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MANAGEMENT AND DECISIONS IN THE STRUCTURES OF HUMAN ACTIVITIES

This article has been devoted to the key dimensions of decision-making. The main goal of the authors was to point out the role and effect of invariants of nature, logic and conceptual systems of science and management, which are extremely important in decision-making processes. The research hypothesis has been tested that the complexity of decision-making and management are determined by the state of reality (Nature). This hypothesis is related to the fact that in science there is currently no uniform methodology associated with decision-making, just as science is not methodologically uniform. One can even doubt whether it is possible to describe the essential dimensions of decisions undertaken by *Man*, as discussed in this article. These problems are not a novelty to science, since they have been analysed by many scientists in the past. The authors of the article present the complexity and diversity of concepts defining systems of decision-making and management, based on selected fields of knowledge which are generally relevant to this issue, in particular fields associated with ontology and epistemology. Therefore, the text refers broadly to investigating the reality of basic areas of human knowledge and the overlapping relationships between them. This applies to the so-called circle of the sciences proposed and examined by the psychologist J. Piaget. An additional aim of the authors was to create a text presenting contemporary human knowledge about the reality which surrounds us. To understand reality means to be in relative equilibrium with it.

Keywords: *management, decision-making, structure, algorithm, scientific language*

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1. Introduction

Many important human activities are associated with making certain decisions. But it can also be said that almost our every move is a decision. There are, therefore, decisions which are crucial and those which are of no importance to our life. The environment, i.e., the set of conditions in which someone makes decisions, in most cases has an undetermined character[4]. However, there are also situations where decisions are made in which the conditions are specified, although not necessarily simple. This aspect of decision-making is not easier than the previous one. What connects these two dimensions of decision-making? Generally, it can be said that these aspects of decision-making and their structure, to some extent, can be represented within the framework of logic. At this point, the following question appears: why only to a certain extent? The answer is: because every decision-making process has content and form. Fundamentally, logic does not deal with content, only with form[5]. However, this has a very important advantage – it is possible, often by one and the same logical form, to represent one decision-making process subject to determined conditions and other process subject to undetermined conditions, i.e., where decisions are associated with risk. Moreover, the same logical form can represent both fundamental and less significant decisions. This constitutes the strength and the ability of logic to abstract from content.

One can look at the decisions we make according to their type (category). Various disciplines of knowledge deal with these categories separately and are able to analyse them in fine detail, and sometimes even in the form of a precise algorithm. In particular, these fields include mathematics [27, 38] and psychology [35, 36], as well as many other fields of science. Each of these and other fields of science recognize the issue of decision-making in terms of their own language (concepts, terms and definitions specific to a given field). However, it is often said that this issue should be studied in an interdisciplinary manner. For this reason, the authors have undertaken an interdisciplinary approach in this text. It is easy to propose verbal theories, but very difficult to find proofs of them, or, in particular, verify them in real life in an operational way. This enables the reliable transmission of such theories to the level of philosophy. This last thought makes the link between formal theories and philosophy.

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[4]This is an extremely important assertion with regard to human behaviour, because you can, e.g., ask how today's Poland would look if Mieszko I had decided to receive baptism not from Rome, but from Byzantium ([41], p. 8).

[5]Although semantics is very widely discussed in logic, it is to a significant extent logical semantics (cf. [4]). In addition, logic covers syntax and logical semantics ([10], p. 113). However, there is also a third dimension, so-called pragmatism ([8], p. 35–58).

2. Cognitive patterns in management

The formulation of the category of decision-making made by the authors in the introduction will now be framed in a more general and, at the same time, interdisciplinary aspect.

2.1. The nature of schemes and patterns in decision-making

Decisions taken by people are rational, particularly in the Cartesian sense, when they are based on knowledge, or, even better, on mathematical formulas [5, 6]. The basis of knowledge from a logical point of view (in the form of truth) is constituted by the concept of a tautology. In practice, a tautology means the presentation of something via the same thing or, colloquially speaking, an object that represents itself ([25], p. 190). However, in logic, the concept of a tautology has a different meaning, namely a logical expression which is always true, regardless of the logical value of the individual sentence variables of a given logical expression. This understanding of a tautology is important for our further considerations. In logic, especially in propositional calculus, there are several known operationally effective methods of checking whether the course of a decision-making process occurs in accordance with the laws of logic: that is to say, whether it has the logical structure of a tautology or not. One of them is called the method of natural deduction or, in other words, the propositional method. The other method is the resolution principle[6]. Both methods play, in a practical sense, a very important role in management and decision-making processes. That is to say, they enable the effective demonstration of whether given conditions imply a given conclusion, i.e., form a law of logic[7]. This is due to the fact that a decision is a conclusion that should constitute the logical consequence of its causes and have a logical explanation for its undertaking. Therefore, at this point, the role and importance for science of the concept of a scheme should be emphasized. The name scheme (invariant) occurs not only in logic, but is also used by such sciences as psychology, linguistics, biology and physics. This concept is an important scientific challenge for knowledge and decision processes in their cognitive and ontological aspects. In the dimension of logic, a scheme corresponds fully to a law (or theorem) of logic. It also reflects, abstractly, the basic patterns of human thinking[8]. Therefore, the authors focus on disciplines of knowledge that, in

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[6]These methods are not the only used in logic, but they have an important practical dimension and reflect, in the dimension of logic, the nature of processes occurring in the human environment (especially their components) very well. The method operates on predicates purely on the basis of tautologies, i.e., on the basis of the laws of logic and its axioms [10], see also [8].

[7]This also often has an effectiveness dimension [3].

[8]This issue is analysed in ([8], p. 35–58), where the role of axiomatisation in relation to decision-making and obtaining knowledge has been specifically highlighted. It is also associated with the concept of reductionism proclaimed in science ([32], p. 573–574).

the dimension of content, make up the so-called circle of the sciences. At this point, it may be further asked: what does the concept of a cognitive scheme bring to decision-making processes?

