria of scientific cognition and which of the model's assumptions are based only on speculation, personal feelings and intuition.

To date, no criteria for how to distinguish between what is a scientific method and what is not (the demarcation problem) have been defined and this problem has been debated by philosophers since antiquity[14]. For example, Popper [33, p. 81] describes the method of science in the form of a personal manifesto, stating that *it is the method of bold conjectures and ingenious and severe attempts to refute them*. Nagel [14, p. 13] is enigmatic, arguing that *the practice of scientific methods is the persistent critique of arguments in the light of tried canons for judging the reliability of the procedures by which evidential data are obtained, and for assessing the probative force of the evidence on which evidential data are obtained*. The authors of this paper dare to say that the problem of defining the concept of science is inherently irresolvable. An objective def-

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[11]See [46] for a discussion on applications of the scientific method as far as it concerns operations research.

[12]A review of the philosophical dilemmas associated with verification is given by Naylor and Finger [31], Kleindorfer et al. [21], and Ijeoma et al. [18].

[13]In contrast to the idealistic postulate of Thomas Aquinas that *veritas est adequatio rei et intellectus*, one can find a more pragmatic approach in the words of Aristotle, who taught, (A) *model is valid if it achieves that degree of accuracy which belongs to the subject matter*, *Ethica Nicomachea*, cited by Georgescu-Roegen [14, p. 333]. In some sense, Aristotle's standpoint on verification is followed by Reichenbach [33, p. 249]: *if a man does his best, what else can you ask of him?*. The authors of this paper hold that the following phrase encapsulates the dilemma facing the verifier: *You will not find what you are seeking, but you should look for it as if you could find it* (this phrase is inspired by Mt 7,7–7,12).

[14]Mayer [29, p. 12] holds, *Philosophers have so far been unable to specify a criterion about how to distinguish between science and non-science*. Also Blaug states [2, p. 139]: *After all, there is little agreement among philosophers of science about the necessary and sufficient conditions that a scientific statement must satisfy to qualify it as a law of science*.

inition of science can be made only from the position of an independent observer, standing outside science, having at least a neutral attitude (not accepting a priori) toward the Kuhnian scientific paradigm. However, scientists reject by definition non-scientific statements by regarding them as unjustifiable, unfounded, and for these reasons – worthless. It is appropriate at this point to recall Gödel's incompleteness theorems, Tarski's undefinability theorem, and the words credited to Einstein, *no problem can be solved from the same level of consciousness that created it*.

Nevertheless, some characteristics allow us to distinguish the scientific approach to the cognition process from other, non-scientific, intellectual activities. The school of Logical Positivism holds (the verifiability principle) that a sentence is cognitively meaningful (informative) in terms of science only if it is either empirically verifiable by experience or tautologically verifiable by deductive reasoning[15]. Because our verification problem concerns not a theory but a model, we pave a third avenue (in addition to empiricism and deductive reasoning) to verification, namely "coherentism". As a theory of truth, coherentism, which has its roots in the semantic theory of truth ([44], earlier Descartes *Meditation on First Philosophy*), restricts the set of true sentences to those that cohere with some specified set of sentences. To the authors' minds, in modelling and simulation, sentences about reality are scientific if one is able to show the coherence of a sentence derived via simulation with a sentence which is a part of a scientific theory that as a part of a model under study, a sentence is meaningful only if: it is either empirically verifiable by experience, logically verifiable by deductive reasoning or verifiable as being coherent with the laws of well-accepted theory.

In the search for elements of scientificity on the road to verifying a model of an economic system, arguments based on deductive reasoning, coherency and empiricism are usually employed concurrently.

By necessity, any verification procedure is based on a limited number of tests, which might not be even vaguely comparable to the multitude of situations that economic reality can create. Thus, a positive result from a verification test does not ensure the scientificity of a model, no matter how many tests are applied or from which standpoint of

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[15]Deductive methods are applied in relation to sentences whose veracity can be unambiguously deduced from the laws of mathematics and logic. In the empirical approach, sentences about a real-life economic system obtained via simulations are scientifically justifiable if they correspond to direct observation of an economic system, performed with the use of so-called intersubjective methods. Popper [23] defines intersubjective criticism *as the idea of mutual control by critical discussion*. Also, Hegel states: *If something is valid for anybody in possession of his reason, then its grounds are objective and sufficient* (*Phänomenologie des Geistes*).

verifiability they were done[16, 17]. Moreover, from a cognitive understanding of a proposed simulation, such positive tests tend to have much less informative value than experiments that lead to falsification of the model. In the case of falsification, a modeller is forced to perform a scrupulous analysis of the model to trace the sources of any errors exposed. Thereby, the modeller gains a deeper insight into the management problems for which the model is designed[18]. For these reasons, the aim of the verification procedure should not be confirmation of a model, but, on the contrary, we should attempt to falsify the model by using all of the available methods and tools, because, in some sense, *ex falso omnia sequitur*.

