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ON THE ONE-SHOT TWO-PERSON ZERO-SUM GAME IN FOOTBALL FROM A PENALTY KICKER'S PERSPECTIVE

A penalty kicker's problem in football has been modelled. The study took into consideration different directions in which the ball can be struck and goalkeepers' success at defending shots. The strategic form of the game that can be used to predict how the kicker should optimally randomise his strategies has been modelled as a non-linear game-theoretic problem from a professional kicker's viewpoint. The equilibrium of the game (i.e., the pair of mutually optimal mixed strategies) was obtained from the game-theoretic problem by reducing it to a linear programming problem and the two-phase simplex method was adopted to solve this problem. The optimal solution to the game indicates that the kicker never chooses to kick the ball off target, to the goalpost or to the crossbar, but rather chooses to kick the ball in the opposite direction to the one where the goalkeeper is most likely to successfully defend from past history.

Keywords: *linear programming, mixed strategy, penalty, kick, strategic game, two-person zero-sum game*

1. Introduction

Game theory has found applications in many real life situations where the outcome of one person's choice depends not only on his/her own behaviour, but also on the choices of other individuals involved [7]. The game of football (soccer) has gained prominence in the sports entertainment industry. Several studies have been carried out on the game of football (for example, the job security of a footballer [13], racism [4], penalty kicks [2, 5, 11, 12] and domestic violence associated with professional football games [1]). This study is research related to penalty kicks. A penalty kick is a simultaneous-move strategic game that involves two players (a kicker and a goalkeeper) and the actions of the players are governed by a precisely defined set of rules. The outcome, which is a goal or no goal, is decided immediately after the kick. From a technical point of view, penalty kicks are described as a one-shot two-person zero-sum game between

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the kicker and the goalkeeper [11]. This is because the game involves two players wherein the ball is kicked once and no second chance given to the kicker when no goal is scored. The kicker gets a bonus point if a goal is scored; otherwise the goalkeeper gets the point. Penalty kicks arise after certain offenses are committed by a team in its own penalty area or at the end of the match in a "no-ties-allowed" tournament when no winner has emerged after ninety minutes of regular time followed by extra-time. The "no-ties-allowed" situation in football is common in a knockout competition. The knockout system is a well-known procedure for managing sports competitions [15]. In such a system, the team that loses a match is eliminated from the competition. In a penalty shootout, a kick is shot from the penalty spot, which is located 11 m from the goalpost, by the kicker to the goalkeeper (who is the only one allowed to defend the ball). According to [11], the 12-yard (11 m) distance between the penalty spot and the goal line is too short for players to choose not to move simultaneously.

It has earlier been found that the goalkeeper's move should depend on the kicking leg of the kicker [11]. That is, the goalkeeper's optimal choice when facing a right-footed kicker is to choose right more often than left; otherwise he chooses left more often than right. The goalkeeper's ability to defend the ball is a subject of concern to the kicker. The kicker's dilemma arises from the uncertainty in predicting the action of the goalkeeper coupled with the fact that he has to choose his actions independently of and simultaneously with the goalkeeper.

The action of the kicker can be observed after the kick from the direction of the ball. There are four possible directions that the ball can go when hit by the kicker. It can be shot wide or hit the goalpost or crossbar (all these constitute a "shot of regret"), at the middle, at the right hand side or at the left hand side of the goalpost. These directions are denoted using the symbols O, M, R, L, respectively. A kick in the direction O does not include saves made by the goalkeeper. The goalkeeper may jump towards the right or left, R or L, or maintain his position, M. Earlier studies assumed that there were only three directions in which the ball can go (M, R, L) to construct the strategy space [2, 5]. If the kicker has a reputation of kicking the ball in a particular direction (that is, he adopts a pure strategy), he will fare poorly, as his action will be anticipated and countered by the goalkeeper. For this reason, the kicker should deliberately randomise his choice of action. The process of selecting an action on a particular occasion is associated with some (fixed) probabilities and such a decision process is known as a mixed strategy.

