O P E R A T I O N S   R E S E A R C H   A N D   D E C I S I O N S

No. 1 2019

DOI: 10.5277/ord190106

Bogdan RĘBIASZ[1]

THE VALUATION OF REAL OPTIONS IN A HYBRID ENVIRONMENT

The aim of this paper is to present the possibilities and purposefulness of the application of fuzzy set theory to the valuation of real options. Owing to temporal fluctuations in the market, some input parameters in a model of a real option cannot always be expressed in a precise sense. Therefore, it is natural to consider them as a fuzzy numbers. Such an approach allows us to keep more information about the possible value of real options. A hybrid (fuzzy-stochastic) model for valuing a switch option is presented. Under these assumptions, the value of a switch option will be a fuzzy random set. This article assesses the incremental benefit of product switch options in steel plant projects. Such options are valued by Monte Carlo simulation and modelling the prices of and demand for steel products using fuzzy geometric Brownian motion. Finally, the value of a product switch option is defined by the upper and lower probability distribution function

Keywords: *switch options, fuzzy sets, random fuzzy sets, investment decision, Monte Carlo simulation*

1. Introduction

Methods of valuing real options take into account the presence of a high level of uncertainty and managerial flexibility in the process of investment decisions. The approach of real options, as a strategic decision making tool, borrows ideas from financial options, because it explicitly accounts for the value of future flexibility. The analysis of real options is based on the assumption that there is an underlying source of uncertainty, such as the price of a commodity or the outcome of an investment project. Over time, the outcome of the underlying uncertainty is revealed, and managers can adjust their strategy accordingly.

[1]Faculty of Management, AGH University of Science and Technology, ul. Mickiewicza 30, 30-059 Kraków, Poland, e-mail address: brebiasz@zarz.agh.edu.pl

It is apparent that financial parameters (cash flow, profit, etc.) are affected by the precision of input data. The valuation procedure is usually carried out under the assumption of a deterministic or stochastic environment, but uncertainty (vagueness) is most commonly neglected [36].

Here one might quote Wu regarding the interest rate [27]. He states: *When the financial analyst tries to price an European option, the interest rate is sometimes assumed as a constant. However, the interest rate may have different values in the different commercial banks and financial institutions although the difference is so small. Therefore, the choice of a reasonable interest rate to price a European option may cause a dilemma. But one thing that can be sure is that the different interest rates may be around a fixed value within a short period of time. For instance, the interest rates may be around 5% in the different commercial banks and financial institutions. The phrase around 5% might have a problem to be modelled by using the probability theory. Therefore, the fuzzy sets theory plays an appropriate role to tackle this kind of fuzziness. In this case, the interest rate may be regarded as a fuzzy number* $\tilde{5}\%$ *when the financial analyst tries to price an European call option using the Black–Scholes formula*. It can be seen here that an appropriate description of reality in the pricing of options requires the use of fuzzy sets.

In models for valuing real options, the uncertainty of selected variables (market size, product prices) is usually modelled with the help of the Black–Scholes (B–S) formula [15, 18, 27, 28]. A fluctuating market and lack of detailed information mean that many parameters used to value a real option cannot always be described in a precise sense. For example, the volatility, a crucial parameter in the standard formula for valuing an option, is too abstract (and unstable) to be set correctly [31]. Very often, the imprecision we encounter when estimating future cash flows is not solely stochastic in nature. Therefore, most models for valuing real options involve uncertainty arising from a lack of knowledge or from inherent vagueness, apart from random uncertainty. The use of probability theory alone in such cases gives us a misleading level of precision and a notion that consequences are somehow replicable [7]. As Zadeh stated, it is difficult to measure impreciseness using the concept of probability, because probability is used to measure randomness. Randomness is relevant to the occurrence or non-occurrence of an event, while fuzziness is relevant to the degree of an event [4, 32].

Very often an analyst estimates parameters based on historical data and then corrects their value by subjectively predicting a change in the future economic environment [17, 18]. For example, the authors of the study [17] state*: A historical series analysis of the steel product prices (from January 2000 to April 2009) shows an average growth rate of 4.67% per year. Nevertheless, we use 2.5% as drift for steel product prices as it is assumed that a structural change has occurred in the sector after the end of the economic crisis which occurred in 2008/2009. Therefore, we assume that for the projected* *five years during which the cash flows will be calculated, a growth rate in price lower than that experienced during 2000–2009.*

In many sectors of the economy (e.g., engineering industry, mining industry, oil industry and metallurgical industry) we have a specific context for applying methods for valuing real options using fuzzy numbers. Investments in such industries normally have a duration of 10–12 years. It should be clear that the relevance of historic data diminishes very quickly after 2–3 years and that it is not worthwhile to claim that time series have any predictive value 5 years into the future [7]. Therefore, expert evaluations are necessary.

Demand for different products is very often interactive and dependent; that is, correlations between the market demands for different products are observable and measurable. Because the demand for any product is generally affected by demand for other products within the same product line, the demand for a single product cannot be determined individually in isolation [12]. Likewise, relationships between the prices of selected product lines can also be observed.

In models for valuing real options, these dependencies are usually not taken into account [17, 26, 33, 35]. Separate forecasting and estimation would likely increase the inaccuracy [12]. Such biased forecasts inevitably impact the precision of valuations of real options. A few models that include such correlations between parameters based on a probabilistic description of B–S models can be found [12, 18]. However, these methods fail to accurately capture or manage the components of random and fuzzy uncertainty in these parameters.

In conclusion, one can say that there exist many cases of valuations of real options where both types of uncertainty are present [4, 18]. An analyst usually depends on an expert's judgment to derive the level of uncertainty regarding correlated parameters in models of real options. Thus, an investor is forced to use both random and fuzzy elements as a basis to assess uncertainty. Experts' opinions or imprecise estimates should be introduced into the model in the form of fuzzy numbers.

This article proposes a new method of valuing real options which is better adapted to the real conditions of decision making. As stated above, an investor usually faces the problem of implicit fuzziness. Therefore, it is difficult to use the traditional probabilistic B–S model to define uncertainty in many practical problems involving the assessment of investment projects. Taking the above into consideration, this article presents a method which uses fuzzy numbers in the assessment of real options and interpretation of the results obtained. The B–S model is applied here with fuzzy parameters. Furthermore, a method is developed for taking into account the correlation between parameters of B–S models when the uncertainty of these parameters is described using probability theory and the theory of fuzzy numbers. In order to model the relationship between fuzzy parameters, it is proposed to use interval regression. The proposed method evaluates investment projects in a more realistic way. Since this method takes into consideration imprecision/vagueness and randomness (a hybrid environment), it is able to give investors a better understanding of a problem when analysing investment decisions.

This concept was verified on the basis of a switch option. A switch option refers to changes in the raw materials used, products manufactured and other production factors, or even an entire technological process. Such switches have the goal of adapting to changes in the market situation.