When the causes of the malfunctioning of a model have been identified, then the model should be either amended or completely discarded. However, this situation can be confusing for the verifier when, despite the negative result of a test, we are not able to identify the roots of the deviations observed. Any objections against the model of an economic system derived from the standpoint of coherency can be overruled by cooking up new conjectures that are designed to just avoid the problem revealed by early testing [15] and/or adding additional, specific *ad hoc* adjustments, constraints and variables, deflating by another, and so on (immunizing stratagems, see [33]). Also, any negative outcome of empirical tests can be called into question, because one can state that it was caused, for example, by erroneous background assumptions (the so-called Duhem –Quine problem [16])[19]. It is safe to say that only deductive falsification is unambiguously decisive for us to be able to prove in this way that the construction of the model is evidently faulty.

To put it bluntly, no positive or negative test done from the stance of coherency or empiricism can be regarded as *experimentum crucis* as far as the model's scientificity is considered[20]. In consequence, there is the need to introduce criticism as an additional feature of the scientific approach to the study of real-life economic phenomena using a model. Derived from the belief that the capabilities of the human mind are limited and that man's senses are imperfect and unreliable, criticism should be understood as the conscious acceptance – so far as verification is concerned – that a model of a real-life firm can never been positively proven, but will certainly be falsified at some time.

[16]Popper [33, p. 16] concludes *...no amount of true sentences (or) experience can justify the claim that a universal explanatory theory is true*. Einstein is said to have stated, *no amount of experimentation can ever prove me right...*

[17]Moreover, a long series of tests that all end with positive results might only lead to suspicion that the verification procedure has been disabled in some way. This result is particularly likely when verification is performed personally by the creator of the model. It is an unfortunate matter of fact that in such cases, the verification procedure is often distorted (occasionally unwittingly) due to the intense emotional relationship between the creator and his model (so-called love of one's own brainchild).

[18]Travestying the words of Ackoff [1], *models of a firm are senseless, but modelling of a firm is priceless.*

[19]See Russell [40, p. 669]: *hume has proved that pure empiricism is not a sufficient basis for science.*

[20]This is an obvious consequence of the thesis about the insolvability of the demarcation problem.

A critical approach is understood as the principle of ongoing observation of the model's performance and a ceaseless search for potential errors, followed by appropriate modifications and improvements[21].

Acceptance of the standpoint of ceaseless criticism implies that the verification process cannot be considered only as a part of the construction of the model. Rather, it should be regarded as a process that accompanies a model during the entire period of its utilization until the moment when it is replaced by a new one that better describes an economic system. Based on Popper's [33, p. 32] argumentation, it is clear that the game called verifying the model of a firm is a never-ending story; *He who decides one day that scientific sentences do not call for any further test, and that they can be regarded as finally verified, retires from the game*.

In the light of the above discussion, perforce a very general one and close to a tautology, one can formulate postulates that entail implications for the proposed, in Section 1.2, procedure of verification for a model of a firm:

1. The process of verifying a model of a firm is an analysis of its scientificity.
2. Analysis of the scientificity of the model of a firm should be carried out from the standpoints of deductive reasoning, coherency and empiricism.
3. To increase the cognitive value of economic simulations, the aim of the verification procedure is not to confirm the scientificity of the model, but – quite the opposite – its falsification using all of the available means.
4. Acceptance of the stance of ceaseless criticism converts verification into a never-ending process, which accompanies a model of a firm from the beginning of its construction until the time when it is superseded by a better and more appropriate one.

Based on the above, following the principles of the Wiener Circle verificationists, it is obvious that a model of an economic system can only be taken seriously by a community of scientists if it has been subject to verification[22]. It follows that, without any results from verification trials, a model, no matter how sophisticated formally, must be classified as non-scientific (like poetry or painting). Thus, the value of such models should be assessed based solely on criteria appropriate for these types of intellectual activities (e.g., aesthetic beauty, colourfulness of narration, or moral values).