A mixed strategy Nash equilibrium is the usual solution to penalty kicks in football. This concept of solution gives accurate predictions of the direction of kicks and jumps in an actual penalty shootout [2]. The concept of mixed strategy equilibrium has been shown to be consistent with zero-sum games [5]. Research has shown that professional players in football play penalty kicks in such manner that their behaviour mimics equilibrium play [11], but this is not true for non-professionals [12]. It has been found using real life data that (i) winning probabilities are statistically identical across the strategies of players and (ii) players' choices are serially independent [11].

Kaneko and Liu [10] studied the process of iterated elimination of (strictly) dominated strategies and inessential players for finite strategic games. Such a process may lead to a reduction of the game. It is usually assumed that decision makers pursue well-defined exogenous objectives (they are rational) and take into account the knowledge or expectations of other decision makers' behaviour (they reason strategically). A decision maker is rational if he is aware of his alternatives, forms expectations about any unknown, has clear preferences and chooses his action deliberately after some process of optimisation.

This study was aimed at devising a mixed strategy that could aid a penalty kicker to select an action from the finite set of pure strategies $\{O, M, R, L\}$ in a 4×3 game in a strategic interaction situation. The theoretical underpinning to achieving this aim hinges on formulating a non-linear game-theoretic model. The model is reduced to a linear programming (LP) form. This reduced form is encouraged by the well-developed methods of solution in the literature for LP problems and the success of LP models in diverse and substantive areas [3, 6, 8, 9, 14, 16]. Rather than using real life data, this study relies on data obtained from simulation. This is because naturally occurring phenomena are typically too complex to be empirically tractable [12]. For instance, in a penalty shootout, the real life data set may be affected by the state of the match (such as the time of the penalty), the place of the match (e.g., the goalkeeper is in the home team), score difference (e.g., match tied, goalkeeper's team ahead by one or more goals or behind by one or more goals), etc.

The work of [11] is very inspiring and a significant contribution to the one-shot two-person zero-sum game. The study modelled the game using the kicker's probabilities of scoring and provided a theoretical analysis of the 2×2 game matrix. As opposed to [11], this paper considers a 4×3 game with twelve possible expected payoffs (each corresponding to a pair of actions). The payoff matrix herein is constructed within the cognitive context that the kicker keeps records of the goalkeeper's successes at saving penalties. From the literature available to the author, this cognitive behaviour, which was earlier reported in [11], has not been used anywhere to model the one-shot two-person zero-sum game in football. Penalty kicks have been modelled in the literature as either a 2×2 game [11] or a 3×3 game [2, 5] based on the probability that a goal is scored. This study contributes to the literature by modelling penalty kicks using the probability of success of the goalkeeper. From a methodological viewpoint, this study illustrates why a kicker should in the absence of communication between him and the goalkeeper choose a particular strategy more often, based on the goalkeeper's previous successes in saving penalty kicks.

2. Methods

The intuition behind formulating the model in this section is straightforward. Take a kicker who keeps records of goalkeepers' saving penalty kicks. Such a kicker may be described as a professional [11]. In this case, it is not the natural side of the kicker that matters, but the goalkeeper's potential as measured by the proportion of penalty kicks saved. The goalkeeper is assumed to be at M on the goal line, facing the kicker, until the ball has been kicked. It is assumed that that the kicker reasons strategically and that the strategy chosen currently by him depends on the goalkeeper's past history. This assumption is valid as professionals keep records of goalkeeper's successes [11]. The assumption that kickers play as if all goalkeepers are identical [5] is relaxed. Learning effects are not considered. This is because professionals have sufficiently trained before they are selected for a match. It is further assumed that the state of the match before a penalty is awarded, the place of the match, the score difference, the effect of a team's supporters during a kick and the kind of competition are exogenous to the penalty kick.

