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SPECTRAL ANALYSIS OF BUSINESS CYCLES IN POLAND AND ITS MAJOR TRADING PARTNERS

The properties of business cycles in Poland and its major trading partners have been examined. The business cycle synchronization (BCS) between Poland and other countries was studied in order to assess the impact of international trade on BCS. The author applies a modification of the Fourier analysis to the estimation of cycle amplitudes and frequencies. This allows more precise estimation of the cycle characteristics than the traditional approach. Cross-spectral analysis of the cyclical components of GDP for Poland and its major trading partners enables us to study the relationships between business cycles in these countries. Comparing the international structure of Polish trade with that of EU members with the cross-spectral characteristics of GDP series allows us to investigate the links between international trade and business cycle synchronization.

Keywords: *business cycle, synchronization, spectral analysis, Fourier representation*

1. Introduction

The level of economic activity and its changes over time have been subjects of great interest to researchers from the beginning of the study of economics as an academic discipline. Scholars initially focused on the long-term equilibrium, and short term changes in the growth rate were treated as random disturbances from long-term growth.

Studies into changes in economic output are conducted in two ways. The first is research on long-term growth, while the second is the analysis of cyclical economic development. Theories of economic growth explain long-run movements in production. In conjunction with surveys of long-term growth, studies on movements of economic variables, especially gross domestic product (GDP), have been conducted [23, 6, 26, 37]. Modelling

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business cycles depends on the definition of fluctuations in economic activity. This determines the choice of the mathematical and statistical methods used to identify business cycles. The author of this paper presents different approaches to the analysis of business cycles and proposes a modification of the Fourier analysis that can be applied to accurately measure cycle characteristics.

The empirical part of the paper includes an analysis of GDP movements in Poland and its major trading partners. The choice of the countries analysed is the result of theoretical and empirical studies on synchronized cycles. The level of trade between countries is indicated to be the main factor affecting the level of synchronization [16, 5]. The author verifies the hypothesis about the positive impact of trade on business cycle synchronization in Poland and its trading partners. In order to verify this hypothesis, the following areas have been investigated: (1) the international structure of Polish trade with the EU, (2) measurement of business cycles in Poland and its major trading partners, (3) the synchronization of business cycles in Poland and other countries, and (4) comparison between the size and structure of Polish trade and BCS.

The paper is organized in the following way. In Section 2, there is a literature review of research on the mathematical description of business cycles and synchronization of cycles. Section 3 presents the methods of univariate and multivariate spectral analysis which are used to identify business cycles and to assess their synchronization between countries. In Section 4, the author applies methods of spectral analysis to identify cyclical patterns in GDP series and to study co-movements between GDP in Poland and its major trading partners. Then, the levels of business cycle synchronization are compared with shares in the value of trade between Poland and other countries to draw conclusions about the links between international trade and BCS between Poland and its partners, and, in consequence, verify the hypothesis.

2. Studies on cycles

Burns and Mitchell [9] proposed the definition of a cycle that is most often quoted by scholars conducting studies of business cycles. *Business cycles are a type of fluctuation found in the aggregate economic activity of nations that organize their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time in many economic activities, followed by similar general recessions, contractions, and revivals which merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic; in duration business cycles vary from just over one year to ten or twelve years; they are not divisible into shorter cycles of similar character with amplitudes approximating their own.* The most important conclusion is that a cycle consists of four distinct phases that evolve from one stage into the next: expansion, recession, depression and revival.

A different view of cycles was presented by Lucas [28] in his article *Understanding business cycles*. He argued that business cycles are fluctuations around trends in the gross national product. In contrast to Burns and Mitchell's view, Lucas does not interpret cycles as inevitable transitions between different phases of a cycle; rather, he treats them as a process where the GNP oscillates around a long-term trend. In this definition, Lucas does not specify the concept of a trend, but at the same time does not use the terms: fixed or average level. Hence, a trend should be understood as a path of long-term growth.

Due to the periodic nature of changes in economic production, its analysis requires the estimation of components which reflect both long-term economic growth and the business cycle. Initially, the dominant approach to modelling long-term growth in economic activity was to treat it as a polynomial deterministic trend and fluctuations were interpreted as stochastic deviations from this trend, i.e. they were considered to be a residual cyclical component. The deviations from the trend were assumed to be stationary, which in turn led to the construction of trend-stationary models. In the 1980s, a difference-stationary process became an alternative to trend-stationary models. Autoregressive integrated moving average (ARIMA) models were applied to macroeconomic variables. Authors used ARIMA models to describe the long-term trend in real GDP and its link with the business cycle [7, 29, 10]. The use of a deterministic trend to model the characteristics of long-term growth results in the assignment of too much variation to cyclical fluctuations, while the use of a stochastic trend leaves too little variability for fluctuations [40].

A compromise is to use the unobserved components models proposed by Harvey [20] and Clark [13]. In their approach, the trend is smoothed and cycles have a high amplitude and are stable. Watson [39] and Stock and Watson [36] analysed the difference between the Beveridge-Nelson approach and the unobserved components model. In the unobserved components model, restrictions are imposed on the structure of the stochastic trend and the cyclical component, as well as on cross-correlations between innovations in both components. Harvey [21, 22] analysed and comprehensively described the structure of these models.

Another approach to extracting the cyclical component of a series, filter based methods, applies frequency filters to decompose the time series. The essence of this method is to extract various components acting at different frequencies. Baxter and King [4], Hodrick and Prescott [24] and Christiano and Fitzgerald [12] presented such methods and the results of an analysis of cycles in the United States using a frequency filter.

For assessment of the degree of cycle synchronization, various methods are applied. One of the most popular tools is the Pearson correlation coefficient, which measures the degree of co-movement in the time domain. Its main drawback is a loss of information about frequency horizons. Forni et al. [15] and Lee [27] used a dynamic factor model to explore the co-movements of business cycles across countries in Europe. On the other hand, Artis et al. [2] applied Markov switching vector autoregression models to determine the dynamics of a European business cycle. Harding and Pagan [19] presented methods of measuring and testing the degree of synchronization. A different approach to studying business cycles, in both time and frequency domains, is the wavelet analysis [14, 1].