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[21]One can cite the following statement related to scientific theories, but it is also appropriate for assessing the scientificity of a model of an economic system: *The issue of demarcating scientific theory from a pseudo-scientific approach is in fact a discussion about the distinction between critical and dogmatic thinking. Dogmatic thinking is prescientific* (...) *A theory ascribed to be scientific should be subject to the possibility of refutation and should predict situations in which it cannot be applied (i.e., those that might disprove the theory). The more situations prohibited by a theory, the wider is the set of potential refutations, and the better is the theory in question. The criteria sine qua non of science is thus the possibility of refuting theories and their vulnerability to criticism. Criticism is the most important factor in the world of science* [17, p. 76].

[22]*If a sentence has no possible method of verification, it has no meaning*, Godfrey-Smith [15, p. 27].

1.2. A procedure for verifying the model of a firm – the RAD-VER procedure

This procedure for verifying a model of a firm is based on the methodological recommendations formulated in Section 1.1, in which we concluded that verification (in fact, attempting to refute) is a ceaseless process of the evaluation of a model's scientificity from the standpoints of deductive reasoning, coherency and empiricism. In the RAD–VER procedure, verification has been divided into two steps: the verification of the simulator[23] and the verification of the assumptions underlying the model of an economic system, in our case, a firm.

Verifying the simulator. From the point of view of sentential calculus, a computer program, i.e., simulator, is composed of a set of analytical sentences. At this stage of the verification procedure, it must be shown that the transformations – carried out in the course of a simulation – of the synthetic sentences in the form of the model's underlying assumptions, together with the input data (synthetic sentences), via the computer program (analytical sentences) into output data (synthetic sentences) may be regarded as a flawless, tautological chain of deductive implications. Therefore, the deductive criteria derived from mathematics and formal logic can be accepted as appropriate foundations for verifying the internal consistency of the simulator. Considering that the system under study is an economic one, deductive accounting rules are also applied[24].

In our case, the final form of the model is given by a set of discontinuous differential equations. Each equation is a mathematical representation of a statement that has been taken for granted by the modeler. In particular, the model (continuous-discrete type) of a firm has the following structure[25]:

$$M^{(c-d)} = (T, G, Z, f, \beta) \tag{1}$$

where: T – is an interval of real-time base, in which (t_p, t_k) represent the boundary moments of the simulation, $G = \{g(t) = (g_1(t), ..., g_n(t))\}$ is a set of functions defined on (t_p, t_k) and $g_1(t)$ is the *j*-th input function.

The set of its points of discontinuity of the function $g_j(t)$ is denoted by T_j^d thus $T_j^d = \{t_j^1, ..., t_j^{l(j)}\}$. Furthermore, let $T^d = \bigcup_{j=1}^{n} T_j^d = \{t_1, ..., t_i, t_{i+1}, ..., t_m\}$, where m is the number of all the points of discontinuity of $g(t) \in G$. $Z = z(t) = \{z_1(t), ..., z_s(t)\}$ is a set of functions defined on the interval (t_p, t_k), and $z_j(t)$ denotes the *j*-th state function.

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[23]Henceforth, the model in the form of a computer program will be referred to as the simulator.

[24]In this article, accounting rules (based on the BSM method) are used in a specific way for verification purposes only and do not comply strictly with the assumptions of the classical double entry accounting system.

[25]This form of notation was borrowed from Zeigler [48].

On the interval (t_i, t_{i+1}) with t_i and t_{i+1} being two consecutive elements of T^d, $z(t)$ is a solution of the following differential equation:

$$\frac{dz(t)}{dt} = f(z(t), g(t)) \text{ with initial condition } z(t_i) = z_i \quad (2)$$

where f is a state transition function $f : Z \times G \to Z$. For every $t_i \in T^d$, there exists a static function: $\beta_i : R_Z \to R_Z$, also for t_k is $\beta_k : R_Z \to R_Z$, where R_Z represents the range of the function Z.