Let $G = (X, Y, \mathbf{A})$ be a game modelling penalty kicks, where $X = \{O, M, R, L\}$, $Y = X/\{O\}$, as the goalkeeper may choose to jump to the left (L) or right (R), or remain in the middle (M), and $\mathbf{A} = (a_{ij})$ is a real-valued function defined on $X \times Y$ with

$$a_{ij} = \begin{cases} -1 & \text{for } i = O \\ (1-2p_j) & \text{for } i = j \neq O, \quad i \in X, \; j \in Y \\ 1 & \text{otherwise} \end{cases}$$

where p_j is the probability that the goalkeeper successfully defends the ball in direction j. This probability, p_j, is used as a proxy for the goalkeeper's ability (or potential) to save a shot on target in direction j. The elements of X or Y are referred to as pure strategies and the matrix $\mathbf{A}$ is the game (or payoff) matrix with element a_{ij} being the payoff of the kicker. In a_{ij}, i denotes the kicker's action and j denotes the goalkeeper's action. The set $\{O, M, R, L\}$ is the strategy space of the kicker with R and L viewed from the standpoint of the goalkeeper (for instance, R is the right hand side of the goalkeeper).

Suppose that $p_j = 0$ for at least one $j \in X/\{O\}$. Then the kicker would always kick the ball in a direction j for which $p_j = 0$, regardless of the decision of the goalkeeper. In this case, there exists at least one $a_{ij} = 1$ which defines a saddle point and the penalty shootout has an equilibrium in pure strategies. Nonetheless, this kind of situation is not obtained in practice, as the goalkeeper has the ability to defend the ball in different directions. Thus the value of the game, v, should be less than 1. When $0 < p_j < 1$, the value of the game, v, satisfies the relation

$$\max_{j \in Y} (1-2p_j) < v < 1$$

As the kicker is assumed to reason strategically, the kicker would maximise his expected payoff by taking into account the distribution of the goalkeeper's choice of action to minimise his expected payoffs. Let $\boldsymbol{\sigma} = (x_O, x_M, x_R, x_L)'$ be a 4-tuple of probabilities defining the mixed strategy of the kicker. These probabilities, the x_i's, can be determined by solving the following maximin problem

$$\max_{x_i} \left(\min_{j \in Y} \sum_{\forall\, i \in X} a_{ij} x_i \right)$$

subject to

$$\sum_{\forall\, i \in X} x_i = 1$$

$$x_i \geq 0 \;\; \forall\, i \in X$$

Note that the constraints in the maximin problem are linear, but the objective function is not a linear function of the probabilities, the x_i's, because of the min operator. The problem is made linear by setting

$$v = \min_{j \in Y} \sum_{\forall\, i \in X} a_{ij} x_i$$

The variable v is unrestricted in sign, since the lower bound of the interval $\max_{j \in Y}(1 - 2p_j) < v < 1$ is negative when $p_j > 0.5$. It follows that the variable v can be represented using a pair of non-negative variables as $v = v^+ - v^-$, $v^+ \geq 0$, $v^- \geq 0$. Revising the constraints of the original problem by introducing both slack and artificial variables as needed, the kicker's problem is transformed to the following linear programming (LP) problem,

Maximise

$$z = \mathbf{e}_{1\times 6}\mathbf{d}_{6\times 1} + \mathbf{0}_{1\times 3}\mathbf{S}_{3\times 1} + \mathbf{0}_{1\times 4}\boldsymbol{\alpha}_{4\times 1}$$

subject to

$$\left[\boldsymbol{\Delta}_{4\times 6},\ \begin{bmatrix} \mathbf{I}_{3\times 3} \\ \mathbf{0}_{1\times 3} \end{bmatrix},\ \mathbf{I}_{4\times 4} \right] \begin{bmatrix} \mathbf{d}_{6\times 1} \\ \mathbf{S}_{3\times 1} \\ \boldsymbol{\alpha}_{4\times 1} \end{bmatrix} = \mathbf{b}_{4\times 1}$$

$$\mathbf{d}_{6\times 1} \geq \mathbf{0}_{6\times 1}\,,\ \mathbf{S}_{3\times 1} \geq \mathbf{0}_{3\times 1}\,,\ \boldsymbol{\alpha}_{4\times 1} \geq \mathbf{0}_{4\times 1}$$

where $\mathbf{0}_{1\times n}$ is the $1\times n$ null vector, $\mathbf{S}_{3\times 1}$ is a 3×1 vector of slack variables, $\boldsymbol{\alpha}_{4\times 1}$ is a 4×1 vector of artificial variables, $\mathbf{I}_{n\times n}$ is the $n\times n$ identity matrix,