Polish researchers have also undertaken the analysis of business cycles in Poland and their synchronization with the cycles of other European countries. Gradzewicz et al. [17] and Skrzypczyński [35] applied spectral analysis to modelling Polish business cycles. Skrzypczyńska [33] compared the results of using three methods (unobserved components model, Christiano–Fitzgerald band-pass filter and Markov-switching model) to model business cycles in Poland. Skrzypczyński [34], Bruzda [8], Konopczak and Marczewski [25] studied the synchronization of Polish business cycles with other European countries using various methods: multivariate spectral analysis, wavelet analysis and an extension of the Blanchard–Quah model.

3. Methods

Spectral analysis is a method of estimating the spectral density function or spectrum of a given time series. Its goal is to determine the contributions of various periodic components with different frequencies in the time series. This method analyses the properties of the series in the frequency domain and is complementary to analysis of the time domain based on the covariance function. Any covariance-stationary process has both a time-domain representation and a frequency-domain representation, and any feature of the data that can be described by one representation can equally well be described by the other representation [18].

3.1. Fourier analysis and its modification

Fourier analysis is basically concerned with approximating a function of time by a sum of sine and cosine terms [31], called the Fourier series representation.

When T is even, the Fourier representation of the time series, y_t, is

$$y_t = \hat{\mu} + \sum_{j=1}^{T/2-1} \left\{ \hat{\alpha}_j \sin\left(\omega_j t\right) + \hat{\delta}_j \cos\left(\omega_j t\right) \right\} + \hat{\delta}_{T/2} \cos\left(\omega_{T/2} t\right) \tag{1}$$

where: $\omega_j = 2\pi j/T$, $\hat{\mu}$ is the sample mean and $\hat{\alpha}_j$, $\hat{\delta}_j$ are coefficients.

If T is odd, then the time series can be expressed as

$$y_t = \hat{\mu} + \sum_{j=1}^{(T-1)/2} \left\{ \hat{\alpha}_j \sin\left[\omega_j \left(t-1\right)\right] + \hat{\delta}_j \cos\left[\omega_j \left(t-1\right)\right]\right\} \tag{2}$$

Since the components in the equation are mutually orthogonal, each one explains a specific part of the variance of the series. It is worth noting that the Fourier analysis partitions the variability of a series of even length T into components of frequencies $2\pi/T$, $4\pi/T$, ..., π. These frequencies are discrete and depend on the length of the time series. This means that the analysis is limited to pre-established frequencies and consequently to pre-established wavelengths, which is not always appropriate and has no substantive justification. If one wants to study the contribution of cycles of given frequencies to variation in the series, the traditional Fourier representation is of little use. For example, it is impossible to examine the contribution of a cycle with a length of 7 years based on an 18-year time series.

A simple but ineffective solution is to adjust the length of the time series to the length of the analysed cycles. This requires lengthening or shortening the series. It should be noted that lengthening the series is not always a possible solution, and that shortening results in a loss of some information.

The author of this paper proposes a solution to these limitations. It involves estimating a model using significant harmonics at frequencies that provide an explanation for the highest level of variance of the variable:

$$y_t = \mu + \sum_{i=1}^{p} \alpha_i \sin(\omega_i t) + \sum_{j=1}^{q} \delta_j \cos(\omega_j t) + \varepsilon_t \tag{3}$$

where $\omega_i,\ \omega_j \in \left(0,\ \pi\right]$.

In this model, the number of parameters is lower than the number of observations, because it only includes significant harmonics. This approach allows the precise estimation of the frequencies and amplitudes of the cycles that have the greatest contribution to the variance of the time series.

In order to estimate the parameters of the above model, the following procedure is applied:

1. *Estimation of the Fourier series coefficients*. In this way, the contribution of each harmonic to the total variance of the series is determined.

2. *Estimation of the parameters of the model using a constant and the harmonic with the greatest contribution to the total variance*. To estimate the parameters, we determine the frequency of the harmonic in the interval $(\omega_j - \pi/T,\ \omega_j + \pi/T]$, which minimizes the sum of squared residuals.

3. *Estimation of the parameters of the model using the constant and harmonic component determined in the previous step together with the harmonic with the second greatest contribution to the total variance*. To estimate the parameters, we determine the frequency of the harmonic in the interval $(\omega_j - \pi/T, \omega_j + \pi/T]$ with the second greatest contribution, i.e. minimizes the sum of squared residuals. If there are insignificant parameters in the model, they are removed.

4. *Further steps*. New harmonics enter the model using a procedure analogous to the previous step. The procedure ends when the model cannot be improved by the introduction of a significant harmonic.

The advantage of the proposed approach is that the model is not limited to the harmonics with discrete frequencies resulting from the length of the time series. Using the traditional approach, any variation in the time series data that is in reality due to cycles with frequencies other than these particular values is attributed to one of these discrete frequencies. Using the proposed modification, the frequencies of cycles can be precisely calculated. On the other hand, this method for determining the frequency of harmonics means that they are not orthogonal and, in consequence, they do not explain any specific part of the variance of the series.

3.2. Periodogram analysis

The primary tool for spectral analysis is the power spectrum of the process, which is a function that attributes portions of variance in the process to various frequencies. For a covariance-stationary process Y_t with absolutely summable autocovariances, the power spectrum at frequency ω can be expressed as:

$$s_Y(\omega) = \frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \gamma_j \mathrm{e}^{-i\omega j} \tag{4}$$

The obvious estimator of the power spectrum is the sample periodogram which for a given ω is calculated as

$$\hat{s}_y(\omega) = \frac{1}{2\pi} \sum_{j=-T+1}^{T-1} \hat{\gamma}_j \mathrm{e}^{-i\omega j} = \frac{1}{2\pi} \left[\hat{\gamma}_0 + 2\sum_{j=1}^{T-1} \hat{\gamma}_j \cos(\omega j) \right] \tag{5}$$

The periodogram considers discrete frequencies in the interval [0, π]. Although it is an asymptotically unbiased estimator of a population spectrum, it is not a consistent one. Although the periodogram is itself an inconsistent estimator, there are procedures for smoothing this factor.