2. Related works

Fuzzy numbers are used in the valuation of financial options and real options. The literature on the use of fuzzy numbers to value financial options is more developed. Many works use the B–S model with fuzzy parameters for pricing financial options. Guerra et al. [10] consider the B–S model for option pricing, and present a sensitivity analysis based on a study of the option price when the parameters are assumed to be fuzzy numbers. Zdenek [33] proposed a generalized hybrid fuzzy-stochastic binomial model of an American real option using fuzzy numbers and a decomposition principle where the input data are in the form of fuzzy numbers. Zmeskal [36] applied B–S methodology for appraising the equity of a European call option by using input data in the form of fuzzy numbers. Appadoo and Thavaneswaran [2] derived the membership function of the price of a European call based on the B–S model with fuzzy volatility. They fuzzified the maturity value of the stock price using adaptive fuzzy numbers. The B–S model in a fuzzy environment was analysed by Wu [28] and further adapted in [27]. An application of the extension principle in fuzzy set theory to the B–S formula was proposed in that paper. The authors considered a fuzzy interest rate, fuzzy volatility and fuzzy stock price in a financial market. Under these assumptions, the prices of European call and put options will be fuzzy numbers, and the extension principle is invoked to generate the pricing boundaries of European call and put options [27, 28]. Lee et al. [15] adopted fuzzy decision theory and Bayes' rule as a basis for measuring fuzziness in the practice of analysing options. Their study also employed a fuzzy decision space consisting of four dimensions, i.e., a fuzzy state; fuzzy sample information, fuzzy action and evaluation function to describe an investment decision, which is used to derive a model for fuzzy B–S option pricing in an uncertain environment.

Another authors used a jump diffusion model with fuzzy parameters. Zhang et al. [35] derived a fuzzy pricing formula for a European option based on Kou's diffusion model for a double exponential jump. They also proposed a formula for crisp possibilistic pricing of a mean option by using the possibilistic mean value of a fuzzy number. Weidong et al. [26] discuss analytical solutions for a European option using a fuzzy normal model of jump-diffusion and possibility theory. Under the assumption that the risk-free rate, the volatility, and the average jump intensity are fuzzy numbers, Xu et al. [29] present a jump-diffusion approach to pricing options in fuzzy environments. They also provide a model of crisp possibilistic mean jump-diffusion to price vulnerable European call options.

The literature also discusses applications of fuzzy set theory to variability models. This problem was investigated by Thavaneswaran et al. [23] and Thiagarajah et al. [24]. The purpose of this research is to introduce a class of models of coefficient volatility based on fuzzy theory and probability theory. Fuzzy option values and the superiority of fuzzy forecasts over minimum mean-square forecasts are discussed in some detail. Furthermore, the authors use fuzzy set theory to price binary options. Specifically, they study binary options by fuzzifying the maturity value of the stock price using trapezoidal, parabolic and adaptive fuzzy numbers.

Several papers describe the possibility of using fuzzy numbers to value real options. Carlsson and Fuller [7] and Carlsson et al. [8] use possibility theory to study fuzzy valuations of real options. The authors define the present values of expected cash flows and expected costs of investment by trapezoidal fuzzy numbers. Carlsson and Fuller [7] determined the optimal exercise time with the aid of the possibilistic mean value and variance of fuzzy numbers. Carlsson et al. [8] also developed a methodology for valuing options on R&D projects. In particular, they presented a fuzzy mixed integer programming model for the problem of optimal R&D portfolio selection. The authors point out that in R&D and large infrastructure projects fuzzy numbers are an appropriate way to express uncertainty. Garcia [9] used a model for the fuzzy valuation of real options in a real investment project from the energy sector. In Garcia's paper, the model suggested by Carlsson and Fuller [7] was applied to a timing decision and selection of a power station for various investment alternatives. Similarly, Zeng et al. [34] analysed the uncertainty involved when investing in the power grid and discussed how to make an investment decision when the cost of an investment and cash flow are both fuzzy numbers. Furthermore, they compared the application of the classical method of net present value with real options for evaluating investments. In the above works, the authors assume that the present values of expected cash flows and expected costs of investment are fuzzy values. However, they do not indicate how to designate them.

Another idea for analysing real options using fuzzy numbers was proposed by Kahraman and Ucal [14]. They used the approach of certainty equivalence to value real options in the oil sector with fuzzified data.

Allenotor and Thulasiram [1] used a fuzzy trinomial model of real options for pricing grid resources and proved the feasibility of the model through experiments. An important topic of this research is the modeling of uncertainty in the area of quality of service using fuzzy logic. Tao et al. [22] developed a comprehensive methodology based on fuzzy risk analysis and the approach of real options to evaluate investments in information technologies in a nuclear power station. By linking the variability of expected payoffs to specific sources of risk factors, this method could help decision makers to achieve more reliable valuations of investments affected by multiple sources of risk. Shiu and Shu [21] propose a fuzzy binomial model of options pricing for valuing an investment project in uncertain environments. The proposed approach reveals the value of the flexibility embedded within a project and defines a method to compute the mean value of a project's fuzzy expanded *NPV* that represents the entire value of project.

It should be noted, however, that none of these studies deal with the problem of a hybrid environment (a fuzzy-stochastic environment) for pricing real options. As stated above, decision-makers often have to take both types of uncertainty into account in the investment process. Furthermore, in the case of a hybrid environment, there exists the problem of correlating the variables described by B–S models. The prices of individual assortments of products and raw materials are strongly correlated. Similarly, the sales volume of particular ranges of product manufactured by an enterprise are usually correlated.

3. Model and methodology

This section introduces the model and methodology for valuating the option of a product switch in a hybrid environment. Firstly, the modelling of uncertainty using correlated fuzzy geometric Brownian motion (GBM) is described. Next, a model for estimating the value of a product switch option in a hybrid environment is presented.

3.1. Modelling uncertainty by correlated fuzzy geometric Brownian motion (GBM)

GBM is a special case of Brownian motion or a Wiener process. The variable q follows GBM if it satisfies the following diffusion equation [3, 16, 17, 25]:

$$dq_t = \mu q_t dt + \sigma q_t dw_t \tag{1}$$

where $dw_t = \varepsilon_t \sqrt{dt}$ is the standard increment of a Wiener process, and μ and σ are the drift parameter and standard deviation parameter, respectively. The deviations, ε_t, are independent realizations from the standard normal distribution.

Let Δ be the interval of time between two successive observations. Based on Eq. (1), we obtain the following formula for predicting q_t [3, 16, 17, 25]:

$$q_t = q_{t-1} \exp\left[\left(\mu - \frac{\sigma^2}{2}\right)\Delta + \varepsilon_t \sigma \sqrt{\Delta}\right] \tag{2}$$

Very often the statistical data available are not sufficient or appropriate for estimating μ and σ. In this case, they are estimated by experts or, as stated above, these parameters are first estimated on the basis of these statistical data and next corrected by an expert. In this case, μ and σ are imprecise quantities and can be described by fuzzy numbers. Thus, if the statistical data available are not sufficient to define a stochastic process, one has to define a fuzzy-stochastic process. In such a process, q_t will be described by a fuzzy random variable. The appearance of a fuzzy random variable makes the combination of randomness and fuzziness more persuasive, since probability theory can be used to model randomness, and the theory of fuzzy sets can be used to model imprecision. This gives us a more realistic description of the actual decision-making environment.