$$\mathbf{e}'_{1\times 6} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},\ \boldsymbol{\Delta}_{4\times 6} = \begin{bmatrix} 1 & -1 & 1 & 1-2p_M & -1 & -1 \\ 1 & -1 & 1 & -1 & 1-2p_R & -1 \\ 1 & -1 & 1 & -1 & -1 & 1-2p_L \\ 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\mathbf{d}_{6\times 1} = \begin{bmatrix} v^+ \\ v^- \\ x_O \\ x_M \\ x_R \\ x_L \end{bmatrix} \text{ and } \mathbf{b}_{4\times 1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The prime is used to denote a matrix or vector transpose. The kicker's transformed problem can be solved using the two-phase method. As the name suggests, the two-phase method involves the use of two objective functions in two phases, the objective function *w*, which is the sum of the artificial variables, and the original objective function, *z*. In the first phase (phase I) attention is geared towards finding whether a basic feasible solution exists by minimising the sum of the artificial variables subject to the revised constraints. The computational efforts involved in finding the mixed strategy of the kicker encompass iterations with 6 decision variables, 3 slack variables and 4 artificial variables with 4 functional constraints. The initial simplex tableau is given in Table 1.

Table 1. Initial tableau

Basic variable	$\mathbf{d}_{6\times 1}$	$\mathbf{S}_{3\times 1}$	$\boldsymbol{\alpha}_{4\times 1}$	Solution
$\boldsymbol{\alpha}_{4\times 1}$	$\boldsymbol{\Delta}_{4\times 6}$	$\begin{bmatrix} \mathbf{I}_{3\times 3} \\ \mathbf{0}_{1\times 3} \end{bmatrix}$	$\mathbf{I}_{4\times 4}$	$b_{4\times 1}$
z	$-\mathbf{e}_{1\times 6}$	$\mathbf{0}_{1\times 3}$	$\mathbf{0}_{1\times 4}$	0
w	$-\mathbf{1}_{1\times 4}\boldsymbol{\Delta}_{4\times 6}$	$-\mathbf{1}_{1\times 3}$	$\mathbf{0}_{1\times 4}$	-1

where $\mathbf{1}_{1\times 4}$ is a 1×4 vector of ones. The second phase (phase II) searches for an optimal solution by applying the simplex method using the original objective function, *z*. The optimal solution gives the kicker's equilibrium mixed strategy $\boldsymbol{\sigma} = (x_O, x_M, x_R, x_L)'$. This distribution would only hold for a particular game matrix depending upon the realisation of (p_M, p_R, p_L), as the payoff matrix is game-specific.

The most similar model compared to the present study has been presented by Chiappori et al. [5] in the sense that both studies constructed a match-specific game matrix and adopted a mixed strategy whose optimal solution is contained in the strategy space $\{M, R, L\}$. Yet there are some differences. For instance, the goalkeeper's ability to defend the ball in various directions was not considered in [5]. The authors constructed a 3×3 game matrix based on the probability that a goal is scored, so that all of the entries in the matrix are non-negative. The population in [5] was characterised by some distribution with five parameters, whereas this study utilises a distribution with three parameters for the same purpose. Though the results obtained in this study are consistent with the ones in the literature, the intuition and simplicity of the formulation of the model are a significant contribution to knowledge. What is more, the strategy of a kicker is observed after the kick. This implies that the strategy space can be constructed from the possible directions in which the ball can go. This conceptual underpinning leads to a delineation of the strategy space of the kicker as $X = \{O, M, R, L\}$. All the same, this strategy space becomes $\tilde{X} = \{M, R, L\}$, as in [5], when the principle of eliminating dominated strategies is applied to the game.

Another dissimilarity between this study and [5] is the interval in which the optimal value of the game, v, lies. To see this, consider the game with the payoff table that was analysed in [5], where M is used here instead of C to denote the centre of the goal (Table 2).