In psychology, a cognitive scheme consists of various patterns and rules of behaviour. However, the basic role of a scheme, in the cognitive sense, lies in the fact that it would be possible to transfer it from one psychological situation to others ([14], p. 77, and [22, 29]). In biology, the concept of a scheme refers to studying the organisation of organisms and species. A pattern or scheme is defined primarily in the sense of an invariant. This lies in the fact that although a system as a whole is subject to change (in a mathematical sense, disturbances in relation to the steady state), some of its properties – its elements – remain constant ([1], p. 111, [31], p. 43–44). In addition, the role of a scheme in biology is understood just as in psychology. That is to say that when nature can copy a given scheme, then it is eventually transferred to another process in a biological sense. However, when we are dealing with a novel scheme, that is to say a new biological situation (quality), then a new pattern is created – an invariant [7]. This new situation has a certain feature in common to both of these fields of knowledge analysed from the position of a pattern or scheme – namely it serves to regulate forms, e.g., in a cognitive sense – the psychological equilibrium of the subject with the environment. In the case of biological regulation, the goal is to preserve the life of an organism, because if biological regulation is impossible, then an organism would die ([1], p. 8).

In relation to physics, schemes are understood rather as the laws of physics ([1], p. 187), i.e., the laws for which science seeks invariants.

In mathematics, in turn, a special form of pattern is presented by the concept of an algorithm[9]. In our context, an algorithm plays a central role in the concept of a scheme, because only the mathematical definition of a scheme, i.e., the concept of an algorithm, defines an order, that is to say a hierarchy of the rules used in a given pattern or scheme.

2.2. Place and role of algorithms in activities

So far, a few scientific disciplines have been taken into consideration in the analysis of the role of an invariant when making decisions. Actually, the concept of an algorithm operates within each of these areas of knowledge. However, in practice, this concept is ambiguous, and thus different ways of using it can be seen in practice. This raises the following question: what does an algorithm really represent? Let us consider the following example of creating an algorithm defining the behaviour of a system:

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[9]The word algorithm derives from the old English word algorism, meaning to perform actions with Arabic numerals (cf. [20]).

Example 1. In theory, an infinite sequence consisting of zeros and ones should be analysed, but in reality only a part of the sequence is given in the form:

0	1	0	1	0	1	0	1	0	1	0	1	...

Based on the above representation of the sequence, its general properties should be deduced. That is to say that an algorithm describing the functioning of the sequence should be elaborated. It is easy to note that in any part of this sequence zeroes alternate with ones and the sequence begins with the 0 symbol. From a logical analysis of these elements, one can see that we could easily indicate which symbol is located, for example, in the 16th, 97th or the 3756th place of the analysed system. It is enough to determine whether the number of the position is even or odd, as the ones are located in the even positions, and the zeros, in turn, are in the odd positions. The structure governing the appearance of ones and zeros in this sequence can therefore be given in a strict and unambiguous way.

In common language, set procedural rules are called algorithms. Mathematics and other sciences operate with algorithms of various forms of essence, i.e., procedures that relate to different areas of knowledge and life. Often, therefore, they are defined (understood) in various ways. Many types of algorithms such as logical (e.g., for finding one's way out of a labyrinth), numerical (computational), genetic, and many others have been developed [12, 26]. Generally, an algorithm of any essence should fulfil the following properties:

1. Be a strictly defined procedure, calculation etc., which means that the order in which operations are applied should be clearly defined and as a result any degree of latitude is eliminated.

2. Be generally applicable to a class of problems. An algorithm must be applicable to a complete set of variants, i.e., cannot refer to individual, though sometimes complex, cases.

3. Oriented towards obtaining results. The application of an algorithm must involve a finite number of operations from which an unambiguous result should be obtained.

Classical propositional calculus, for example, operates using such an algorithm to determine the tautological properties of its expressions ([10], p. 79–80). However, as noted above, the concept of an algorithm is understood in various ways in science. Therefore, a Russian mathematician Markov [27] formulated a definition of this term in a strict mathematical sense. According to Markov, an algorithm is understood as a finite set of defined elements, known as an alphabet, together with a set of substitution rules (for replacing symbols) for converting a word, a sequence of symbols from the predetermined set, on the basis of this set of generation rules. These rules do not have to be known (the case of a so called black box), only the transformation of a given word. In each step of the procedure, only one rule is used until all the possibilities of converting a given word are exhausted.

Example 2. The alphabet *A*, the set of substitution rules *P* and the word *x* formed from the elements of the alphabet *A* are given below. The word *x* will be converted into the word x' on the basis of the set *P*. Let:

$$A = \{1, +\}$$

$$P: \begin{cases} 1+ & \rightarrow & +1 \\ +1 & \rightarrow & 1 \\ 1 & \rightarrow & 1 \end{cases}$$

and let the initial word have the form: $x = 1111+11+111$. Realisation of Markov's algorithm, according to the adopted set of rules *P*, transforms the word *x* after a finite number of steps into the form[10] $x' = 111111111$.

1	1	1	1	+	1	1	+	1	1	1
1	1	1	+	1	1	1	+	1	1	1
1	1	1	+	1	1	1	+	1	1	1
1	+	1	1	1	1	1	+	1	1	1
+	1	1	1	1	1	1	+	1	1	1
+	1	1	1	1	1	+	1	1	1	1
+	1	1	1	1	+	1	1	1	1	1
+	1	1	1	+	1	1	1	1	1	1
+	1	1	+	1	1	1	1	1	1	1
+	1	+	1	1	1	1	1	1	1	1
+	+	1	1	1	1	1	1	1	1	1
+	1	1	1	1	1	1	1	1	1	
1	1	1	1	1	1	1	1	1		
1	1	1	1	1	1	1	1	1		

[10]This can be easily verified by using the set of rules *P* step-by-step in the order of their occurrence, i.e., in a given step where one transformation can be made, apply that one, but if two or more can be used, then apply the first one.

Consequently, the result of this algorithm is a sequence of ones whose length is the number of ones contained in the initial word *x*.

2.3. The concept of a structure in science

At this point, the question may be asked as to whether the presented concepts: invariant, pattern and algorithm, require the adoption of something which is their common feature. This property exists and occurs under the name: structure. The concept of a structure has a strict mathematical definition[11]. In the process of the analysis associated with decision-making, an explanation of the genesis and the content of a structure plays a more important role than a presentation of the mathematical concept of a structure.