At present, there is no general method of analytical solving the set of equations given by (2). Consequently, an exact solution is beyond our reach. However, we can utilize numerical methods to find approximate solutions of (2). If we apply, for example, the forward Euler method, then we should re-define some sets in the structure given by:

$$M_E^{(c-d)} = (T, G, Z, f, \beta, \Delta t) \quad (3)$$

Let $(t_k - t_p)/\Delta t = b_k$, then b_k is replaced by n'_k, where n'_k is the nearest integer to b_k. It follows that: $t_k \cong t_p + n_k \times \Delta t$, where $|t_k - t'_k| < \Delta t/2$. Then $(t_i \to t'_i)$, where: $t_i \cong t'_i = t_p + n_i \times \Delta t$, $|t_i - t'_i| < \Delta t/2$, $T^{d'} = \{t'_1, ..., t'_i, t'_{i+1}, ..., t'_m\}$. In each time interval (t'_i, t'_{i+1}) we have r_i points $t'_{i,h}$ such that $t'_{i+1} = t'_i + (r_i + 1) \times \Delta t$. Thus for $h \in \{0, 1, ..., r_i\}$

$$z(t'_{i,h+1}) = z(t'_{i,h}) + \Delta t \times f(z(t'_{i,h}), g(t'_{i,h})) \quad \text{with initial condition } z(t'_i) = z_i \quad (4)$$

A similar situation applies to the interval (t_p, t'_1).

$$\text{For every } t'_i \in T^{d'} \text{ is } \beta_i : R_Z \to R_Z;\ \text{also for } t'_k \text{ is } \beta_k : R_Z \to R_Z \quad (5)$$

We should be aware that the proposed numerical solution (4) for the set of Eqs. (2) is inaccurate, because of, e.g., numerical integration and truncation errors. Moreover, for a certain unknown-in-advance set of input data, there is no guarantee that the imprecision resulting from a global error approximation would not be accelerated rather than damped. Such acceleration could cause the global deviation from the exact form of the antiderivative to grow exponentially. In consequence, the iterative process of numerical integration would not be stable numerically. This weakness of numerical methods substantially reduces the confidence of a researcher in the continuous-discrete simulator as a scientific tool of operations research. Thus providing another reason that, according to the principle of ceaseless criticism formulated in Section 1, each cognitive experiment using the simulator should also be a verification experiment[26].

[26]The unpredictability of simulation, especially when it involves numerical computation, as a scientific method is well illustrated by the problem with finding the Hardy–Littlewood constant. Although the existence of

Verification of the assumptions. The scientificity of a model's assumptions can be evaluated from the standpoint of deductive reasoning, coherency and empiricism. This evaluation process is of the utmost importance in building up our confidence in a model of a firm because simulation is not directly a cognitive experiment, but rather is only a mechanical transformation of the input data and the model's assumptions into output results. These assumptions are nothing more than a petrified image of our knowledge (or ignorance) about a firm, because, to adapt what genetic empiricists have said, *nihil est in modo simulare, quod non prius fuerit in intellectu*. The authors of this paper share the views of those who believe that simulation in itself reveals nothing[27]. It only presents, in a new way, information already contained in the model's assumptions and input data. However, this new form of presentation might encourage researchers to creative self-reflection about the problem under study and to formulate new questions about future research. To the authors' minds, the value of modelling and simulation as a means of learning more about the reality around us lies precisely here. As Reichenbach [36, p. 320] writes, *it merely makes explicit some consequences which are contained implicitly in the premises. It unwraps, so to speak, the conclusions that were wrapped up in the premises.*

The accepted route to scientific verification is the deductive-nomological approach known as the Hempel–Oppenheim–Nagel method of explanation. This method can be applied if we have at our disposal an *explanans*, by which we mean a system of general laws (at least one) that covers an *explanandum*. This system, called a scientific theory, must be approved by the *communus omnia doctorum* and remains valid until falsified by an empirical experiment or a deductive argument. A wide-ranging review of the literature leads us to reflect that economists have so far failed to create a body of knowledge that can claim to be a theory of the firm. Scientists representing radically different economic schools, such as Robinson [38] and Kornai [24], uphold the idea that the theoretical approach is strongly limited as far as economics is concerned. *There is no mature economic systems theory* [24, p. 43]. *There have been many accounts of the behaviour of particular firms. These have mainly been pure descriptions without the benefit of theory or they have befuddled themselves with attempts to fit into an inappropriate analytical scheme. A theory of the firm appropriate to a dynamic economy is in its infancy* [38, p. 107]. If we map micro-economic phenomena in the form of a model