Table 2. A 3×3 game matrix modelling penalty kicks

$i \backslash j$	L	M	R
L	p_L	π_L	π_L
M	μ	0	μ
R	π_R	π_R	p_R

In the payoff table, p_i is the probability that a goal is scored when the kicker and the goalkeeper choose the same side, $i \in \{R, L\}$, π_i is the probability that a goal is scored when the kicker chooses i and the goalkeeper chooses differently, $i \in \{R, L\}$ and μ is the probability that a kick to the centre is a goal when the goalkeeper jumps to one side. The study [5] assumes that if both players choose the centre, then the goalkeeper always saves the ball. Thus the value at the centre of the principal diagonal is zero. Just as in [2], the assumption that if both players choose the centre, the goalkeeper always saves the ball is relaxed in this study. The study (see page 1141 of [5]) also assumed that $1 > \pi_i > p_i$,

$\pi_i > p_j$, $i \in \{L, R\}$, $i \neq j$, and $\pi_i > \mu$ as a kick in direction i may go off target. Using the minimax-maximin principle, the optimal value of the game lies in the interval

$$\max(p_L, p_R) < v < \min(\pi_R, \max(\pi_R, \pi_L), \pi_L) = \min(\pi_R, \pi_L)$$

The bounds may be determined according to the kicking leg of the kicker, that is, the natural side of the kicker. For example, if L is the natural side of the kicker, then $\pi_L \geq \pi_R$ and $p_L \geq p_R$ so that

$$p_L < v < \pi_R$$

It is now clear that the bounds on the value of the game in [5] require information on the natural side of the kicker, in addition to the probability that a goal is scored, whereas the one herein only depends on the probabilities of the goalkeeper's saving a shot given that he moves in the correct direction.

In another study which is similar to [5], a 3×3 game matrix was built using the sub-strategy space $\tilde{X} = \{M, R, L\}$ and the probability that a goal is scored [2]. The payoff matrix of [2] is analogous to the one in this study, except that the payoffs to the kicker when he strikes to the middle and the goalkeeper moves in the wrong direction are always less than 1. The reason for this difference may be attributed to the fact that the probability of a "shot of regret" when the kicker's choice is, e.g., M is embedded in the expected payoffs in [2], whereas this action is disaggregated and denoted as O in the current study.

3. Numerical experiment

The utility of the game-theoretic LP model for penalty kicks is illustrated by way of simulating the goalkeeper's ability to save a shot. Simulation is a helpful tool to gain insight into the nature of a game. The simulations and all other computations are carried out in the MATLAB environment. For illustrative purposes, the probability that the goalkeeper defends the ball on target is simulated 20 times using the MATLAB command "rand". This simulation is assumed to reflect the match-specific scenarios of 20 goalkeepers. In this illustration, the number of simulations does not matter, as the object of this study is not to provide a statistical test regarding the parameters that characterise the population or assumptions made in the construction of the model, but to show that under the present setting of penalty kicks, equilibrium strategies may be used to predict how the kicker should randomise his actions. To make the simulated results realistic, goalkeepers' histories of successes may be obtained from a series of penalty kicks taken from the archives of television channels on competitive football games (such as leagues, FIFA World Cup, African Nations Cup, etc.) and the likelihood of a goalkeeper saving in a specific direction is computed as the number of saves made by the goalkeeper divided by the number of kicks in the case where both the kicker and the goalkeeper choose that specific direction. Each case of these simulated values is used to construct the game matrix and the resulting system is solved using the two-phase method. The results obtained are presented in Table 3.