Taking into account uncertainty about the values of μ and σ, Eq. (2) can be written as follows:

$$q_t = q_{t-1} \exp\left[\left(\tilde{\mu} - \frac{\tilde{\sigma}^2}{2}\right)\Delta + \varepsilon_t \tilde{\sigma}\sqrt{\Delta}\right] \tag{3}$$

where $\tilde{\mu}, \tilde{\sigma}$ are fuzzy numbers characterizing μ and σ, respectively.

It should be taken into account that the prices of many products and of raw materials, as well as the market volume of selected products, can be modelled in this way. The prices of various products and raw materials are usually correlated. The volume of the markets for individual products is also correlated. To incorporate these correlations into the model, let us suppose that we are analysing I primary variables, which will be predicted by Eq. (3). Additionally, it is assumed that subsets M^K of correlated variables, $M^K = \{i, i \in K\}$, $K \in K_s$, may be defined. Here, K is the subset of indices of the correlated variables, and K_s is the set of indices of the selected subsets of corelated variables.. From the above, when the upcoming period is t, Eq. (3) can be expressed as the following forecasting model for the primary variables i.

$$q_{it} = q_{i,\,t-1} \exp\left[\left(\mu_i - \frac{\sigma_i^2}{2}\right)\Delta + \eta_{it}\sigma_i\sqrt{\Delta}\right] \tag{4}$$

Here, it is assumed that μ_i and σ_i, $i = 1, 2, ..., I$ are fuzzy variables, the values of which are limited by the respective fuzzy numbers $\tilde{\mu}_i$, $\tilde{\sigma}_i$, $i = 1, 2, ..., I$. To take into consideration the correlation betweenthe imprecisely known variables, the set of independent variables ε_{it}; $i = 1, 2, ..., I$ should be replaced by the set of correlated variables η_{it}; $i = 1, 2, ..., I$. Correlated values η_{it} can be derived on the basis of the values ε_{it}. For this purpose, one can use the method of Cholesky decomposition of the correlation matrix described in the [30]. These matrixes should be developed for the primary variables $i \in K$, separately for each set $K \in K_s$.

When performing operations on fuzzy numbers, we often use the α-levels of these numbers. When using α-level notation, the constraints on the fuzzy variables μ_i and σ_i can be defined as follows:

$$\inf\left(\tilde{\sigma}_i\right)_\alpha \le \sigma_i \le \sup(\tilde{\sigma}_i)_\alpha \text{ for } \alpha \in [0,1],\ i \in I \tag{5}$$

$$\inf\left(\tilde{\mu}_i\right)_\alpha \le \mu_i \le \sup(\tilde{\mu}_i)_\alpha \text{ for } \alpha \in [0,1],\ i \in I \tag{6}$$

where: $(\tilde{\sigma}_i)_\alpha$ – α-level of the fuzzy number $\tilde{\sigma}_i$, $\left(\tilde{\mu}_i\right)_\alpha$ – α-level of the fuzzy number $\tilde{\mu}_i$.

The correlation between μ_i for different products can be modelled by interval regression [11]. Equations that define the correlation relationships between these variables, described using α-level notation, are presented below:

$$\mu_i \ge \inf\left(a_1^{iz}\right) \times \mu_z + \inf\left(a_2^{iz}\right) \text{ for } i \in K,\ z \in K,\ K \in K_s \tag{7}$$

$$\mu_i \le \sup\left(a_1^{iz}\right) \times \mu_z + \sup\left(a_2^{iz}\right) \text{ for } i \in K,\ z \in K,\ K \in K_s \tag{8}$$

where: a_1^{iz}, a_2^{iz} – the coefficients of the interval regression equations determining the dependencies between the variables $\tilde{\mu}_i$ and $\tilde{\mu}_z$.

The coefficients a_1^{iz}, a_2^{iz} may be determined using the method proposed by Hladik and Černy (the crisp input–crisp output variant) [11].

Equation (4) and Ineq. (5)–(8) can be used in simulations to estimate the static *NPV*. When estimating the value of a product switch option, a Monte Carlo simulation must be carried out under the assumption that the uncertain variables involved follow a risk-neutral GBM.

In this case, the following equation should be used instead of Eq. (4) [3, 5]:

$$q_{it} = q_{i\,t-1} \exp\left[\left(\mu_i - \pi_i - \frac{\sigma_i^2}{2}\right)\Delta + \eta_{it}\sigma_i\sqrt{\Delta}\right] \tag{9}$$

Estimation of the risk-premium (π) is usually done as described by Hull [13] and has been used in several works, such as Blank et al. [5]. At the same time, it is commonly known that risk premia are very hard to estimate with precision. Therefore, they should often be presented in the form of fuzzy variables. In this case, the constraints on the fuzzy variables π_i can be defined as follows:

$$\inf\left(\tilde{\pi}_i\right)_\alpha \le \pi_i \le \sup(\tilde{\pi}_i)_\alpha \text{ for } \alpha \in [0,1],\ i \in I \tag{10}$$

where: $(\pi_i)_\alpha$ – α-level of the fuzzy number $\tilde{\pi}_i$.

3.2. Model for estimating the value of a product switch option in a hybrid environment

In this section, the model used for valuating product switch options in a hybrid environment is defined. This model is an extension of the model presented in [18]. It takes into account the occurrence of fuzzy parameters in a model of GBM. Using this model and Monte Carlo simulation, a product switch option is valued in a hypothetical production setup (see Fig. 1).

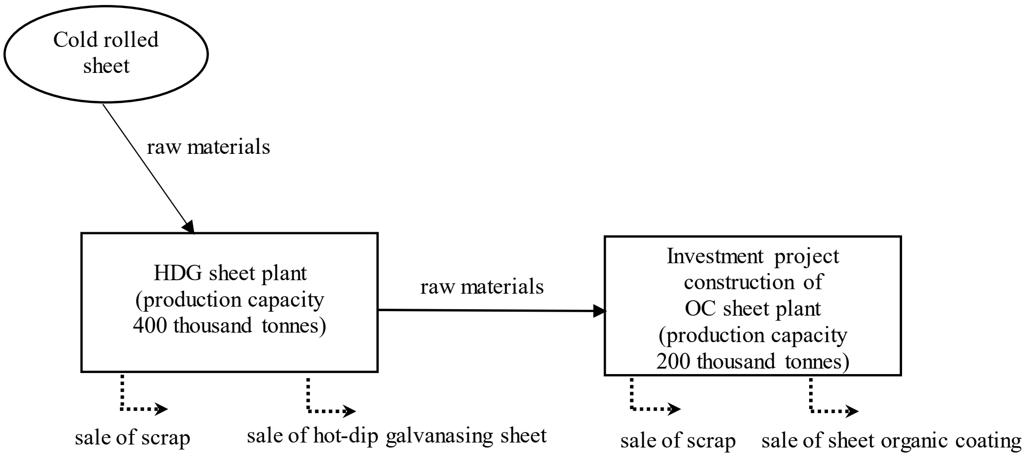

Fig. 1. Description of the analyzed production setup

In this case, the effectiveness of the project – the construction of a new organic--coated sheet (OC sheet) plant – is analyzed. This plant can produce OC sheet – a product made from hot-dip galvanised sheets (HDG sheet) with greater added value and several uses. For the analyzed production setup, cold–rolled sheets (CR sheets) are the basic raw material. CR sheets are converted into HDG sheets. HDG sheets are partly sold and partly converted into OC sheets. The latter are all sold. Steel scrap is a waste product in the production of HDG sheets and OC sheets and is sold.