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this constant has been proven analytically, to date it has not been possible to find this constant by using a computational approach alone (see [37]). Also, the validity of the Riemann hypothesis has been demonstrated by use of computation for more than three million zeros of the zeta function (see [39]). However, this result is not treated by mathematics as an acceptable proof equivalent to deductive reasoning. In light of the above examples, some scientists place numerical calculations in doubt as a scientific method. Moreover, Chattoe [4, p. 94] argues, *most simulations have so many parameters, it is claimed, that the designer can use them to produce almost any desired result.*

[27]In fact, computational errors are the only brand new information generated in the course of simulation.

of a firm, then the deterministic, and even probabilistic principle of empirical verification proposed by Reichenbach [36] does not hold. According to Georgescu-Roegen [14, p. 278) *the validity of statistical tests, even non-parametric ones, requires conditions which a rapidly changing structure, such as the economic process, may fulfil only by sheer accident*[28, 29]. In economics, there are a number of hypotheses concerning different aspects of a firm's economic performance. However, there is no consensus on how the methods derived from these hypotheses might prove to be comprehensive.

The authors of the paper dare to say that the ability of economics to formulate even a single general law about the firm, such as one requiring the deductive-nomological method, is placed in doubt. As Nagel [30, p. 436] notes, *the differences between the required ideal conditions, for which an economic law was formulated, and real-life market conditions are so great* (…) *that the value of the nomological approach in economics is still put in question*. In other words, economics, as a science, does not offer a dependable system of quantitative economic laws, in either a deterministic or probabilistic form, that for given specific initial conditions would be accepted as an *explanans* for an *explanandum*, i.e., the assumptions of the model of a firm[30, 31]. We end this part of the discussion by conceding that regarding the economics of a firm, a standpoint based on coherency is unjustifiable, which substantially limits the credibility of verification in comparison to other sciences with well-established theoretical backgrounds[32, 33].

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[28]He continues, *by cleverly choosing one's chisels, one can always prove that inside any log there is a beautiful Madonna* (…) *there is no shortage of econometric tools by which an economist can carve as good a fit as he may please* [14, p. 340].

[29]Proposing a set of tests for building confidence in economic models of system dynamics, Forrester and Senge [13, p. 94] hold that *conventional statistical tests of model structure are not sufficient ground for rejecting the causal hypotheses*.

[30]As stated by Kobrinskij [23, p. 54], *Their large number of assumptions significantly distinguish economic theories from scientific theories, in which a relatively small number of assumptions can provide a large number of results*. This is illustrated by the neoclassical theory of the firm.

[31]From a rational standpoint, it appears that we can only assess whether the assumptions of the model of a firm are correct from the point of view of finance and, to some degree, production. In finance, such relationships are not economic laws *sensu stricto*, but are mostly analytical sentences that arise from the accounting system applied, which is in fact completely virtual. In turn, the production process is subject to relationships that have their roots in well-known theories from natural science.

[32]In summary, concerning the extensive studies of a firm's production functions, Czerwiński [5, p. 199, 200] argues, *in many cases, we have obtained good approximations to the results observed. However, these approximations are limited to a narrow range in both time and space. The accuracy of econometric models has turned out to be local, and in this respect, such models do not resemble "universal" laws. Quantitative descriptions of local regularities are clearly of use for short-term forecasting. However, econometrics has not so far detected quantitative, universal laws of economics* [41, p. 1539] remarks that years of experience have taught him *how treacherous are economic "laws" in economic life.*

[33]For economists, it is somehow reassuring that physicists are confronted with the same dilemma. Einstein [9, p. 110] argues, *for the time being* (…) *we do not possess any general theoretical basis for physics, which can be regarded as its logical foundations.*

Abandoning the deductive-nomological approach, we will rely on the hypothetical--deductive method as an alternative for verifying a model's assumptions by combining creative thinking with mathematical tools and observational methods. In the first step of the RAD-VER procedure (Fig. 1), we provisionally take for granted a set of sentences that consist of the assumptions of the model. Thereafter, verification simulations are performed.