Smoothing a periodogram is carried out in both the time and frequency dimensions. In the former, the population spectrum might be estimated by a weighted average of discrete values of the periodogram in a neighbourhood around ω_j where the weights depend on the distance to ω_j. Thus,

$$\hat{s}_Y\left(\omega_j\right) = \sum_{m=-h}^{h} \kappa_m \hat{s}_y\left(\omega_{j+m}\right) \tag{6}$$

where κ_m are weights and h is a bandwidth parameter indicating the number of frequencies useful for estimating the power spectrum. The weights sum to one:

$$\sum_{m=-h}^{h} \kappa_m = 1 \tag{7}$$

A possible approach to determining the weights is to assume that they are proportional to $h + 1 - |m|$ as in Hamilton [18]

$$\kappa_m = \frac{h+1-|m|}{\left(h+1\right)^2}, \; m = -h, -h+1, \ldots, h-1, h \tag{8}$$

Another type of estimation procedure consists of taking the Fourier transform of the truncated weighted sample autocovariance function. Thus,

$$\hat{s}_Y\left(\omega\right) = \frac{1}{2\pi}\left[\hat{\gamma}_0 + 2\sum_{j=1}^{q} \kappa_j \hat{\gamma}_j \cos\left(\omega j\right)\right] \tag{9}$$

where κ_j are weights and q is a truncation point which together define the lag window. Two popular set of weights are given by Barlett [3] and Parzen [30]. The Barlett window is as follows:

$$\kappa_j = \begin{cases} 1 - \dfrac{|j|}{q+1} \text{ for } |j| \le q \\ 0 \text{ for } |j| > q \end{cases} \tag{10}$$

and the Parzen window is calculated as:

$$\kappa_j = \begin{cases} 1-6\left(\dfrac{j}{q}\right)^2 + 6\left(|j|/q\right)^3 \text{ for } |j| \le q/2 \\ 2\left(1-\dfrac{j}{q}\right)^3 \text{ for } q/2 < |j| \le q \\ 0 \text{ for } |j| > q \end{cases} \tag{11}$$

The choice of the bandwidth parameter *h* and truncation point *q* is difficult and little advice is available in the literature. It is crucial to find the optimum balance between bias and variance. This choice can be made arbitrarily, so as to obtain the most reliable assessment, or using statistical criteria. A useful rough guide is to choose *q* to be equal to int(2√*T*), which ensures the fact that as $N \to \infty$, so does $M \to \infty$, but in such a way that $M/N \to \infty$ [11].

3.3. Cross-spectral analysis

The extension of one-dimensional spectral analysis to a two-dimensional situation is represented by cross-spectral analysis. By analogy with the equation for the power spectrum, the cross-spectrum of a discrete bivariate process is defined as the Fourier transform of the cross-covariance function:

$$s_{YX}(\omega) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty} \gamma_{YX}^{(k)} \mathrm{e}^{-i\omega k} = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty} \gamma_{YX}^{(k)} \left[\cos(\omega k) - i\sin(\omega k)\right] \tag{12}$$

where $\gamma_{YX}^{(k)} = \mathrm{cov}(X_t, Y_{t-k})$ is the cross-covariance of two discrete time series *X* and *Y* lagged by *k* periods. The cross spectrum can be written in terms of its real and imaginary components as

$$s_{YX}(\omega) = c_{YX}(\omega) + iq_{YX}(\omega) \tag{13}$$

The real part of the cross spectrum, called the co-spectrum, is given by

$$c_{YX}(\omega) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty} \gamma_{YX}^{(k)} \cos(\omega k) \tag{14}$$

and the imaginary part, called the quadrature spectrum, is in the form

$$q_{YX}(\omega) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \gamma_{YX}^{(k)} \sin(\omega k) \tag{15}$$

The co-spectrum and quadrature spectra are used to determine the following cross--spectral statistics: coherence, gain and phase shift. All these measures are functions of frequency and thus allow us to study the relationship between the variables at various frequencies. Coherence, gain and phase shift are calculated as follows [32, 18]:

$$h_{YX}(\omega) = \frac{\left[c_{YX}(\omega)\right]^2 + \left[q_{YX}(\omega)\right]^2}{s_Y(\omega)\, s_X(\omega)} \tag{16}$$

$$g_{YX}(\omega) = \frac{\left[\left[c_{YX}(\omega)\right]^2 + \left[q_{YX}(\omega)\right]^2\right]^{1/2}}{s_X(\omega)} \tag{17}$$

$$\phi_{YX}(\omega) = \left(\tan\left[-\frac{q_{YX}(\omega)}{c_{YX}(\omega)}\right]\right)^{-1} \tag{18}$$

The coherence is the square of the linear correlation coefficient between the two series at the frequency ω. It is interpreted as the coefficient of determination for the linear regression of one variable with respect to the other. Thus, coherence measures how much of the variance in one variable is explained by the variance of the other variable at the frequency ω. The gain is the absolute value of the regression coefficient for the process Y with respect to process X at the frequency ω. It provides information about the relationships between the amplitudes of the two processes at a given frequency. The phase shift indicates the shift between two series at the frequency ω.

An important property of the gain and phase shift is that they are more informative for high values of coherence [38]. This is due to the fact that the error in estimation for these measures is inversely proportional to the square of the coherence. This means that a decline in the value of the coherence increases the estimation error for the gain and phase shift. Thus, a sufficiently high level of coherence for a given frequency is needed to accurately estimate the relationships between amplitudes and the time shifts between processes.