Firstly, the basic case is analyzed. It consists of a standard valuation of a cash flow from which a static *NPV* (Eq. 11) for the projected construction of a new OC sheet plant is obtained. The annual cash flows obtained by investing in the OC plant, used to calculate the static *NPV*, can be estimated from Eqs. (12)–(32):

$$NPV = \frac{\sum_{t=0}^{T} ICF_t}{\left(1+r_{ris}\right)^t} - I \tag{11}$$

$$ICF_t = \left(ZNo_t + DAh + DAo + ZKOo_t\right) - \left(ZNh_t + DAh + ZKOh_t\right) \text{ for } t = 0, 1, \ldots, T-1 \tag{12}$$

$$ICF_t = \left(ZNo_t + DAh + DAo + ZKOo_t + RVo_t\right) - \left(ZNh_t + DAh + ZKOh_t + RVh_t\right) \text{ for } t = T \tag{13}$$

$$ZNo_t = SPo_t - Ko_t - \max(SPo_t - Ko_t, 0)Ta \text{ for } t = 0, 1, \ldots, T \tag{14}$$

$$ZNh_t = SPo_t - Kh_t - \max(SPh_t - Kh_t, 0)Ta \text{ for t} = 0, 1, \ldots, T \tag{15}$$

$$SPo_t = SRo_t \times So_t + SRh(2)_t \times Sh_t \text{ for } t = 0, 1, \ldots, T \tag{16}$$

$$SPh_t = SRh(1)_t \times Sh_t \text{ for } t = 0, 1, \ldots, T \tag{17}$$

$$SRo_t = \min(SFo_t; CAPo) \text{ for } t = 0, 1, \ldots, T \tag{18}$$

$$SRh(1)_t = \min(SFh_t; CAPh) \text{ for } t = 0, 1, \ldots, T \tag{19}$$

$$SRh(2)_t = \min\,(SFh(1)_t; (CAPh - \mathrm{Mo} \times SRo_t)) \text{ for } t = 0, 1, \ldots, T \tag{20}$$

$$SFh_t = ACh_t \times MSh \text{ for } t = 0, 1, \ldots, T \tag{21}$$

$$SFo_t = ACo_t \times MSo \text{ for } t = 0, 1, \ldots, T \tag{22}$$

$$Ko_t = SRo_t \times Cc(2)_t - SRh(2)_t \times Cc(1)_t - OPCh \times SRh(2)_t - SRo_t \times OPCo - DAh - DAo - GAh - GAo \text{ for t} = 1, 2, \ldots, T \tag{23}$$

$$Kh_t = SRh(1)_t \times Cc(1)_t - OPCh \times SRh(1)_t - DAh - GAh \text{ for } t = 0, 1, \ldots, T \tag{24}$$

$$Cc(1)_t = Mc \times Sc_t - (Mc - 1) \times Sz_t \text{ for } t = 0, 1, \ldots, T \tag{25}$$

$$Cc(2)_t = Mc \times Mh \times Sc_t - (Mc \times Mh - 1) \times Sz_t \text{ for } t = 0, 1, \ldots, T \tag{26}$$

$$ZKOo_t = KOo_t - KOo_{t-1} \text{ for } t = 0, 1, \ldots, T \tag{27}$$

$$ZKOh_t = KOh_t - KOh_{t-1} \text{ for } t = 0, 1, \ldots, T \tag{28}$$

$$KOo_t = \frac{SPo_t}{Sq} + \frac{SPo_t}{Cna} + \frac{Ko_t - DAo_t - DAh_t}{Cz} - \frac{Ko_t - DAo_t - DAh_t}{Czb} \quad \text{for } t = 0, 1, \ldots, T \tag{29}$$

$$KOh_t = \frac{SPh_t}{Sq} + \frac{SPh_t}{Cna} + \frac{Kh_t - DAh}{Cz} - \frac{Kh_t - DAh}{Czb} \quad \text{for } t = 0, 1, \ldots, T \tag{30}$$

$$RVo_t = \frac{SPo_t}{Sq} + 0.7\frac{SPo_t}{Cna} + 0.7\frac{Ko_t - DAo_t - DAh_t}{Cz} - \frac{Ko_t - DAo_t - DAh_t}{Czb} \quad \text{for } t = T \tag{31}$$

$$RVh_T = \frac{SPh_t}{Sq} + 0.7\frac{SPh_t}{Cna} + 0.7\frac{Kh_t - DAh_t}{Cz} - \frac{Kh_t - DAh_t}{Czb} \quad \text{for } t = T \tag{32}$$

The prices of HDG sheet, OC sheet, CR sheet, steel scrap and apparent consumption of HDG sheet and OC sheet are defined on the basis of Eq. (4). In the simulation process determining the *NPV*, additionally Eqs. (5) and (6) are taken into account for all the prices and apparent consumption of goods. The correlation between the prices of the listed products and raw materials and the correlations between the apparent consumption of HDG sheet and of OC sheet are defined on the basis of Eqs. (7) and (8).

The following notation was used in these equations:

ICF_t – cash flow in year t for the project – construction of a new OC sheet plant,
r_{ris} – weighted average cost of capital,
I – capital expenditure on the project,
ZNo_t – net profit in year t in the scenario where the project is implemented,
ZNh_t – net profit in year t in the scenario where the project is not implemented,
SPo_t – revenue in year t in the scenario where the project is implemented,
SPh_t – revenue in year t in the scenario where the project is not implemented,
Ko_t – total cost in year t in the scenario where the project is implemented,
Kh_t – total cost in year t in the scenario where the project is not implemented,
$CAPh$ – installed capacity of the HDG sheet plant,
$CAPo$ – installed capacity of the OC sheet plant,
SFh_t – sales forecast for HDG sheets in year t,
SFo_t – sales forecast for OC sheets in year t,
MSh – market share for HDG sheets,

MSo – market share for OC sheets,
ACh_t – forecasted apparent consumption of HDG sheets in year t,
ACo_t – forecasted apparent consumption of OC sheets in year t,
SRo_t – sale of OC sheets realized in year t in the scenario where the project is implemented,
$SRh(1)_t$ – sale of HDG sheets realized in year t in the scenario where the project is not implemented,
$SRh(2)_t$ – sale of HDG sheets realized in year t in the scenario where the project is implemented,
Sh_t – price of HDG sheet per ton in year t,
So_t – price of OC sheet per ton in year t,
Sc_t – price of CR sheet per ton in year t,
Sz_t – price of steel scrap per ton in year t,
$Cc(1)_t$ – cost of CR sheet per ton of HDG sheet in year t,
$Cc(2)_t$ – cost of CR sheet per ton of OC sheet in year t,
Mc – per unit consumption of CR sheet when producing HDG sheet,
Mh – per unit consumption of HDG sheet when producing OC sheet,
$OPCh$ – other (with the exception of the cost of CR sheet) annual variable production costs per ton for HDG sheet,
$OPCo$ – other (with the exception of the cost of CR sheet) annual variable production costs per ton for OC sheet,
GAh – annual fixed costs for HDG plant,
GAo – incremental annual fixed costs for OC plant,
DAh – annual amortization for HDG plant,
DAo – annual amortization for OC plant,
KOo_t – net working capital in year t in the scenario where the project is implemented,
KOh_t – net working capital in year t in the scenario where the project is not implemented,
$ZKOo_t$ – change in net working capital in year t in the scenario where the project is implemented,
$ZKOh_t$ – change in net working capital in year t in the scenario where the project is not implemented,
RVo_t – residual value in year t in the scenario where the project is implemented,
RVh_t – residual value in year t in the scenario where the project is not implemented,
Sq – cash in hand turnover,
Cz – inventory turnover,
Czb – debtor turnover,
Cna – receivables turnover,
T – economic life-cycle of project,
Ta – tax.