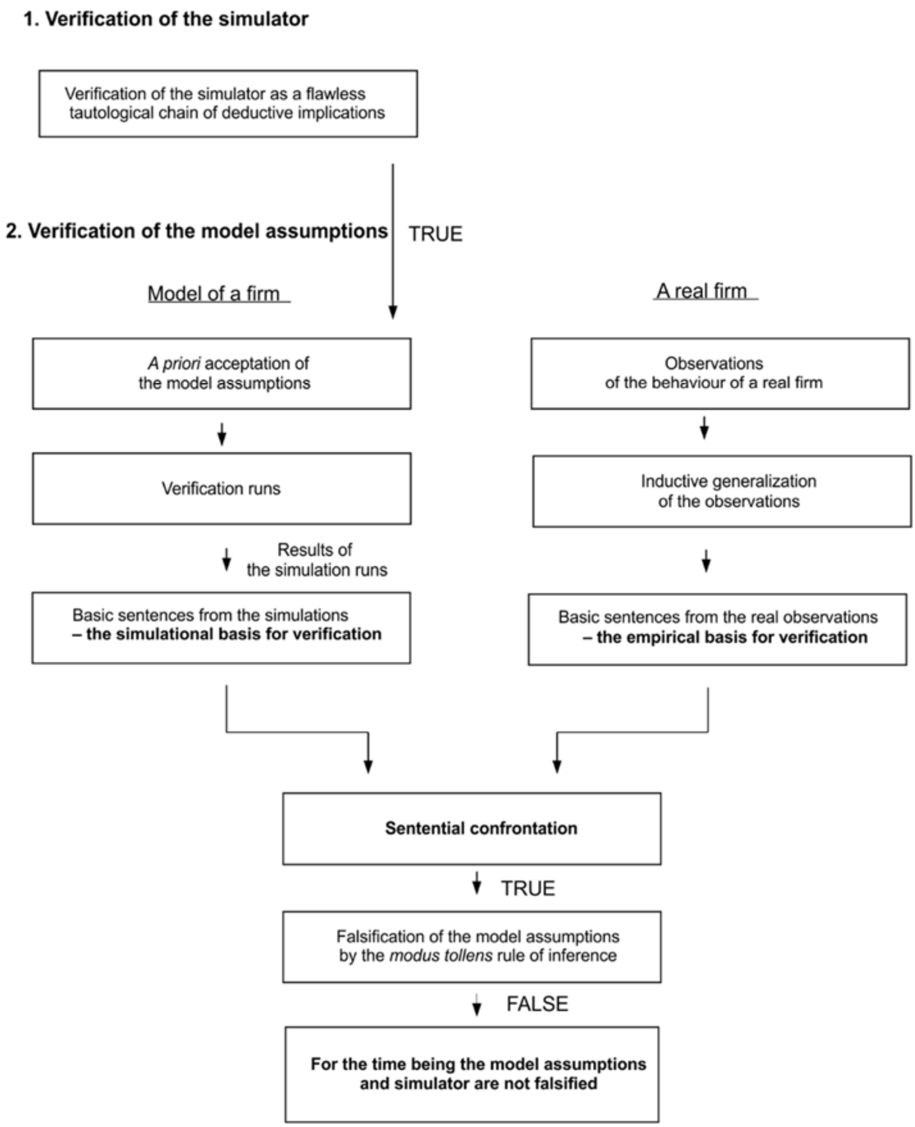

Fig. 1. Verification of the computerized model of a firm – the RAD-VER procedure

The outcomes of these runs are subjected to derivation that can lead to implications in the form of basic, simulational sentences. This set consists of the simulational basis of verification. Simulational sentences should be formulated as similarly to the form of observational statements as possible[34]. In this way we are able to compare (so called sentential confrontation) a particular simulational statement directly with its counterpart in the empirical base.

Positive verification of a simulator, as a deductive machine, and the positive results of sentential confrontation, allow us to state the correctness of the model's assumptions on the basis of the *modus tollens* rule of inference. In propositional logic, *modus tollens* is an application of the general truth that if a statement is true, then so is its contra positive. In our case, the rule of inference is as follows: model's assumptions → sentential confrontation and ¬ sentential confrontation, then model's assumptions.

In other words: when model's assumptions are accepted a priori as true, it implies that sentential confrontation must be true, but if sentential confrontation is false, it implies that model's assumptions must be false. This means that if sentential confrontation fails, then the statement about the validity of the model's assumptions must be rejected. A negative appraisal from the verification procedures requires performing a critical analysis of the detailed assumptions made during concretization, and in some cases it might be necessary to rebuild the general hypotheses formulated at the stage of abstraction. In turn, positive confrontation of the simulation outcomes with reality allows us to state that, so far as it concerns the RAD-VER procedure being accepted, the statement about the validity of the model's assumptions has not been falsified.[35]

In the operational stage, the validity of each cognitive experiment is also systematically evaluated by the verification submodel, according to the previously accepted postulate that each simulation experiment is also a verification experiment.