According to Piaget, one of the most outstanding scientists in the field of psychology in the twentieth century, structure lies in the so-called concept of an operation of the intellect. For Piaget, a unit of logical thinking is an operation – the basic unit of human action. An operation cannot be considered in isolation because it is always an element of a certain system. As a result of such an understanding of an operation, Piaget proposes the need to create a specific logic of the whole which would be able to express operations in its own language. Based on this, a structure is a system of transformations, which has its own laws. It is neither innate nor is it an arrangement of any permanent features. This idea is based on the results of long-term empirical experiments associated with the mental and intellectual development of children ([34], p. 33, [35], p. 9). A structure exhibits three fundamental characteristics: it forms a whole, constitutes a transformation system and is self-controllable (transformations are the result of specific regulatory processes)[12]. Each real structure has its own genesis (the cause of its emergence) and vice versa, any genesis derives from a certain structure. This is the so called way of structure formation ([33], p. 146–149). One of these structures in human intellectual development, which is dynamic, can be interpreted as the invariant that in the mathematical sense is an element of the group with the name (symbols) INRC [36]. The mathematician H. Poincaré, who explained the formation of displacements in geometry, dealt with such a mathematical structure earlier than Piaget ([36], p. 198).

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[11]More broadly described in, e.g., [2, 43].

[12]This approach to the understanding of structure is similar to W. Ross Ashby's view on understanding structure from the point of view of diversity, which also states that only diversity can destroy (change) a given variety. Every law of nature is a restriction of diversity and a world without limits on diversity would be completely chaotic. For example, continuity is a common, but very strong, constraint on diversity ([1], p. 190).

Group theory is an important branch of mathematics [23] and, in addition, is a field that has developed its own axiomatics. Hence, it finds numerous applications in mathematics itself, psychology, physics and chemistry, as well as in other areas of science. Accordingly, there is a need to introduce a formal definition of the group theory[13].

The concept of structure was also a subject of interest for another great philosopher, logician and mathematician, Bertrand Russell. This scientist stated, on the topic of structure, that a lot of speculative arguments could be avoided, if only one becomes well aware of the importance of structure and difficulties in grasping the essence of this concept. It is often said that we perceive phenomena subjectively, but their causes lie in things in themselves and moreover, these things must have differences corresponding to the differences between the phenomena causing them. However, if this hypothesis were true, then subjective counterparts would create a world having exactly the same structure as the world of phenomena. This world would allow us to conclude from phenomena about the truth of all sentences that could be expressed in terms detached from them, but we know are true in reference to phenomena. In brief, any sentence that has meaning expressible by a language statement must be either true in reference to both worlds, or is not true in either of them [37]. This problem is quite extensively analysed in relation to the importance of structure in human communication ([21], p. 120–121).

These two approaches to the concept of structure are presented by two great scientists of seemingly different interests and with two different languages for expressing thoughts about the structure of objects. Now the question is whether their approach to understanding the concept of the structure of objects is identical or different. Bearing in mind that William Ross Ashby also assumed that a structure is something which has the same diversity in different situations, so is, within certain limits, invariant. Therefore, different language is used by various scientists in relation to this scientific concept. The authors do not intend to resolve here the problem of which language to use, because this is not the issue. Decisions are the central dimension of the analysis being performed. It seems that each of these three scientists had in mind the same thing – structure is represented by the variation (dynamics) based on the relationships occurring between elements (systems) in relation to time.

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[13]Definition. A group is a structure consisting of a set of elements G together with a two-argument operation $\circ$ that satisfies the following conditions:

1. $\forall_{a,b,c\in G}\,(a\circ b)\circ c = a\circ(b\circ c)$, i.e., the operation is associative.
2. $\exists_{e\in G}\forall_{a\in G}\, e\circ a = a\circ e = a$, where e is the identity element of the operation,
3. $\forall_{a\in G}\exists_{b\in G}\, a\circ b = b\circ a = e$, where b is the inverse element of the element a and e is the neutral element. The inverse element is also often denoted by a^{-1} or a depending one on whether it is a multiplicative or additive group.

Moreover, if for all $a, b\in G$ $a\circ b = b\circ a$, then such a structure is called an Abelian group. The above conditions constitute the axioms of the group theory.

So far, there has been talk of patterns, invariants, the laws of logic and schemes. Further, the role of logical schemas in relation to decision-making should be described, and in a more utilitarian than theoretical form. In logic, logical laws are presented in the form of formulas called logical schemas.

Example 3. The law of the conjunction of implications, given in the following analytical form:

$$\left[(p \to q) \wedge (r \to s)\right] \to \left[(p \wedge r) \to (q \wedge s)\right]$$

can be presented as the following logical construction:

$$\begin{array}{c} p \to q \\ r \to s \\ \hline (p \wedge r) \to (q \wedge s) \end{array}$$

Substituting meaningful sentences for the logical variables *p*, *q*, *r* and *s*, on the basis of this logical scheme, multiple real events occurring around us in time may be represented. Since such schemes replace the laws of logic, the logical operation of substitution can be used in relation to these schemes. In this way, a variety of so called secondary schemes can be obtained. In these schemes, each logical variable is replaced by any logical expression (appropriately created), which gives great opportunities for expressing knowledge in a strict logical form (especially knowledge concerning simple facts).

In this place there is special need to accent the logical relation between premises (evidence) and their conclusions. In fact, one set of premises may generate many different conclusions. Therefore, for the set of above premises, i.e., $\left[(p \to q) \wedge (r \to s)\right]$, we can find other logical conclusions that together with their premises create a law of logic. One of these can be derived in the following way:

$$\begin{array}{c} p \to q \\ r \to s \\ \hline (p \vee r) \to (q \vee s) \end{array}$$

There will be a discussion about this scheme in the part of this text about the logical structures of games. The latter scheme is called the law of the constructive dilemma. This law is used to make arguments regarding many different real processes, for example, connected with the weather: (if (it is cold – *p*), then (you need to dress warmly – *q*) and if (it is raining – *r*), then (you need an umbrella – *s*) so, if (it is cold – *p* or raining – *r*), then (you need warm clothes – *q* or an umbrella – *s*).

3. Analysis of the main dimensions of knowledge

Schemes, structures and other objects which are not only very important for science but also in practice for management, are defined on the basis of the concept of sets and therefore special attention is focused on this dimension of science.