The estimator of the cross-spectrum is the sample cross periodogram from X to Y at the frequency ω and this is calculated as:

$$\hat{s}_{yx}(\omega) = \frac{1}{2\pi} \sum_{k=-T+1}^{T-1} \hat{\gamma}_{YX}^{(k)} e^{-i\omega k} = \frac{1}{2\pi} \sum_{k=-T+1}^{T-1} \hat{\gamma}_{YX}^{(k)} \left[\cos(\omega k) - i \sin(\omega k)\right] \tag{19}$$

Thus, a sample co-spectrum and quadrature spectrum are estimated as follows:

$$\hat{c}_{yx}(\omega) = \frac{1}{2\pi} \sum_{k=-T+1}^{T-1} \hat{\gamma}_{YX}^{(k)} \cos(\omega k) \tag{20}$$

$$\hat{q}_{yx}(\omega) = \frac{1}{2\pi} \sum_{k=-T+1}^{T-1} \hat{\gamma}_{YX}^{(k)} \sin(\omega k) \tag{21}$$

Estimation of the cross-spectral characteristics requires smoothing of the power spectrum, co-spectrum and quadrature spectrum. The method of smoothing is analogous to that used in one-dimensional cases.

4. Results

The analysis of the cyclical behaviour of GDP was conducted for Poland and current EU member countries with the highest shares in foreign trade with Poland in the years 1995–2015. The six countries with the highest share of imports and exports in total imports and exports are Germany (27.8%), Italy (6.2%), France (5.6%), the Netherlands (4.6%), the United Kingdom (4.5%) and the Czech Republic (4.2%). The methods presented in the previous section were used to analyse the periodicity of a time series of GDP.

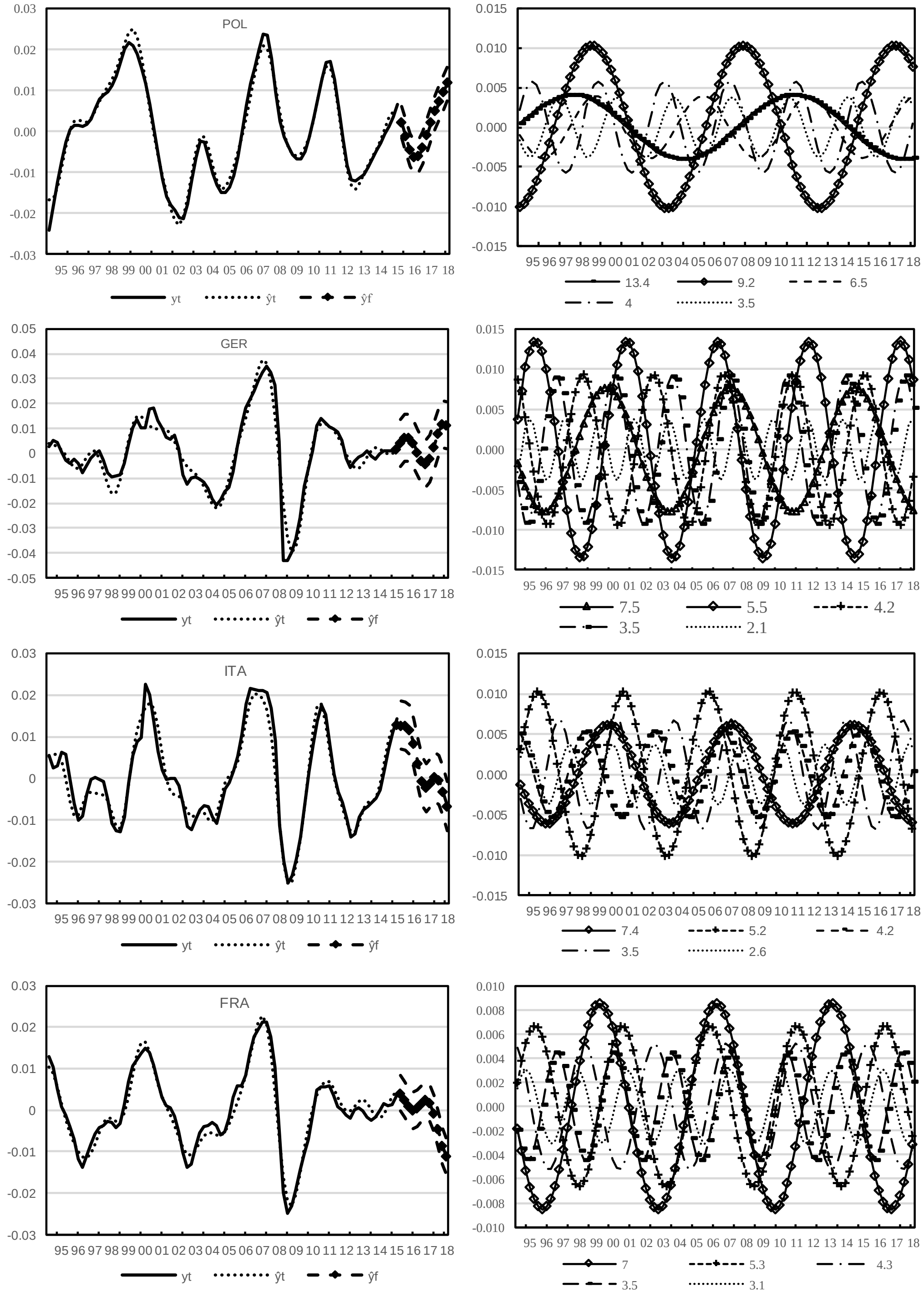
The study deals with quarterly time series data of GDP for Poland, Germany, Italy, France, the Netherlands, the United Kingdom and the Czech Republic for the period 1995 (Q1)–2015 (Q4). Initially, the raw GDP values are logarithmised and deseasonalised using the Tramo–Seats procedure. Then the Hodrick–Prescott filter is applied to extract cyclical components from the series.

The models with harmonics describing the cyclical components were constructed using the method proposed by the author. The introduction of harmonic components into the models was performed using a stepwise procedure with a significance level of 5%. The obtained models were used to determine forecasts for GDP in 2016–2018, together with 95% confidence intervals. The estimates of the parameters in the models for GDP are reported in Table 1. The cyclical components of GDP, theoretical values and forecasts, together with the corresponding cyclical harmonics, are shown in Fig. 1. The left panel of the chart shows the cyclical components of GDP, theoretical estimates obtained from the models and forecasts for cyclical components. The right panel of the chart visualizes the distribution of the five harmonics with the highest amplitudes.