Thus, at least a temporary foundation has been set up for presuming that the construction of a simulator and the assumptions behind the corresponding model of a firm are acceptable, to the extent that the criteria of deductive reasoning, empiricism and, to a degree, coherency derived from the hypothetical-deductive method have been applied. Therefore, we have no reason to discard the simulator under verification. If we accept the correctness of the RAD-VER procedure, the simulator can be viewed as a scientific tool of operations research explaining the real phenomenon under study with a degree of accuracy which belongs to the subject matter.

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[34]It is by no means obvious that the following question arises: when is a basic sentence, in terms of meaning and form, appropriate for sentential confrontation and when is it not? Popper [34] writes about *the relativity of basic statements* and that convention plays a crucial role in accepting a statement as a base for empirical falsification.

[35]In this part of the verification process, we can trace some aspects of the coherency approach to truth. However, note that in Section 1.1, we restrict true sentences to those that cohere with sentences of a well-established theory of the firm.

If we accept the hypothetical-deductive method, in the form of the RAD-VER procedure, as an approach to verification, we must again return to a discussion about the potential applications of a simulator of a firm. In general, it can be applied to:

- explaining, in the form of sentences, the dynamic relationships between causes and effects (so-called what-if analysis) observed in the economic system under study,
- forecasting, in the form of sentences, about the future of an economic system.

If we are going to utilize a simulator for making forecasts, then we accept the following two fundamental assumptions:

- the ontological assumption that relationships between the past, the present and the future of a firm are embedded somewhere in the system structure, but are not directly visible,
- the epistemological assumption that these hidden relationships can be revealed, i.e., identified, described and measured.

There is also a metaphysical assumption that real time and simulation time are of the same substance[36]. All of the above assumptions (ontological, epistemological and metaphysical) are what Kant would have termed a priori sentences that are not verifiable by the criteria of coherency, deductive logic or empiricism[37].

For these reasons, it remains open to question, moreover, whether an economic model can be utilized as a scientific tool for management forecasting. Observing the discussion between Bacon (*Novum Organum*) and Hume (*An Enquiry Concerning Human Understanding*), the authors of this paper are inclined to opt for a view that we cannot have any knowledge about the future and that therefore forecasting methods are unjustifiable, especially in economics[38].

We would like to point out that there is an inverse relation between the amount of informative content in a sentence and the probability that it will be shown to be true to the extent that it addresses the future. Sentences about the future whose probability is

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[36]The assumptions that real time and simulation time are equivalent concepts must be regarded as a radical simplification. For example, a simulation model posits the linearity and sequentiality of events and processes. A comprehensive analysis of this problem, crucial to verification, is not possible within the scope of this paper.

[37]The basic assumption in dynamic models of an economic system that the behaviour of a firm can be mapped by a set of differential equations, can also be put in question. This assumption is also *a priori* a Kantian sentence and has been arbitrarily formulated based on a highly speculative analogy between economic systems and fundamentally different physical systems, particularly mechanical and electrical ones (see [12]]. As Żurawicki [48, p. 83] states *when arguing for the use of (dynamic) models as a cognitive tool in economics, we recall the numerous successes of this modelling approach in physics, but it seems that the uncritical transplantation of concepts from the field of mechanics to economics is unjustifiable, due to many reasons.*

[38]Schoeffler states [43, p. 94]: *scientific predictions are only possible when there are universal laws unrestricted as to circumstances, and since the economic system is always open to noneconomic forces and the play of chance, there can be no economic laws and hence no economic predictions as such*. Also, Blaug states [2, p. 247]: *Clearly, there are still serious limitations on the capacity of economists to predict the actual course of economic events and hence ample room for scepticism about mainstream economics.*

almost equal to 1 have a cognitive value close to nil, therefore, as near tautologies, they are almost worthless[39]. Correspondingly, when a sentence contains an extremely important message about the future, then the probability that it will happen is usually close to nil, making it also valueless. In any field of knowledge, including economics as a science, there are some "grey zones" in which, due to the efforts of scientists, we are able to formulate sentences about the future with substantial informative value and a reasonable probability of being fulfilled[40]. However, concerning the reliability of a model as a scientific tool of operations research, we limit its potential field of application to explanatory (what if), dynamic experiments performed in *all other things being equal* (*ceteris paribus)* mode[41, 42].