3.1. The role of concepts in scientific research (The understanding of processes)

Talking about the results of studies and praxeological research, which are important to science, generally comes down to terms like knowledge and invariants, as well as discussing broadly the role of schemes in decision-making. One should refer to the problem of classifying the concepts used by science[14]. One may ask, why is there a need to do it?

Human knowledge depends largely on the method of classifying and defining concepts, and then understanding them in science and everyday life. In practice, this impacts our way of taking decisions. The concepts of: definition and the nature of language constitute the essential basis for scientific terminology. Here, the need to define precisely meets the arbitrariness of language/symbols, i.e., the relation between convention (form) and the nature (content) of a symbol. Therefore, the problem of defining concepts and classifying them symbolically appears only in outline in this article. Generally, it is still an unexplored dimension of methodology, logic and philosophy. There are already existing definitions of the division of scientific concepts into categories, which are even very accurate from the point of view of scientific studies. However, these classifications still do not solve the problem of what the definitions of the classes, in themselves, should be [32].

If everything in human life were based solely on science, very few real-life problems could be properly solved and analysed. For this reason, practically every field of science considers problems which are very important in everyday life and those that are of secondary importance. The reasoning used is applicable at the level of the concepts relevant to a given field of knowledge. In respect to knowledge analysed from the point of view of methodology and logic (general philosophy), the issue of concepts can be essentially expressed in two dimensions. One is constituted by the concepts necessary for calculations (accounting), including complicated ones. The second category includes the important relational connections which form between various important concepts

[14]Analysing the three forms of structure presented here through various concepts, one may raise the question as to whether the understanding of structure by these three great scholars is, mathematically speaking, isomorphic in content?

and even fields of knowledge[15]. With regard to the system of two-valued logic (visible on the plane of axiomatic systems), each theorem of logic (except axioms) is directly associated with a number of other theorems. In other words, theorems can be derived from other theorems by logical reasoning – deduction, because they are linked together through logical relations. Philosophically speaking, for the pragmatics of knowledge, this means that a man basically uses number (quantity) and language (description), i.e., the quantitative and qualitative dimensions – measure and content. This first concept has been mathematically formalized [9].

Nowadays, the aim is to ensure that everything can be measured[16]. The key question is whether content can be measured, and if so, what are the appropriate measures? One may also ask: what do the concepts of metrics and dimension bring to the achievements of science? The definitions of these categories have allowed a new approach to and improved understanding of many processes taking place in the real world. Returning to number and language, language is unmeasurable in a quantitative sense. Such important fields of knowledge as linguistics, psychology, sociology and axiology deal with language. The following question arises: which of these two dimensions is more important in relation to problems, fields of science and concepts in the area of obtaining knowledge? On the other hand, it is possible to doubt the validity of such a question, because both dimensions are components of knowledge and of the human mind and only together can they form a unity of cognition[17]. Therefore, a conceptual system of science is presented here from two perspectives: the measurable dimension and a second dimension which may have other metrics, because, in the end, everything must be understood in some way. There have been attempts to define quantitative measures for concepts that do not belong to the measurable class. Perhaps we should consider creating approaches in the form of quality measures corresponding to the nature of this conceptual category, which cannot be represented by quantitative numerical values. Currently, natural language represents (measures) the substantial dimension of concepts, terms and immeasurable objects in the best way. On the other hand, natural language is subject to mathematical measures (formal grammars, the topological formulation of language, or its statistical analysis). At the same time, such an analysis refers naturally to the field which describes this natural language, i.e., linguistics.

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[15]The relations between mathematical fields in the dimension of metalogic (metamathematics) are an example of this ([10], p. 490–492).

[16]There have been philosophers like Thomas Hobbes, who proclaimed that Man, despite the fact that he was the most important subject in the world, is only worth so much as can be paid for him.

[17]The anatomical construction of the human brain is asymmetrical, one hemisphere is more responsible for discrete processes – measurable (algebraic) objects and the other for substantial – semantical, and therefore continuous (topological) objects. However, a man has only one brain. How our brain transfers measurability into immeasurability, discreteness into continuity, or number into language, is still an open issue for science [15].

One dimension is associated with premises (causes), and the other with conclusions (the consequences of causes). These two aspects can be considered as a game and represented by a logical structure, which includes the decisions taken and their risk, i.e., the consequences of decisions.

3.2. Human activities in the terms of games

Introduction to Cybernetics [1], written by the already quoted William Ross Ashby, is a modern classic, which has played a very important role in the popularization of cybernetics, a scientific discipline created by Norbert Wiener [45], which allows the transfer of knowledge from one science to another. The author expresses thoughts that bind cybernetics (now also called system theory) to game theory [28]. Namely, he states that the game theory will play a central role in the development of cybernetics, and thus of science ([1], p. 335–338). It could be said that all decisions are the result of a game with at least two players. The authors note that logic and, in particular, the normal form of any logical expression (propositional calculus) plays an important role in respect to this problem[18]. One of man's procedures in obtaining knowledge about the reality around him is constituted by the concept of a game. Why is this concept important? The answer is simple. Man has always conducted games with Nature. The whole process of evolution constitutes some form of game. Game theory, as a branch of science, considers games in various analytical forms[19]. There are always at least two sides in a game. The following example concerns only two-person zero-sum games with players G1 and G2, because there are clearly defined analytical methods of solving them. Both players have a number of strategies at their disposal. Even if one of the players is Nature, then a clear and simple logical picture of such a game can be presented in normal form. In addition, there are more games played with nature than have been created by human civilization. In order to maintain the transparency of the presentation of analytical games and the possibility of their graphical representation, our considerations are limited to games of type 2×2. So let such a game be represented by its payoff matrix

$$\mathbf{M} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

[18]In mathematics, even games with a continuum of players are considered ([30], p. 68).

[19]Game theory brings many important theoretical and practical benefits to science. Statistical games are one of the important dimensions of this theory and they should be included in the group of games against Nature. In our further considerations, attention will be focused on the role of statistics in acquiring knowledge. The importance of statistics as an important partner that enables us to recognize the behaviour of the reality surrounding man will be highlighted.