Table 1. Estimates of parameters in models with cyclical components for GDP in Poland, Germany, Italy, France, the Netherlands, United Kingdom and Czech Republic

Poland			Germany		
Cyclical component	Cycle length [years]	Parameter estimate	Cyclical component	Cycle length [years]	Parameter estimate
sin(0.037π)	13.4	0.004[b]	sin(0.067π)	7.5	–0.008[b]
cos(0.055π)	9.2	–0.01[b]	sin(0.091π)	5.5	0.014[b]
sin(0.077π)	6.5	–0.004[b]	cos(0.119π)	4.2	0.009[b]
sin(0.125π)	4	0.006[b]	sin(0.144π)	3.5	–0.009[b]
cos(0.135π)	3.7	–0.003[a]	sin(0.179π)	2.8	–0.003[b]
sin(0.143π)	3.5	–0.004[b]	sin(0.194π)	2.6	0.003[b]
sin(0.255π)	2	–0.002[b]	cos(0.238π)	2.1	–0.004[b]
Italy			France		
Cyclical component	Cycle length [years]	Parameter estimate	Cyclical component	Cycle length [years]	Parameter estimate
sin(0.067π)	7.4	–0.006[b]	cos(0.069π)	7.3	0.003[b]
sin(0.096π)	5.2	0.01[b]	sin(0.072π)	7	–0.009[b]
cos(0.12π)	4.2	0.005[b]	sin(0.095π)	5.3	0.007[b]
sin(0.144π)	3.5	–0.007[b]	cos(0.117π)	4.3	0.005[b]
sin(0.166π)	3	0.002[b]	sin(0.144π)	3.5	–0.005[b]
sin(0.194π)	2.6	0.004[b]	sin(0.163π)	3.1	0.003[b]
sin(0.357π)	1.4	–0.002[b]	sin(0.197π)	2.5	0.002[b]
Netherlands			United Kingdom		
Cyclical component	Cycle length [years]	Parameter estimate	Cyclical component	Cycle length [years]	Parameter estimate
sin(0.043π)	11.7	0.003[b]	cos(0.044π)	11.4	0.008[b]
sin(0.065π)	7.7	–0.012[b]	sin(0.071π)	7.1	–0.008[b]
cos(0.083π)	6	0.003[b]	sin(0.092π)	5.5	0.004[b]
sin(0.093π)	5.4	0.005[b]	cos(0.118π)	4.2	0.006[b]
cos(0.119π)	4.2	0.005[b]	sin(0.145π)	3.4	–0.006[b]
sin(0.142π)	3.5	–0.008[b]	cos(0.188π)	2.7	0.003[b]
sin(0.194π)	2.6	0.003[b]	sin(0.216π)	2.3	–0.003[b]
Czech Republic					
Cyclical component	Cycle length [years]	Parameter estimate	Cyclical component	Cycle length [years]	Parameter estimate
sin(0.05π)	10.1	0.01[b]	sin(0.142π)	3.5	–0.012[b]
sin(0.066π)	7.6	–0.005[a]	cos(0.164π)	3	–0.004[b]
cos(0.074π)	6.8	0.004[a]	sin(0.213π)	2.4	–0.006[b]
sin(0.095π)	5.3	0.013[b]			

[a]Statistical significance at 1% level.
[b]Statistical significance at 0.1% level. Source: Author's calculations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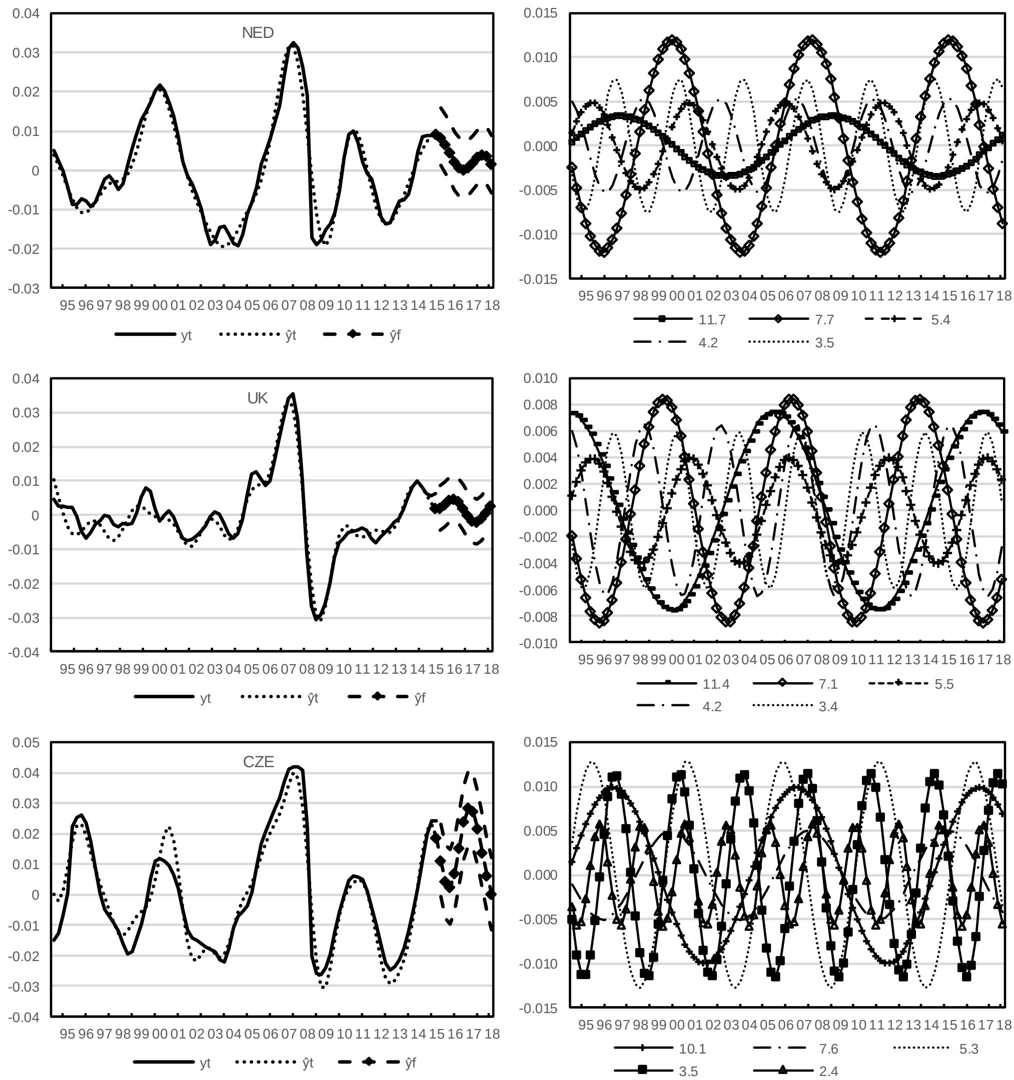