When defining management problems to which the model of a firm can be successfully applied, we should also take into account the unsolved problems of observing, categorizing and measuring economic variables at the microeconomic level. Formulating a relationship in a quantitative form, as required by any computer simulation, is a burdensome task because many of the crucial factors that determine a firm's dynamics are extremely difficult to identify and measure. For example, factors such as the position of an organization in its business environment, the authority and aspirations of management, and confidence in the stability of the market all have a significant effect on management processes and subsequently the performance of a firm[43]. However, plausible methods for quantifying these features have not yet been found[44].

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[39]This problem draws Mayer's [29] attention, but in a slightly different context – probability *vs*. testability, also Popper [35] considers the relation testability – informative content.

[40]It is a personal opinion of the authors of this paper that effective economic forecasting is possible only under the condition that the assumptions of scientific determinism are fulfilled. A comprehensive analysis of this crucial question of verification is not possible within the scope of this paper.

[41]Note that despite the existence of huge and reliable databases covering past stock prices, it has not even been shown that the best models, based on sophisticated methods of fundamental or technical analysis, forecast better than a random walk. Moreover, to the best of the authors' knowledge, none of the numerous energy-forecasting models managed to predict the rapid fall in oil prices in 2009 and 2014. The authors of this paper dare to say that any attempt to formulate a precise economic forecast is a road to nowhere. The more the forecast is quantitative and accurate in terms of numbers, the smaller the chance that it will have any cognitive value as a prognosis. It is worth mentioning the phrase attributed to Galbraith that *the only function of economic forecasting is to make astrology look respectable*.

[42]Discussing the scientific sense of forecasting models, we should address the problem of mental control by the modeller over a model's performance. Georgescu-Roegen [14, p. 340] pointed this out in a slightly different context: *the more complicated the model and the greater the number of variables involved, the further it moves beyond our mental control, which in the social sciences is the only possible control.* It is beyond the scope of this paper to present a thorough discussion on this crucial problem regarding verification.

[43]It is needless to remind the reader that so far no satisfactory answer has been found to the fundamental question: is economics written in the language of mathematics?

[44]It is not a rare case that a modeller overcomes a problem regarding the measurement of economic variables in a way that makes a travesty of Kelvin's statement: *If you cannot measure, measure anyhow*.

Because of the problems regarding identification and measurement, together with the unverifiability of some crucial assumptions underlying the structure of a model of a firm, it should not be expected that the dynamic characteristics generated via simulations could be verified empirically against reality using quantitative criteria. Once again, having in mind the ideal of a model as a scientific tool of operations research, we should oppose the hypothesis that a model has the power to explain the performance of a real-life firm quantitatively. Thus, we postulate that, when assessing the scientificity of such models, the verification criteria, and therefore conclusions about a real-life firm derived from simulations, should be restricted to qualitative ones[45].

Based on this discussion, we would emphasize the following conclusions to the extent that it applies to the proposed method of

• Verifying a model:

– the RAD-VER procedure based on the hypothetical-deductive method will be applied as a method of verifying the simulator of a firm,

– the verification of outcomes of simulations can be done in qualitative terms only.

• Constructing a model, the verification modules should be an integral part of the simulator software.

• Applying a model as a scientific tool of operations research:

– a simulator of a firm will only be applied for explanatory *ceteris paribus* experiments,

– only qualitative conclusions will be derived from simulation.

As one of the elements of the RAD-VER verification procedure, it was assumed that the verification modules are an integral part of the simulator software. As a consequence of this assumption, a specific procedure for constructing simulation models should be proposed. This approach, called the abstraction – gradual concretization – verification procedure, will be presented in a forthcoming paper *Construction of a flexible simulational model of a corporation*. This procedure will enable one to fulfil in practice the principle that each simulation is simultaneously a verification trial. In that paper, the practical application of the procedures for constructing and verifying models will be demonstrated on the basis of the example of a simulation model of a firm.

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[45]Postulating a limit on the potential field of applications for a simulator of a firm, note that those scientists who are able to control their cognitive desire will discover that they will gain much more than they lose.

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