It is assumed that G_1 wishes to maximise the payoff and G_2 aims to minimise it. Both players can choose between two strategies. If the first player's choice of strategy is represented by the logical variables p_1, p_2, and the second player's – accordingly by the variables q_1, q_2, then strategy selection from the point of view of logic is presented in Table 1.

Table 1. The logical form of the game

G_1/G_2	q_1	q_2
p_1	a_{11}	a_{12}
p_2	a_{21}	a_{22}

Source: authors' work.

The logical expression representing the above game is the following:

$$\alpha = \{[p_1 \wedge (q_1 \vee q_2)] \vee [p_2 \wedge (q_1 \vee q_2)]\}$$

Using the appropriate laws of logic, it can be easily demonstrated that the above expression is equivalent to the one presented below:

$$\beta = [(p_1 \vee p_2) \wedge (q_1 \vee q_2)]$$

or

$$\gamma = [(p_1 \wedge q_1) \vee (p_1 \wedge q_2) \vee (p_2 \wedge q_1) \vee (p_2 \wedge q_2)]$$

The expression β represents the logical structure of this game, and this is the normal conjunctive – alternative form. However, this is not a tautology[20], which is quite a normal thing, because the players only play the game from time to time. Only when the game is played, does this expression necessarily take the value of 1 (one), because both players must implement one of their strategies, i.e., one of the p_i and one of the q_j ($i = 1, 2$; $j = 1, 2$) must be equal to 1 (true). Also, it is worth indicating here that the excluding alternative realizes the logical expression β for each player individually. Formally, if the normal form of a logical expression is given, then it is also easy to specify its logical value, but during the game this expression must take the value of one. Thus, not all of the possible combinations of the values of these variables have clearly defined

[20]It is important to say in this place that it is possible to present any game in logical form as a tautology, which is performed in clear form at the end of this part of the text.

logical sense[21]. It makes sense to talk about the form of this analytical game only when the payoff matrix **M** represents a game with mixed strategies, because the parties may vary their approach to the game. It is assumed that G_1 chooses his strategy 2 with probability *x* and G_2 chooses his strategy 2 with probability *y*. Hence, one can interpret such a game as one in which players choose their strategies, represented by *x* and *y*, from an uncountable set. There is an additional element at stake here, of a psychological nature[22]. The expected payoff for this game is:

$$U(x,y) = \begin{bmatrix} 1-x & x \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} 1-y \\ y \end{bmatrix}$$

After carrying out the above matrix multiplication, we obtain the analytical form of the game, i.e., a payoff function of the form:

$$U(x,y) = (1-x)(1-y)a_{11} + (1-x)ya_{12} + x(1-y)a_{21} + xya_{22} \tag{1}$$

Denoting the coefficients of the variables *x* and *y* and the product *xy* by *b*, *c* and *d*, respectively, and substituting the symbol *a* for the constant a_{11}, the analytical form of the payoff function of the game is of the form:

$$U(x,y) = a + bx + cy + dxy \tag{2}$$

Finding an equilibrium point is also a simple mathematical operation. It is enough to calculate the partial derivatives of the function (2) with respect to the variables *x* and *y*. They are given by:

$$\frac{\partial U}{\partial x} = b + dy \text{ and } \frac{\partial U}{\partial y} = c + dx$$

It is possible that these derivatives always have the same sign for values of *x* and *y* in the interval [0, 1]. G_1 should choose his strategy 2 if the partial derivative with respect to *x* is always positive and his strategy 1 if the corresponding partial derivative is always negative. On the other hand, G_2 should choose strategy 1 if the partial derivative with respect to *y* is always positive and strategy 2 if the corresponding partial derivative is always negative, since he is trying to minimise rather than maximise. Otherwise, setting these derivatives equal to zero, we obtain the values of *x* and *y*, corresponding to the

[21]This problem is described in [8, 16].

[22]There are scientists who think that decision-making is not always based on the concept of probability, but rather on certain patterns [17–19].

equilibrium point of the game. The set of possible decisions can be graphically illustrated by taking a specific game. Suppose the game is of the following form:

$$\begin{bmatrix} 4 & 10 \\ 8 & 2 \end{bmatrix}$$

The payoff function of this game, interpreted as a game with a continuum of strategies, is:

$$U(x,y) = 4 + 4x + 6y - 12xy$$

for which $(x, y) \in [0,1] \times [0,1]$, and the corresponding payoff surface is shown in Fig. 1[23].

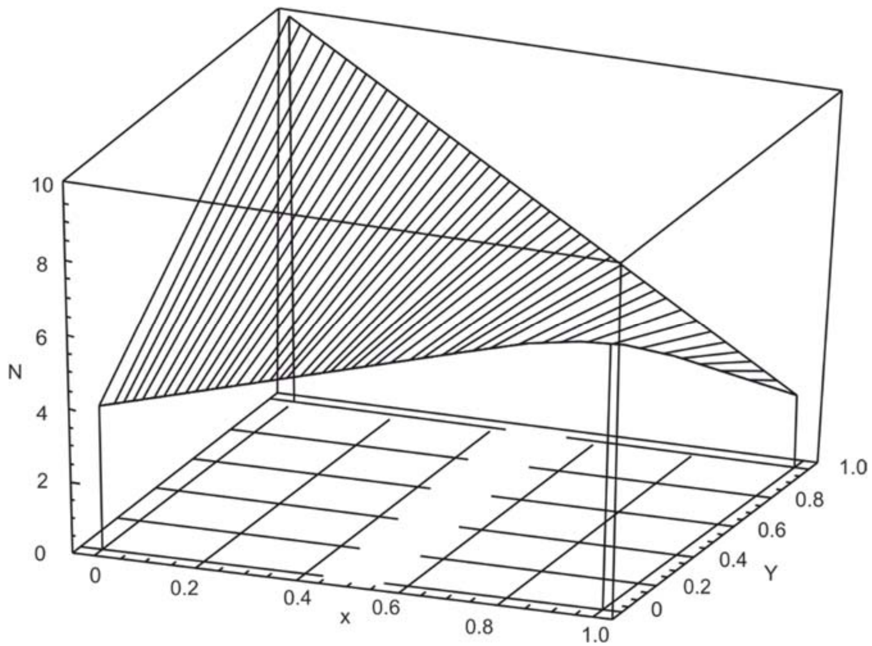

Fig. 1. The payoff surface of the game in analytical form
$U(x, y)= 4+ 4x + 6y – 12xy$. Source: authors' work

The analytical form of the function $U(x, y)$ is logically represented by the expression γ. The values of this function give the expected payoffs of the players given the

[23]The equilibrium point of the game is given by the point: $(x, y) = (1/2, 1/3)$, and the value of the game is: $U(x, y) = U(1/2, 1/3) = U(x, 1/3) = U(1/2, y) = 6$.

mixed strategies they play (their choices of x and y). In such a game, each player can choose from a continuum of strategies, and there is a unique equilibrium.