Equations (12) and (13) are used to compute the cash flow for the project – construction of a new OC sheet plant – in successive years of the project's life-cycle. Equation (12) concerns the years 0, 1, ..., $T - 1$, whereas Eq. (13) concerns the last year T. Cash flow is computed according to an incremental relationship, i.e., the cash flow which would arise without taking into account the investment is subtracted from the cash flow which takes into account the construction of a new OC sheet plant.

Equations (14) and (15) show how to compute the net profit in the scenario where the project is implemented and in the scenario where the project is not implemented, respectively. The key problem in computing these profits is determining the revenue and total costs resulting from the analysed scenarios.

The revenue is computed using Eqs. (16)–(22). The revenue in the scenario where the project is implemented is computed using Eq. (16). Equations (18), (20) and (22) enable computing the level of sales in this case. The revenue in the scenario where the project is not implemented is computed using Eq. (17). Equations (19) and (21) are used to compute the level of sales in this case.

When computing the static NPV, the amount of OC sheets sold (in the scenario where the project is implemented (SRo_t)) is determined as the minimum of the following two values:

- the product of the forecasted apparent consumption of OC sheets and market share,
- the available capacity for producing OC sheets.

The amount of HDG sheets sold in this scenario ($SRh(2)_t$) is determined as the minimum of the following two quantities:

- the product of the forecasted apparent consumption of HDG sheets and market share,
- the available capacity for producing HDG sheets minus the amount of HDG sheets used to produce OC sheets.

In this case, the greatest possible sales of OC sheets are realized according to both production capacity and market conditions. The sales of HDG sheets stem from the market conditions, production capacities and the amount of HDG used to produce OC sheets. On the other hand, the level of sales of HDG sheets in the scenario where the project is not implemented ($SRh(1)_t$) is determined as the minimum of the two following quantities:

- the product of the forecasted apparent consumption of HDG sheets and market share,
- the available capacity for producing HDG sheets.

The remaining quantities required to calculate the NPV are computed on the basis of the revenue resulting from these sales.

Equations (23) and (24) are used to compute the total costs, and Eqs. (25) and (26) to compute the cost of materials, for both of the scenarios analysed.

Equations (27)–(30) enable assessment of the change in the level of working capital in each year of the analysed scenarios. The level of working capital is computed as a function of the levels of cash in hand turnover, inventory turnover, debtor turnover and receivables turnover.

Equations (31) and (32) are used to compute the residual value for the analysed scenarios. The residual value is computed according to the Wilcox formula, which states that the residual value is equal to [18]:

- 100% of the value of means of payment,
- 70% of the book value of supplies,
- 70% of the book value of debts,
- –100% of the value of liabilities.

The static *NPV* is calculated *N* times for each α-level of $\tilde{\mu}_i$ and $\tilde{\sigma}_i$ using Monte Carlo simulation according to Eq. (11). The uncertainty of demand for HDG sheets, OC sheets and the uncertainty of prices for CR sheets, HDG sheets, OC sheets and scrap are taken into consideration. Such a calculation takes into account the correlations between the prices of the following products: steel scrap, CR sheets, HDG sheets and OC sheets. The correlation between the apparent consumption of HDG sheets and OC sheets is also taken into account in the calculation procedure. The result gives an estimate of the upper and lower probability distribution function of the static *NPV*. Formally, the algorithm can be written as follows:

START

Step 1. Define α_0, $\wp$, J

Step 2. Set $j=1$

Step 3. Randomly generate a vector $[\eta_{1t}, \eta_{2t}, ..., \eta_{lt}]$, $t = 1, 2, ..., T$. Take into account the correlation between variables

Step 4. Set $\alpha = \alpha_0$

Step 5. Define α-levels (μ_i), (σ_i) for the prices of the analysed products and raw materials and apparent consumption of HDG sheet and OC sheet

Step 6. Define (sup) and (inf) for α-levels of the fuzzy number defining the *NPV*. Find

$\overline{NPV}_{\alpha,j}(\mu_i, \sigma_i) \to \max$

$\underline{NPV}_{\alpha,j}(\mu_i, \sigma_i) \to \min$

where the problem constraints are specified by Ineq. (5)–(8) and (12)−(32)

Step 7. Set $\alpha = \alpha + \wp$

Step 8. If $\alpha \leq 1$ go to Step 5

Step 9. Set $j = j + 1$

Step 10. If $j \leq J$ go to Step 3

Step 11. Define the set of fuzzy numbers $(\pi_1^{NPV}, ..., \pi_J^{NPV})$

STOP

In this case the π_j^{NPV} for j = 1, 2, ..., J are determined by the intervals $[\underline{NPV}_{\alpha,j}, \overline{NPV}_{\alpha,j}]$, for $\alpha_o, \alpha_o + \wp, \alpha_o + 2\wp$, ..., 1. By this means, we obtain a random fuzzy set defining the static *NPV* of the project. Based on this set, upper and lower cumulative distribution functions, together with the average value of the static *NPV*, is estimated. This uses a method defined in [19].

Next, the switch option is valued. The analysis of the static *NPV* of the project does not take into account the managerial flexibility of being able to switch the output product. In some periods, the production of HDG sheet may be a more interesting and profitable alternative to the company than the production of OC sheet. Therefore, in such cases the switch option is realized. The largest possible sales of HDG sheets are realized according to the available production capacities and market conditions. The sales of OC sheets stem from market conditions, production capacities and the availability of raw materials, i.e., HDG sheets. When the product switch option is realized, the level of sales of OC sheets ($SRo(3)_t$) is determined as the minimum of the following quantities:

- the product of the forecasted apparent consumption of OC sheet and market share,
- available capacity for producing OC sheets,
- the raw materials available, in the form of HDG sheets.

The sales of HDG sheets in this case are determined as the minimum of the following quantities:

- the product of the forecasted apparent consumption of HDG sheets and market share,
- capacity for producing HDG sheets, i.e., it is assumed to equal $SRh(1)_t$.