Table 3. Optimal mixed strategy for various simulations

Case	Goalkeeper's potentials (p_M, p_R, p_L)	Mixed strategy (x_O, x_M, x_R, x_L)	Value of the game
1	(0.1734, 0.9516, 0.0326)	(0, 0.1538, 0.0280, 0.8181)	0.9467
2	(0.3909, 0.9203, 0.5612)	(0, 0.4714, 0.2002, 0.3284)	0.6314
3	(0.8314, 0.0527, 0.8819)	(0, 0.0562, 0.8904, 0.0532)	0.9062
4	(0.8034, 0.7379, 0.6692)	(0, 0.3040, 0.3310, 0.3650)	0.5115
5	(0.0605, 0.2691, 0.1904)	(0, 0.6484, 0.1457, 0.2059)	0.9216
6	(0.3993, 0.4228, 0.3689)	(0, 0.3304, 0.3120, 0.3576)	0.7362
7	(0.5269, 0.5479, 0.4607)	(0, 0.3220, 0.3097, 0.3683)	0.6607
8	(0.4168, 0.9427, 0.9816)	(0, 0.5357, 0.2368, 0.2275)	0.5534
9	(0.6569, 0.4177, 0.1564)	(0, 0.1477, 0.2322, 0.6202)	0.8060
10	(0.6280, 0.9831, 0.8555)	(0, 0.4214, 0.2692, 0.3093)	0.4707
11	(0.2920, 0.3015, 0.6448)	(0, 0.4130, 0.4000, 0.1870)	0.7588
12	(0.4317, 0.7011, 0.3763)	(0, 0.3619, 0.2228, 0.4152)	0.6875
13	(0.0155, 0.6663, 0.1909)	(0, 0.9055, 0.0210, 0.0735)	0.9720
14	(0.9841, 0.5391, 0.4283)	(0, 0.1952, 0.3563, 0.4485)	0.6158
15	(0.1672, 0.6981, 0.4820)	(0, 0.6304, 0.1510, 0.2186)	0.7892
16	(0.1062, 0.6665, 0.1206)	(0, 0.4902, 0.0781, 0.4317)	0.8959
17	(0.3724, 0.1781, 0.5895)	(0, 0.2686, 0.5616, 0.1697)	0.7999
18	(0.1981, 0.1280, 0.2262)	(0, 0.2921, 0.4521, 0.2558)	0.8843
19	(0.4897, 0.9991, 0.3846)	(0, 0.3619, 0.1774, 0.4607)	0.6456
20	(0.3395, 0.1711, 0.5830)	(0, 0.2804, 0.5563, 0.1633)	0.8096

Table 3 shows that the optimal mixed strategies vary according to the goalkeeper's potential and thus counter the assumption that kickers play as if all goalkeepers are identical [5]. The results indicate that the kicker should consciously randomise his strategies over the sub-strategy space $\{M, R, L\}$ so that all shots are on target and the kicker is most likely to shoot in the direction where the goalkeeper is least likely to save the shot. The probability $x_O = 0$ is expected, as any strategy $i \in X/\{O\}$ strictly dominates strategy O. Even though the kicker and the goalkeeper choose their actions independently and simultaneously, the results suggest that, however good the kicker is at kicking the ball in a specific direction by relying on his natural side, he should probably choose the opposite direction if historical records show that the goalkeeper has a high rate of success at saving the ball in the direction of his natural side. This is a novel and a useful contribution to the literature [2, 5, 11, 12].

It is worthy of note that summary statistics (such as the mean, standard deviation, confidence intervals, etc.) may be computed to estimate the value of the game. For instance, Table 3 gives the following summary statistics, $\overline{v} = 0.7501$, σ = 0.1488, $\gamma_1 = -0.2808$ and the 95% confidence interval for $\overline{v}$ is [0.6805, 0.8198], where $\overline{v}$ is the mean, σ is the standard deviation, and γ_1 is the skewness of the value of the game. These results show that the distribution of the values of the game is negatively skewed (i.e., to the left) and that, at the 5% level of significance, the game is not fair.

4. Conclusion

An attempt has been made to gain insight into penalty shootouts. The strategic form of a game was adopted to model the situation. While off target kicks are dominated by kicks on target, the goalkeeper's ability to defend the ball is a subject of concern to the kicker. The study shows that the kicker's dilemma can be evaded if information on the goalkeeper's success rates at defending the ball in different directions is known and then kicks may be more often directed to the side where the goalkeeper has less ability. This means that the kicker should always make a deliberate effort to hit the ball on target and consciously randomise his strategies. The results obtained in this paper only suggest how the kicker should choose a mixed strategy given the goalkeeper's success rates, but do not specify the actual actions of the kicker. This study opens up further research directions. It may be of interest to investigate the assumptions made in this paper and the effects of learning on penalty kicks for amateur teams in a real world scenario using statistical tools. This study assumes that the state of the match before a penalty is awarded, the place of the match, the score difference, the effect of a team's supporters during a kick and the kind of competition are exogenous. These assumptions may be empirically validated using statistical methods (for example, fitting a linear model with these exogenous variables as covariates and success in penalty kicks as the dependent variable).

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