This example of a simple game clearly confirms the role of game theory in decision-making as mentioned at the outset of these considerations. In scientific papers devoted to making decisions, these considerations apply basically to the dimension: a good decision, a misguided decision. This example shows that such thinking is not entirely justified. By selecting combinations of logical variables, one of the possible payoffs is obtained, and by implementing mixed strategies, the expected payoff $U(x, y)$ is obtained. There is a unique equilibrium point, but this does not mean that decisions not corresponding to this point are wrong. They are simply less favourable against another player who is acting in his own interest.

We can ask at this moment, does Nature in every situation realize its optimal solution for a given problem? There are many examples which indicate that the answer is no (for example many natural catastrophes are connected with the weather). They are not optimal, but bounded. What does this mean generally? The following is a question of fundamental importance. Maybe Nature is unable to achieve optimal solutions with regards to certain processes?

Game theory is considered here, because man obtains substantial scientific knowledge about the reality surrounding him and knowledge about himself from studying Nature. In general, it can be said that man constantly plays some sort of game with Nature, and this results in payoffs in the form of knowledge. Symbolically, this can be interpreted as a zero-sum game, because what one player gains the other player loses (man learns about reality, and so its mysteriousness, or analogously diversity, complexity, uncertainty, etc., diminishes). But zero-sum games are a very small proportion of all the games played between man and nature. Therefore, the fact that the payoffs from a game do not necessarily have a numerical value must also be strongly emphasized here[24]. Moreover, any single game is just one of the manifestations of human behaviour.

[24]For example, estimators for examined random processes of Nature, obtained using mathematical statistics, are not numerical values, but random variables from a specific distribution, often with given parameters. In addition, there is a second dimension of payoffs, far more important than statistical payoffs – increased understanding of life. It is sufficient to invoke examples regarding the value systems professed today by various civilizations with respect to their mutual relationships (e.g., crucial differences between the values professed by the world of some religious groups and the world of Christians – the recent events in the Middle East presented in the media and concerning the so-called Muslim state are an example). For a long time, axiology has also dealt with the problem of values. Problems related to axiology are almost as old as philosophy itself. In modern times, the concept of values has been used, among others, by Kant. The term axiology was introduced by P. Lapie in *Logique de la volonté* (1902) and Eduard von Hartmann in *Grundriss der Axiologie* (1908). In Poland, the following authors were specifically interested in the subject of axiology: Florian Znaniecki, *The issue of values in philosophy* (1910), Władysław Tatarkiewicz, *About the ruthlessness of the good* (1919) and Roman Ingarden, *Experience, work, values* (1966).

In his book, Ashby shows a high level of appreciation for game theory, giving a generalized model of games in the form of a diagram, which he interprets in biological and technological terms ([1], p. 335–338). In general, a man always aims to maintain a state of equilibrium with the environment, and understanding constitutes one of the ways of achieving equilibrium. In the cognitive sense, a state of equilibrium means man understanding the behaviour of his surroundings. Hence, this is a game played between man and nature. Games played against nature have their internal logic according to the payoff values. Therefore, at this point we should mention that, due to the connection between game theory and logic, tools from logic are very useful in the process of analysing zero-sum games. In particular, they can be sometimes used to reduce the rank of the payoff matrix (of the game considered). However, this approach is not general. This should be illustrated by an example. Consider the following game:

$$\begin{bmatrix} 3 & 2 & 0 & 3 \\ 1 & 4 & 5 & 2 \\ 0 & -1 & 4 & -1 \end{bmatrix}$$

It is easy to see that last strategy (α_3) of player one is dominated by his second strategy. Hence, we can remove it from our game. Thus the game can be reduced to:

$$\begin{bmatrix} 3 & 2 & 0 & 3 \\ 1 & 4 & 5 & 2 \end{bmatrix}$$

In a similar way, it can be seen that the fourth strategy of player 2 is now dominated by his first strategy and so the last column can be removed. Furthermore, it is known from matrix algebra that this last matrix has rank equal to two. Hence, it must be possible to eliminate another column. Which one? By using a graphical procedure, it is easy to show that the second (β_2) strategy of player G_2 must be rejected as being unprofitable for him (it is assumed that the reader knows this procedure). Finally, our game has been reduced to the form:

$$\begin{bmatrix} 3 & 0 \\ 1 & 5 \end{bmatrix}$$

The same result can be obtained from a procedure based on logic. If player G_1 uses his first strategy (α_1), then his adversary should use his third strategy (β_2). Hence, we are dealing with a logical implication. In place of strategies, we will use logical variables appropriate to the tools described at the beginning of this section:

$$p_1 \rightarrow q_3$$

In the second case, the situation is the following:

$$p_2 \to q_1$$

Player G2 does not use the strategies β_2 or β_4. Both analyses have given the same result, but the one based on logic is shorter[25]. Additionally, it is interesting to notice that the strategies which were dominated, are logically connected by the relation:

$$p_3 \to (q_2 \vee q_4)$$

Now there is a need to summarize our analysis of this game by formulating its logic structures as schemas corresponding firstly to its initial matrix and secondly to its final form. They are:

$$\frac{\begin{matrix} p_1 \to q_3 \\ p_2 \to q_1 \\ p_3 \to (q_2 \vee q_4) \end{matrix}}{(p_1 \vee p_2 \vee p_3) \to (q_1 \vee q_2 \vee q_3 \vee q_4)}$$

$$\frac{\begin{matrix} p_1 \to q_3 \\ p_2 \to q_1 \end{matrix}}{(p_1 \vee p_2) \to (q_1 \vee q_3)}.$$

Both of these structures follow the law of the constructive dilemma. Using the principle of resolution, it is easy to show formally that, for example, the last scheme constitutes a law of logic (tautology).