The values of product switch options can be obtained by simulating the incremental cash flows defined for the level of sales of OC sheets and HDG sheets discussed above in relation to the cash flow defined according to the conditions assumed in the calculation of the static *NPV*. The following equations are used to compute the value of the switch option:

$$OPT = \sum_{t=1}^{T} \frac{ICF_t^*}{\left(1+r_f\right)^t} \tag{33}$$

where OPT – the value of the switch option, r_f – risk free rate, ICF_t^* – incremental cash flow related to the product switch option in year t,

$$ICF_t^* = \max\Big(\big(ZNo(1)_t + DAo + DAh + ZKOo(1)_t \\ - \big(ZNo_t + DAo + DAh + ZKOo_t\big); 0\Big) \text{for } t = 1, 2, \ldots, T-1 \tag{34}$$

$$ICF_t^* = \max\Big(\big(ZNo(1)_t + DAo + DAh + ZKOo(1)_t + RVo(1)_t\big)$$

$$-\big(ZNo_t + DAo + DAh + ZKOo_t + RVo(1)_t\big); 0\Big) \text{ for } t = T \tag{35}$$

The values of $ZNo(1)_t$, $ZKOo(1)_t$, $RVo(1)_t$ correspond to the net profit, change in working capital and residual value after realizing the product switch option, respectively. They are computed on the basis of the values of SPo_t, and Ko_t, which are given by the following formulas:

$$SPo_t = SRo(3)_t \times So_t + SRh\,(1)_t \times Sh_t \tag{36}$$

$$Ko_t = SRo(3)_t \times Cc(2)_t - SRh(1)_t \times Cc(1)_t - OPCh \times SRh(1)_t$$

$$- SRo(3)_t \times OPCo - DAh - DAo - GAh - Gao \tag{37}$$

$$SRo(3)_t = \min\left[SFo_t; CAPo; CAPh - \frac{SRh(1)_t}{Mh}\right] \tag{38}$$

The remaining parameters necessary to calculate the ICF_t^* are defined according to the equations shown above for the calculation of the static NPV.

Formally, the algorithm can be written as follows:

START

Step1. Define α_0, $\wp$, J

Step 2. Set $j = 1$

Step3. Randomly generate a vector $[\eta_{1t}, \eta_{2t}, ..., \eta_{It}]$, $t = 1, 2, ..., T$. Take into account the correlation between variables

Step 4. Set $\alpha = \alpha_0$

Step 5. Define α-levels $(\mu_i)_\alpha, (\sigma_i)_\alpha$ for the prices of analysed products and raw materials and apparent consumption of HDG sheet and OC sheet

Step 6. Define (sup) and (inf) for α-levels of the fuzzy number defining the NPV. Find

$\overline{OPT}_{\alpha,j}(\mu_i, \sigma_i) \to \max$

$\overline{OPT}_{\alpha,j}(\mu_i, \sigma_i) \to \min$

where the problem constraints are specified by Ineq. (5)–(9) and (12)−(15), (17)–(22), (24)–(32), (34)–(38)

Step 7. Set $\alpha = \alpha + \wp$

Step 8. If $\alpha \le 1$ go to Step 5

Step 9 Set $j = j + 1$

Step 10. If $j \leq J$ go to Step 3
Step 11. Define the set of fuzzy numbers $(\pi_1^{NPV}, ..., \pi_J^{NPV})$
STOP

In this case the π_j^{OPT} for j = 1, 2, ..., J are determined by the intervals $[\overline{OPT}_{\alpha, j}$ $\underline{OPT}_{\alpha, j}]$ for $\alpha_o, \alpha_o + \wp, \alpha_o + 2\wp, ..., 1.$ In this way, we obtain a random fuzzy set defining the *OPT* of the project.

Nevertheless, both the simulations and discount rate must assume risk neutrality, as when valuing options, the level of risk will change when these options are exercised. Thus, we must use the risk-free rate for discounting the incremental cash flows when the option is exercised, but these must be simulated using a risk-neutral expectation. Therefore, the GBM process defined by Eqs. (9)−(10) is used here to model the underlying uncertainty. Here, the uncertainty and correlations between the variables were taken into account in the same way as in the estimation of the static *NPV*.

4. Data and results of calculations

4.1. Data used for the calculations

Table 1 presents the correlation matrix for the prices of the products analysed. The coefficient of correlation between apparent consumption of HDG sheet and of OC sheet is 0.501.

Table 1. Correlation matrix for the prices of the products analysed

Product	Scrap	CR sheet	HDG sheet	OC sheet
Scrap	1.000	0.930	0.952	0.936
CR sheet	0.930	1.000	0.839	0.809
HDG sheet	0.952	0.839	1.000	0.828
OC sheet	0.936	0.809	0.828	1.000

Source: author's calculations, based on the historical time series of the prices on Polish market from years 1996−2016.

The coefficients of the interval regression equations depicting the interrelations between the μ_i for the prices of particular product ranges are shown in Table 2, and for apparent consumption in Table 3. The weighted average cost of capital, considering the financial leverage typical of this industry, for this type of steel plant is assumed to be 10% per year in real terms and the risk free rate is 5% [20].

Table 2. Coefficients of the interval regression equations depicting the interrelations between the μ_i for the prices of particular product ranges

Independent variable		Dependent variable			
		Scrap	CR sheet	HDG sheet	OC sheet
Scrap	a_1		[–0.057, 1.930]	[0.373, 1.381]	[0.601, 1.565]
	a_2		[–0.021, 0.001]	[–0.006, –0.002]	[0.003, 0.009]
CR sheet	a_1	[–0.908, 2.992]		[–1.589, 3.317]	[–1.705, 3.848]
	a_2	[–0.011, 0.036]		[–0.017, 0.035]	[–02.029, 0.066]
HDG sheet	a_1	[0.733, 1.516]	[0.411, 1.650]		[0.527, 1.912]
	a_2	[0.003, 0.006]	[–0.010, –0.003]		[0.005, 0.018]
OC sheet	a_1	[0.065, 1.764]	[–2.280, 3.986]	[–0.610, 2.216]	
	a_2	[–0.010, 0.000]	[–0.067, 0.039]	[–0.024, 0.007]	

Source: cf. footnote to Table 1.

Table 3. Coefficients of the interval regression equations depicting the interrelations between the μ_i for the apparent consumption of HDG sheet and of OC sheet

Independent variable		Dependent variable	
		HDG sheet	OC sheet
HDG sheet	a_1		[–0.238, 0.804]
	a_2		[–0.047, 0.157]
OC sheet	a_1	[–1.623, 3.495]	
	a_2	[–0.043, 0.092]	

Source: cf. footnote to Table 1.

Analysis of the time series for the price of OC sheet from 1996 to 2016 in real terms shows an average growth rate of 0.97% per year. For the prices of other products, the average growth rate is as follows: HDG sheet – 1.61%, CR sheet – 1.88%, scrap – 1.72%. Nevertheless, we use the trapezoidal number (0.9, 1.1, 1.3, 1.5%) to describe the drift for all of these products, as it is assumed that this increase was partly a result of the structural changes occurring in this sector during the economic boom, which occurred in 2006–2007.

Analysis of the time series for the apparent consumption of OC sheet and HDG sheet (from 1996 to 2016) shows an average growth rate of 8.00% per year for HDG sheet and 9.26% for OC sheet. Nevertheless, we use the trapezoidal number (6.0, 6.5, 7.0, 7.5%) as the drift for all of these products as it is assumed (as above) that this increase was partly a result of the structural change occurring in the sector during the economic boom, which occurred in 2006–2007.