Fig. 1. Cyclical components of GDP, theoretical estimates, forecasts and cyclical harmonics for Poland (POL), Germany (GER), Italy (ITA), France (FRA), the Netherlands (NED), United Kingdom (UK), and Czech Republic (CZE); left: *yt* – empirical values, *ŷt* – theoretical estimates obtained from models based on harmonics, *yf* – forecast (95% confidence interval); right: the numbers in the legend are the lengths of harmonics (in years). Source: author's calculations

In Poland, the greatest amplitudes belong to 9.2 and 4 year cycles. Only the United Kingdom and the Czech Republic have cycles with the greatest amplitude which are of similar length (11.4 and 10.1 years, respectively) to the longer Polish cycle.

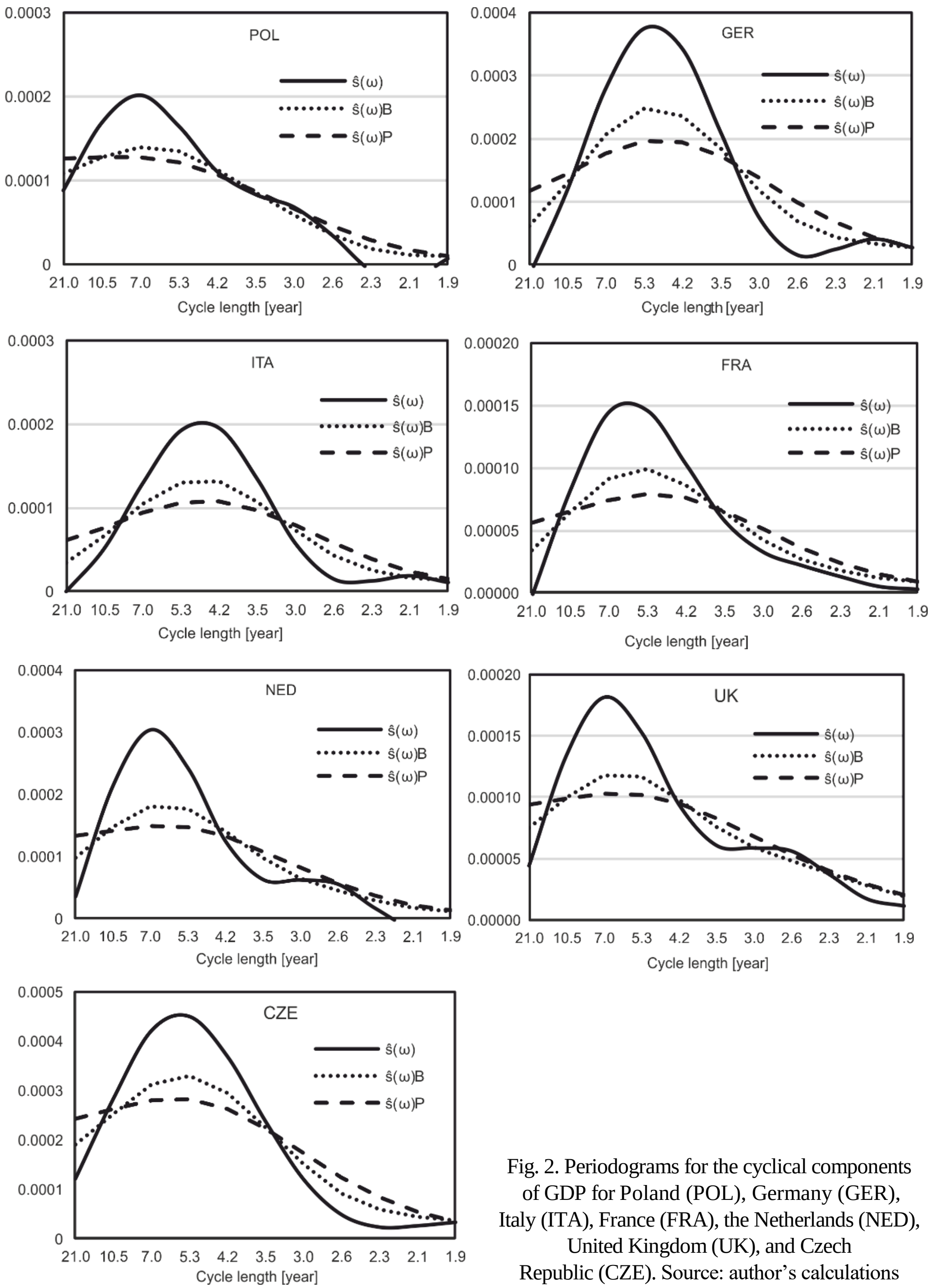

Fig. 2. Periodograms for the cyclical components of GDP for Poland (POL), Germany (GER), Italy (ITA), France (FRA), the Netherlands (NED), United Kingdom (UK), and Czech Republic (CZE). Source: author's calculations

In the other countries, the longest dominant cycles have the length of 7–8 years (France, the Netherlands) or about 5 years (Germany, Italy). This means that in the case of Poland, the share of imports and exports in total imports and exports with major trading partners has no impact on the synchronization of the longest cycles. The 4-year cycles also have high amplitudes in the case of Germany, Italy, the Netherlands and the Czech Republic. These are countries with a wide range of shares of trade with Poland. Periods of growth in long-term cycles coincide in Poland and four other countries (Italy, the Netherlands, the United Kingdom, the Czech Republic). In Germany and France, the economic situation is stable.