In order to interpret this approach, some may ask why should we make descriptions of games based on logic theory? It is interesting that such doubt also has a logical basis. Therefore, this question should receive a clear answer. The role of structure was presented in the section entitled The role of the concept of structure in science. Knowledge about the structure of an investigated process gives the possibility of transferring scientific tools from one discipline of science to other ones. Moreover, this is also close in meaning to the role of analogy, which is another important concept in science. What is the practical conclusion of our analysis? One of the answers is as follows: Compare the text on the structure of the game with the text about the weather. At first sight, there is

[25]The equilibrium of this game is: $x = (4/7, 3/7, 0)$, $y = (5/7, 0, 2/7, 0)$, $v = 15/7$.

no structural relationship between these objects. But the application of logic theory assigns them the same structure. This means, that there exists an analogy between these two situations. In conclusion, we can say that such analysis creates new scientific tools and a generally new approach to scientific investigations of the reality surrounding us.

4. The circle of science and information

4.1. Towards the creation of a uniform language of science

One problem regarding the classification of concepts can be also considered with respect to the connections (relations) occurring, not only within the scope of a single discipline of knowledge, but also between the fundamental branches of science, which include mathematics, physics, biology and psychology. Such an approach to the analysis of knowledge allows us to look at science as a human-made construct of a unified character (like a monolith), because the reality surrounding us, though diverse, is yet one. At this point, the following question should be answered: why are only the branches of science mentioned above taken into account in our analysis? The answer is almost simple. They came into being and developed from the emergence of ancient Greek philosophy. Furthermore, mathematics, physics, biology and psychology also constitute the core of science today.

Mathematics is an abstract science, physicists are interested in inanimate matter, the subject of interest in biology is the structure of the animate world, and psychology studies the nature of the world of the human mind. However, from the point of view of logic, such a system of sciences should not be considered as a line, but as a cycle, because on the basis of continuously gathered knowledge (epistemological, psychosociological, mathematical and of other types), the level of interaction between these sciences is continuously increasing. For example, mathematics and psychology are becoming closer and closer to each other, creating a kind of circle. This circle is understandable from the point of view of psychology. Psychologists try to explain why the development of human intelligence leads to establishing systems of reversible operations, which may be connected into arrangements of relations (structure). Such structure is a necessary form of physical and biological balance (reversible operations have this feature in the psychological sense, they lead from a certain arrangement to a state of balance). This matter should also have a reasonable explanation from the side of mathematics.

In this area, two differing approaches dominate with regard to the origins of mathematical operations. The supporters of one of them (including the mathematician H. Poincaré) think that mathematical operations have a natural intuitive basis, thanks to which they create a direct connection between psychology (its intuitive grounds) and the subject of mathematics. According to mathematicians and the members of various mathematical schools of logic (Russell, Hilbert, Ajdukiewicz and others) representing the second direction of views on the origins of mathematical operations, the problem actually concerns logical analysis itself. They reject a psychological approach and base the adopted axioms, relations and various other mathematical and logical operations (not explaining them essentially, that is, psychologically) on the ground of abstract – conventional relations (it is irrelevant to them where these mental operations come from). This is where the name of this school of thought, conventionalism[26], comes from. Historically, the philosophical views of the so called Vienna Circle fit into this school, especially after the emigration of many of its representatives to the US ([35], p. 144). Currently, the origins of the formal character of scientific opinions on the concept of operation lie in the relation between mathematics and psychology.

The relation between mathematics and biology should be analysed in a similar way and then the remaining arrangements of binary relations. Mathematics and biology complement each other in a very interesting way. Mathematics as a field of science is a deductive science and as it develops, appeals to experience to an ever lesser extent, and because of this it makes use of the activity of the subject to the maximum. Biology, on the other hand, restricts the activity of the subject to a minimal level, because basically it is an experimental science, and uses deduction only in a very simple form[27]. While

[26]In science, there also functions an indirect course, according to which it is assumed that certain operations such as human thinking and behaviour have genetic or inborn origins. Hence, thinking and behaviour derive from intelligence, or actually from genetically predetermined human thinking, or substantially ontological secondary operations. Thinking has no specific character, which can only be created in the process of acquiring human experience in relation to reality. None of the three main approaches to structure provides conclusive evidence, in the scientific sense, for its philosophy of thought, and therefore none of them constitute the dominant direction. However, whichever way we look at the problem of the origin of relations, the question of the nature of abstract relationships still remains to be solved. Are relations a reflection of eternal ideas, an expression of the plain language of convention, or are they an axiomatization of the intellectual operations of a cognitive entity? Regardless of whether the basic concepts of mathematics are directly related to the mental activities of a man, or whether this link is formed in an indirect way through the axiomatization of operations, then in both cases the link is formed between the field of thought prefigured in psychological research (in sociological terms – the sociology of language) and that of abstract objects of mathematics and logic. As stated above, both ends of this chain together aim to connect. This problem lies in the arrangement of the relation between a subject and an object, which takes place at each encounter, which was shown many years ago by Hoeffding. An object is perceived only through the subject thinking, and this subject perceives itself only by way of adapting itself to the object. Hence, the world is perceived by man through logic and mathematics, which are products of the human mind. However, men can only understand how logic and mathematics were established by examining himself from a psychological point of view, in other words, in his relation to the whole universe. In this statement lies the true meaning of the circle of science, namely the unity of science that shows mutual interdependencies between the various disciplines ([36], p. 121–126).

[27]In biology, as in many other sciences, the achievements of mathematics – statistics in particular, are used to develop the results of empirical research, but this is another dimension of the relation between both sciences.

mathematics aims to bring the object to the subject, biology aims to bring the subject to the object. Physics and psychology lie between these two symmetrical extremes. They both act in the direction of ideals, which predominates in mathematics, as well as in the direction of realism, a good example of which is biology. Physics applies mathematics to the description of reality and thanks to that it takes part in adapting the schemes of the human mind to reality. Moreover, physics cannot ever totally separate an object from abstract or specific operations which affect it in the process of cognition. Psychology, on the contrary, adapts realism from biology and the behaviour of organisms influences the process of explaining mental life. As a result of this, psychology tries to explain the basic operations of mathematics and physics by tracing the appropriate stages of mental development. To some extent, it avoids reductions to ideal objects, which dominates in pure mathematics. This means that the human psyche can in its thinking change the role of being a subject to being an object and vice versa, so that it changes position. The observer becomes the observed, which leads to many scientific discoveries [22, 29].