The volatility parameters were estimated using the standard deviation of the log return of the price series and demand series (from 1996 to 2016). These values are equal to:

For prices, %:

Scrap 16.33

CR sheet	21.05
HDG sheet	19.62
OC sheet	13.56

For apparent consumption, %:

HDG sheet	11.77
OC sheet	16.43

For similar reasons as before, in the calculations we adopted the following trapezoidal numbers describing volatility:

For prices, %:

Scrap	13.0, 14.0, 15.0, 16.0
CR sheet	15.0, 17.0, 18.0, 20.0
HDG sheet	15.0, 17.0, 18.0, 20.0
OC sheet	10.0, 11.0, 12.0, 13.0

For apparent consumption, %:

HDG sheet	8.0, 9.0, 10.0, 11.0
OC sheet	12.0, 13.0, 14.0,15.0

The other variable production costs, (energy, manpower and maintenance) amount to approximately USD 114.0 per ton of HDG sheet and USD 174.0 per ton of OC sheet. The fixed costs for the HDG sheet plant are estimated at USD 31.08 million per year. The incremental fixed costs for the OC sheet plant are estimated at USD 12.95 million per year. The initial investment for the OC sheet plant amounts to approximately USD 40 million. The market share for HDG sheets was assumed to be 30% and for OC sheets 25%.

The premiums (π) on the basis of statistical data were estimated as follows:

For prices, %:

Scrap	0.81
CR sheet	0.79
HDG sheet	1.01
OC sheet	0.75

For apparent consumption, %:

HDG sheet	1.32
OC sheet	2.11

For the calculations we adopted the following trapezoidal numbers:

For prices, %:

Scrap	0.77, 0.80, 0.82, 0.85
CR sheet	0.75, 0.77, 0.80, 0.82
HDG sheet	0.97, 0.99, 1.02, 1.05
OC sheet	0.71, 0.73, 0.76, 0.78

For apparent consumption, %:

HDG sheet	1.28, 1.30, 1.33, 1.35
OC sheet	2.07, 2.10, 2.13, 2.15

Simulations were carried out on the basis of these assumptions and the static *NPV* and value of the product switch option for the project of constructing a new OC sheet plant were calculated.

4.2. Results and discussion

Figures 2 and 3 show the upper and lower probability distribution function of the static *NPV* and the upper and lower probability distribution function of the value of the product switch option for the project – construction of a new OC plant. For comparison, Figures 2 and 3 present results based on an approach that takes into account the correlation between variables alongside results based on a variant that does not take these correlations into account. Table 4 summarizes the values found for the project – construction of OC plant containing a product switch option (calculation based on the variant taking into account the correlation between the primary variables).

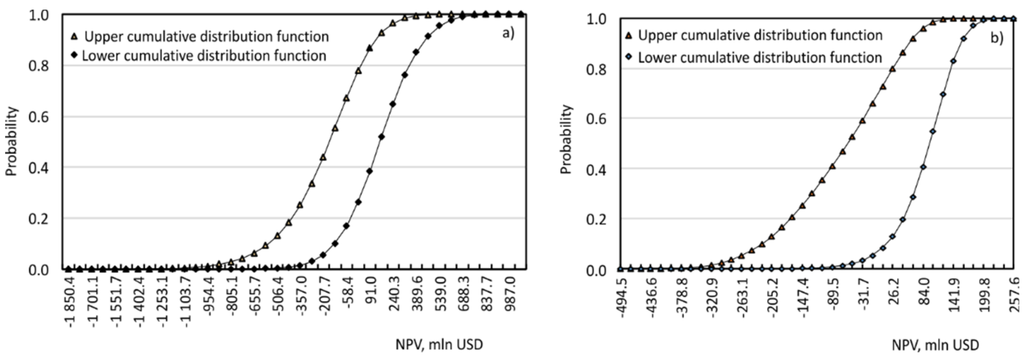

Fig. 2. Upper and lower cumulative distribution functions for the static *NPV* of the project *Construction of a new OC plant*: without (a) and taking into account (b) the correlation between the primary variables

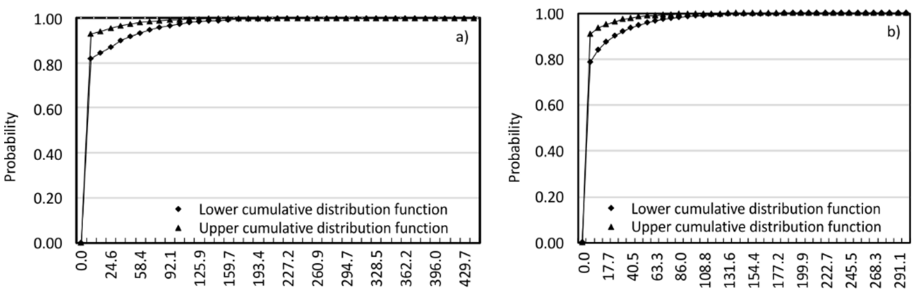

Fig. 3. Upper and lower cumulative distribution functions for the value of a product switch option for the project *Construction of a new OC plant* without (a) and taking into account (b) the correlation between the primary variables

Based on the probability distribution functions depicted in Figs. 2 and 3, one can specify intervals containing the probability of the occurrence of a particular event. For example, according to Fig. 3b, the probability that $OPT \geq 17.4$ million USD is in the range [0.046, 0.126]. The difference between the upper and lower boundary of this range results from the lack of precision in estimating the parameters of the model. If the decision maker considers that this difference is too great, he/she can commission an additional study that will increase the precision of estimating these parameters.

Table 4. Value of the product switch option and static *NPV* of the OC plant

OC plant	Average value [thousand USD]	Lower, upper bound
Static NPV	15 625.7	(–69 044.7, 99 063.5)
Extended NPV	21 804.2	(–26 624.0, 69 739,6)

The average values were calculated on the basis of the probability distribution determined by the average value of the upper and lower probability distribution functions [19]. The lower and upper bounds were calculated on the basis of the lower and upper cumulative distribution functions.

Based on the data in Table 4, it can be concluded that the average value of the product switch option is equal to 6 178.5 thousand USD. This means that the average extended *NPV* of the OC plant is 39.5% greater than the average of its static *NPV*. A comparison of Figs. 2a, 3a and 2b, 3b indicates that the correlations between parameters significantly influence the probability distribution function of the *NPV* and value of the switch option. Hence, ommission of these correlations leads to systematic errors in estimating the *NPV* and value of the option.

5. Conclusion

A new method for valuing switch options has been suggested. This method is based on the probability theory and the theory of fuzzy sets. We assume that some parameters of the model for pricing real options cannot be described precisely and therefore they are introduced into the model as fuzzy numbers. This assumption enables us to consider various sources of uncertainty, not only stochastic ones and to retain more information about the possible value of a real option. This approach is particularly useful in the case of projects with a long economic life. A computational procedure for deriving the value of a switch option is also proposed. Under the assumptions of fuzzy volatility and fuzzy drift parameters, the value of a switch option turns into a random fuzzy number.

The range of applications of this model include long-term financial decision-making and the methodology for valuing real options. Hence, further development and verification of such fuzzy-stochastic models might be useful.

The example analysed indicates that, in the steel industry, inclusion of an investment project's switch option in the analysis has a significant impact on its valuation. In the case of uncertainty described by fuzzy numbers, modelling the relationships between the primary variables is a very important problem. Ignoring these dependencies leads to systematic errors in the valuation of investment projects.