Forecasts for the cyclical component of GDP in Poland indicate that the growth trend will continue in the future. In contrast to Poland, in Germany and the Czech Republic the economic situation is predicted to be relatively stable and in the other countries a change in the trend of the economic and financial situation is forecasted. This downturn is predicted due to a transition to the decline phase in the dominant harmonics.

In order to verify the agreed levels of volatility, periodograms for the cyclical component of GDP were also plotted (Fig. 2). Periodograms were smoothed in the frequency domain using a Barlett and Parzen window, as proposed by Chatfield.

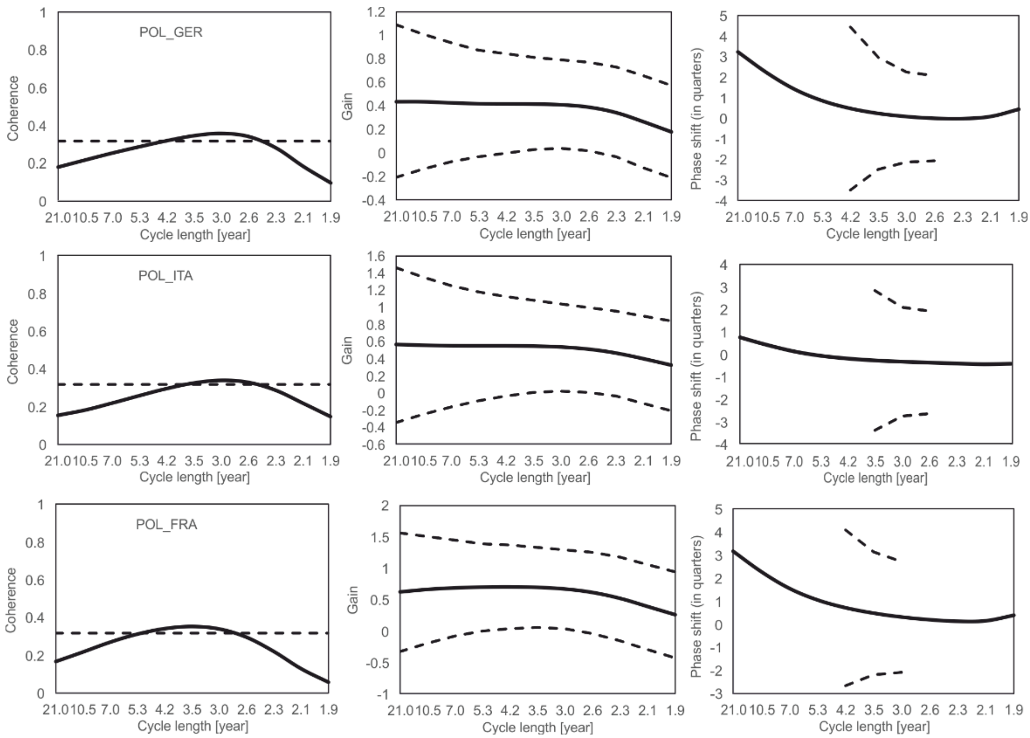

Fig. 3. Coherence data, gains and phase shifts for the cyclical components of GDP for Poland (POL) and its main trading partners such as: Germany (GER), Italy (ITA) and France (FRA)

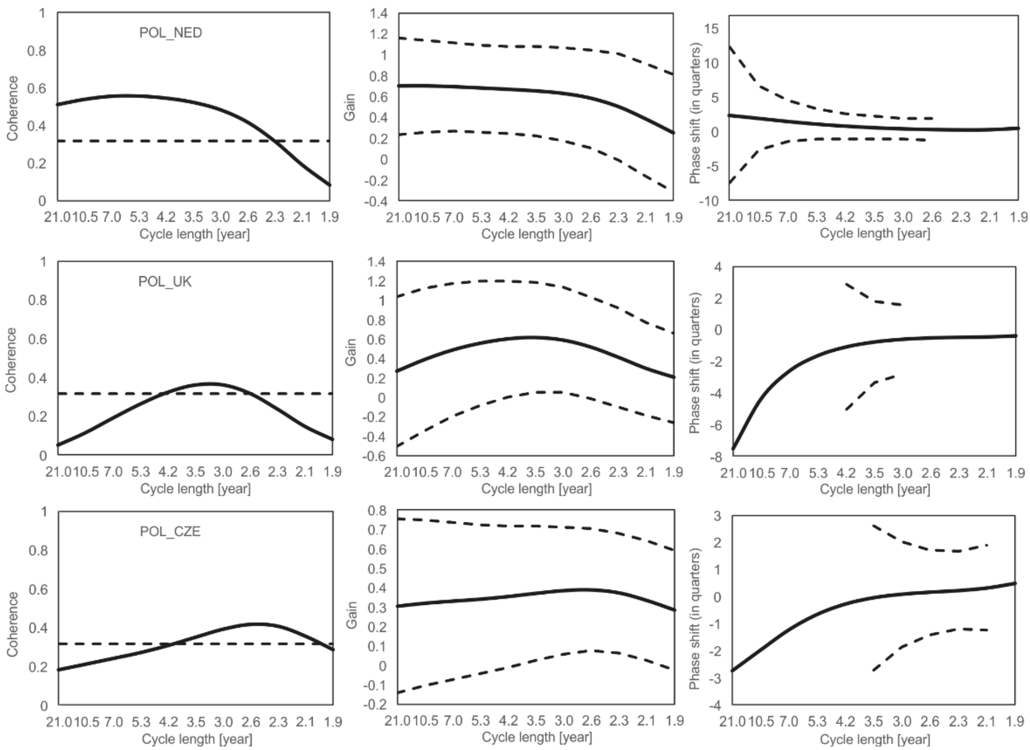

Fig. 4. Coherence data, gains and phase shifts for the cyclical components of GDP for Poland (POL) and its main trading partners such as: the Netherlands (NED), United Kingdom (UK) and Czech Republic (CZE)

Periodograms and smoothed periodograms confirm that in Poland, the United Kingdom and the Czech Republic longer cycles play a crucial role in explaining the variation of GDP. The significance of these cycles was confirmed by both harmonic analysis and periodograms. Application of the models based on harmonics proposed by the author enabled precise estimation of the frequencies and lengths of cycles.