Summing up the analysis above, we can say that such a circle of science points to the close interdependence between subject and object. This formulation is not a new scientific discovery, but indicates that the language characterizing a particular scientific discipline varies in its level of abstraction or realism (in the colloquial sense)[28] depending on its location in the circle of science. Every field of knowledge, not only the four selected, uses both precise terms and ones of a fuzzy character in presenting its underlying concepts. Thus, once again a question of fundamental importance arises. What determines one's perception of reality, the language of the perceiver with thinking (structure), or the nature of the perceived object? This question is, in a sense, known in science, because it has been stated in the form: how perceivable is reality and, on the other hand, to what extent are language and cognition able to perceive the reality surrounding man. Therefore, the circle of science discussed above implies a need to create a new dimension of scientific language and a system of scientific concepts appropriate to its structure. Both systems should primarily convey a better combination of form with content, or in other words, syntax with semantics. This is a major challenge for the future of science and cognition in general. Although Aristotle divided the concept of an object into form and content, the first thinker to see the need to combine content with mathematical form was Descartes. Currently, it seems that the ideas of Aristotle and

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[28]Even Piaget himself asks the question as to which of these languages is true. His answer is: (...) *when biology becomes, if it comes to that, completely mathematicised, it will become clear whether the equation of protoplasm, and consequently the protoplasm itself, are the results of the activity of our mind, or does our mind and the equations that it creates come from the protoplasm. Perhaps, psychology is already sufficiently developed to demonstrate to mathematicians supporting the first thesis and biologists – supporters of the second thesis, that they are saying basically the same thing. However, psychologists would be the first to understand – why this is all happening in this way* ([34], p. 125–126).

Descartes with respect to representation in terms of the language of science, particularly in relation to Piaget's circle of science, have been achieved best in practical terms by Ashby. His book presents a conceptual and linguistic representation of the problems common to mathematics, physics, psychology and biology and hence a solid presentation of the basic circle of science [1]. This circle of science is presented graphically in Fig. 2.

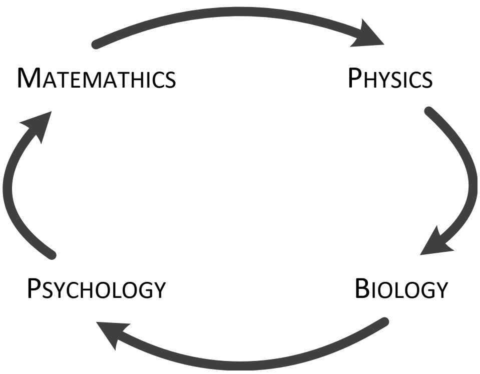

Fig. 2. The cyclic system (circle) of the basic branches of science.
Based on [35], p. 124

4.2. The information society

Above we dealt with the language of science, which plays a general role in the information dimension. In this chapter we look briefly at the dimension of information and the information society [46].

Decisions are made in different dimensions, and each dimension influences the effects of the decisions undertaken [46]. Currently, one of these dimensions is information technology (IT). Information technology enables people to integrate, so creating the information society. Such a society is characterized by the widespread use of IT and telecommunication technologies [39, 44]. These technologies influence the functioning of both organizations and individual citizens. The methods of communicating information to users applied in information technology have resulted in establishing an entirely new concept called cyberspace. Cyberspace has such a property that information placed there becomes available to all users, both theoretically and practically. This characteristic is very important for management.

Universal access to a global network provides an easy way of reaching out with information to large groups of people. In addition, it facilitates organizing people and the mutual provision of information[29]. It can be expected that the twenty-first century will be a period of creating a knowledge society, because information is knowledge when it is verified, i.e., shown that it corresponds to the facts contained in it (is true). However, for truth to be effective and have a usable value, not necessarily monetary, perhaps above all not monetary, verification must be carried out quickly. In terms of usability, the verification of knowledge is the pragmatic dimension of knowledge. IT is already being used to achieve this goal to a great extent. Knowledge is to be transferred to the broad masses in such a way as to be understood and quickly verifiable – confirmable. The fourth power, i.e., the media could play a great and important role in this process. However, there is a problem: what should the media convey to the public: information, knowledge, in short – the truth, or just the point of view of an information centre (organization)?

5. Summary

The problems raised in this article concern the issue of management and decisions. One significant problem was approached in a new light: the multiple dimensions that have been analysed so far in the contemporary literature, usually separately, are logically linked.

All of the concepts collected in this article, related to management, structure and decision, create a kind of a whole, making it possible to comprehensively manage the decision-making process. Knowledge acquired in the course of exploring the reality surrounding us and from the practice of science is the basis for selecting the right decisions. Knowledge eliminates or substantially reduces uncertainty from the decision-making process. Correctly made decisions lead to an increase in the quality of management at every stage of the process. However, for effective management, access to specific numerical or qualitative data is required.

It may seem that this article has considered decision-making only from the point of view of management at the level of production processes or economics. Such decisions are undoubtedly an important element of the behaviour of reality. However, the issue discussed is as broad as reality is diverse, and the most important factor in human behaviour is the aspiration to reach equilibrium with the environment. That is why it is believed that the issues discussed are all fundamental to decision-making. Structures, patterns and invariants play a special role in these areas.

[29]At the end of the first decade of the twenty-first century there was an attempt to reduce inter alia the freedom to use this network. At this point, the following question may be asked, who cares about such limitations, and why?

Therefore, at the end of our remarks about scientific research connected with general scientific language and the role that it plays in management and generally in science, there is a need to mention the fields of knowledge that were applied in our deliberations, such as scientific language, formal logic, general algebra, philosophy in science, the information society, creative information processing. All this testifies to the fact that the overall objective has been achieved.

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