References

[1] ALLENOTOR D., THULASIRAM R.K., *A grid resources valuation model using fuzzy real option*, Lecture Notes in Comp. Sci., Parallel and Distributed Processing and Applications, Springer, 2007, 4742, 622–632.

[2] APPADOO S.S., THAVANESWARAN A., *Recent developments in fuzzy sets approach in option pricing*, J. Math. Fin., 2013, 3 (2), 312–322.

[3] BASTIAN-PINTO C., BRANDÃO L., HAHN W.J., *Flexibility as a source of value in the production of alternative fuels. The ethanol case*, Energy Econ., 2009, 31 (3), 411–422.

[4] BELLMAN R.E., ZADEH L.A., *Decision-making in a fuzzy environment*, Management Science, 1970, 17 (4), 141–164.

[5] BLANK F.F., BAYDIA T.K.N., DIAS M.A., *Private infrastructure investment through public private partnership. An application to a toll road highway concession in Brazil*, Proc. 13th Annual International Conference on Real Options, Braga, Portugal, Santiago, Spain, 2009, 1–21.

[6] BRENNAN M.J., SCHWARTZ E.S., *Evaluating natural resource investments*, J. Bus., 1985, 58 (2), 135–157.

[7] CARLSSON C., FULLER R., *A fuzzy approach to real option valuation*, Fuzzy Sets Syst., 2003, 139 (2), 297–312.

[8] CARLSSON C., FULLER R., HEIKKILA M., MAJLENDER P., *A fuzzy approach to R&D project portfolio selection*, Int. J. Appr. Reas., 2007, 44 (2), 93–105.

[9] GARCIA F.A.A., *Fuzzy real option valuation in a power station reengineering project. Soft computing with industrial applications*, Proc. Sixth Biannual World Automation Congress, Spain, 2004, 281–287.

[10] GUERRA M.L., SORINI L., STEFANINI L., *Parametrized fuzzy numbers for option pricing*, IEEE International Conference on Fuzzy Systems, London 2007, 727–732.

[11] HLADÍK M., ČERNY M., *Interval regression by tolerance analysis approach*, Fuzzy Sets Syst., 2012, 193 (4), 85–107.

[12] HUANG M.-G., *Real options approach-based demand forecasting method for a range of products with highly volatile and correlated demand*, Eur. J. Oper. Res., 2009, 198 (3), 867–877.

[13] HULL J.C., *Options, futures and other derivatives securities*, 6th Ed., Prentice Hall, Englewood Cliffs 2006.

[14] KAHRAMAN C., UCAL İ., *Fuzzy real options valuation for oil investments*, Proc. 8th International FLINS Conference, World Scientific Publishing Co. PTE Ltd., Singapore 2008, 1027–1032.

[15] LEE C.F., TZENG G.H., WANG S.Y., *A new application of fuzzy set theory to the Black–Scholes option pricing model*, Expert Syst. Appl., 2005, 29 (2), 330–342.

[16] MARATHE R., RYAN S.M., *On the validity of the geometric Brownian motion assumption*, Eng. Econ., 2005, 50 (2), 1–40.

[17] OZORIO L.M., BASTIAN-PINTO C.L., BAIDYA T.K.N., BRANDÃO L.E.T., *Investment decision in integrated steel plants under uncertainty*, Int. Rev. Fin. Anal., 2013, 27 (4), 55–64.

[18] RĘBIASZ B., GAWEŁ B., SKALNA I., *Valuing managerial flexibility. An application of real-option theory to steel industry investments*, Oper. Res. Dec., 2017, 27 (2), 91–111.
[19] RĘBIASZ B., *New method of selection of efficient project portfolios in the presence of hybrid uncertainty*, Oper. Res. Dec., 2016, 26 (4), 65–90.
[20] RUEY S.T., *Analysis of Financial Time Series*, Wiley, New York 2002.
[21] SHIU H.H., SHU H.L., *A fuzzy real option approach for investment project valuation*, Exp. Syst. Appl., 2011, 38 (12), 15296–15302.
[22] TAO C., JINLONG Z., SHAN L., BENHAI Y., *Fuzzy real option analysis for IT investment in nuclear power station*, Lecture Notes in Comp. Sci., ICCS 2007, Springer, 2007, 4489, 953–959.
[23] THAVANESWARAN A., APPADOO S.S., FRANK J., *Binary option pricing using fuzzy numbers*, Appl. Math. Lett., 2013, 26 (1), 65–72.
[24] THIAGARAJACH K., THAVANESWARAN A., *Fuzzy coefficient volatility models with financial applications*, J. Risk Fin., 2006, 7 (5), 503–524.
[25] WATTANARAT V., PHIMPHAVONG P., MATSUMARU M., *Demand and price forecasting models for strategic and planning decisions in a supply chain*, Proc. Schl. ITE Tokai Univ., 2010, 3 (2), 37–42.
[26] WEIDONG X., CHONGFENG W., XU W.J., LI H.Y., *A jump-diffusion model for option pricing under fuzzy environments*, Ins.: Math. Econ., 2004, 44 (3), 337–344.
[27] WU H.-C., *Using fuzzy sets theory and Black–Scholes formula to generate pricing boundaries of European options*, Appl. Math. Comp., 2007, 185 (1), 136–146.
[28] WU H.-C., *Pricing European options based on the fuzzy pattern of Black–Scholes formula*, Comp. Oper. Res., 2004, 31 (7), 1069–1081.
[29] XU W., PENG X., XIAO W., *The fuzzy jump-diffusion model to pricing European vulnerable options*, Int. J. Fuzzy Syst., 2013, 15 (3), 317–325.
[30] YANG T.I., *Simulation based estimation for correlated cost elements*, Int. J. Project Manage., 2005, 23 (4), 275–282.
[31] YU S.-E., LI M.-Y.L., HUARNG K.-H., CHEN T.-H., CHEN C.-Y., *Model construction of option pricing based on fuzzy theory*, J. Marine Sci. Technol., 2011, 19 (5), 460–469.
[32] ZADEH L.A., *Fuzzy sets*, Inf. Control, 1965, 8, 338–358.
[33] ZDENEK Z., *Modelling the sequential real options under uncertainty and vagueness (fuzzy-stochastic approach)*, [In:] J. Ramik, D.K. Stavarek (Eds.), Proc. 30th International Conference Mathematical Methods in Economics, Karvina, Czech Republic, 2012, 1027–1032.
[34] ZENG M., WANG H., ZHANG T., LI B., HUANG S., *Research and application of power network investment decision-making model based on fuzzy real options*, International Conference on Service Systems and Service Management, IEEE, Chengdu, China, 2007, 397−402.
[35] ZHANG L.-H., ZHANG W.-G., XU W.-J., XIAO W.-L., *The double exponential jump diffusion model for pricing European options under fuzzy environments*, Econ. Model., 2012, 29 (3), 780–786.
[36] ZMESKAL Z., *Generalised soft binominal American real option pricing model. Fuzzy stochastic approach*, Eur. J. Oper. Res., 2010, 207 (2), 1096–1103.

Received 4 November 2017
Accepted 8 February 2019