Next, cross-spectral statistics (coherence, gain and phase shift) were estimated for Poland and its major trading partners. For the purposes of statistical estimation, the Parzen window was applied with a truncation point of 18. The estimates of cross-spectral statistics are shown in Fig. 3. The dotted line on the graphs of coherence indicates the critical value at a 5% significance level. The graphs of the gain and phase shift include confidence intervals at a 95% confidence level.

The coherence measure for the GDP of Poland and each of its major trading countries has a higher value than the critical one for cycles of 3–4 year length. The low level of coherence between Poland and its three largest trading partners: Germany (27.8% share in trade), Italy (6.2%) and France (5.6%) indicates a low level of synchronization of the business cycles. The situation is also similar for the Czech Republic (4.2%) and the United Kingdom (4.5%). Only in the case of the Netherlands (4.6%), cycles of all lengths are highly related to Polish cycles, but 5–7 year cycles are the most synchronized. This demonstrates the weak relationship between the cyclical fluctuations of GDP in Poland and its trading partners. A comparison of shares in trade and coherence for pairs of countries which include Poland does not confirm the positive impact of trade on business cycle synchronization in Poland and other EU countries.

A level of coherence which is mostly below the critical value means that the gain and phase shift for Poland and its trading partners are not informative, due to relatively high errors in estimation. The values of the gain indicate that the amplitudes of Poland's 3–4 year cycles are nearly half those of other countries. The phase shift between Poland's 3–4 year cycles and those of other countries shows that the Polish cycles are delayed with respect to German cycles (by about 1 month), to French cycles (by about 1.5 months), to Dutch cycles (by about 2 months), but are in front of Italian cycles (by about 1 month) and British cycles (by about 2.5 months). The Polish and Czech cycles almost coincide. The 5–7 year Polish cycles are delayed with respect to Dutch cycles, with which they are most strongly correlated, by about 4 months.

5. Conclusions

The paper has focused on studying the properties of business cycles in Poland and its major trading partners. The author has proposed a method for estimating the amplitudes and frequencies of cycles by modifying the Fourier analysis. It enables measurement of the frequencies of cycles with higher precision. The study confirmed the dominant role of cycles of 9 years or longer, as well as 4 year cycles, in explaining the variation in GDP in Poland, the United Kingdom and the Czech Republic. Periods of simultaneous growth in long-term cycles can be observed in these countries, together with Italy and the Netherlands. Forecasts of cyclical components point to continued growth in the near future only in Poland, stabilization in Germany and the Czech Republic, and economic declines in the other countries.

These conclusions are similar to the findings made by Gradzewicz et al. [17], Skrzypczyński [35] and Skrzypczyńska [33], who report the strongest effect of 7–8 year and 3–4 year cycles on the variation of the analyzed series.

The conclusions on business cycle synchronization are similar to Skrzypczyński [34], who finds that the degree of synchronization of Polish and Eurozone cycles is higher for short cycles (up to 3 years) than for long ones (6–7 years). Konopczak and Marczewski [25] indicate an increase in the interdependency of output between Central and Eastern European Countries and the euro area. In contrast, Poland has experienced a decreasing level of synchronization. The low level of BCS found in this study may negatively influence Polish efforts towards greater economic integration with European

Monetary Union members and full enjoyment of the benefits of accession to the euro area. A high degree of business cycle synchronization between EU countries is a crucial condition for the smooth functioning of the EMU, as it facilitates the coordination of economic policies and, in particular, conducting a common monetary policy.

The synchronization of business cycles of all lengths is only high between Poland and the Netherlands. The Netherlands is the fourth largest trading partner of Poland with a trade share of 4.6%. The level of business cycle synchronization with other countries is low, in particular with Germany (trade share of 27.8%). In the case of Poland, the hypothesis that the main factor affecting the level of synchronization is the level of trade between countries has not been confirmed.

This study enables the assessment of cyclical movements of GDP in Poland and its major trading partners. It has identified significant cycles, predicted future developments and evaluated the degree of business cycle synchronization between these countries. It also allows verification of the impact of trade on business cycle synchronization between Poland and its trading partners.

References

[1] AGUIAR-CONRARIA L., SOARES M.J., *Business cycle synchronization and the euro: a wavelet analysis*, J. Macroecon., 2011, 33, 477–489.

[2] ARTIS M., KROLZIG H. M., TORO J., *The European business cycle*, Oxford Economic Papers, 2004, 56, 1–44.

[3] BARTLETT M.S., *Periodogram analysis and continuous spectra*, Biometrika, 1950, 37 (1–2), 1–16.

[4] BAXTER M., KING R.G., *Measuring business cycles. Approximate band-pass filters for economic time series*, Working Paper No. 5022, National Bureau of Economic Research, Cambridge 1995.

[5] BAXTER M., KOUPARITSAS M.A., *Determinants of business cycle co-movement. A robust analysis*, J. Mon. Econ., 2005, 52 (1), 113–157.

[6] BAXTER M., STOCKMAN A., *Business cycles and exchange rate regime. Some international evidence*, J. Mon. Econ., 1989, 23, 377–400.

[7] BEVERIDGE S., NELSON C.R., *A New approach to decomposition of economic time series into permanent and transitory components with particular attention to measurement of the "Business Cycle"*, J. Mon. Econ., 1981, 7, 151–174.

[8] BRUZDA J., *Business cycle synchronization according to wavelets. The case of Poland and euro zone member countries*, Bank Kredyt, 2011, 42, (3), 5–32.

[9] BURNS A.F., MITCHELL W.C., *Measuring Business Cycles*, National Bureau of Economic Research, New York 1946.

[10] CAMPBELL J.Y., MANKIW N.G., *Permanent and transitory components in macroeconomic fluctuations*, Am. Econ. Rev. (Papers and Proceedings), 1987, 77, 111–117.

[11] CHATFIELD C., *The Analysis of Time Series. An Introduction*, Chapman and Hall, London 1996.

[12] CHRISTIANO L.J., FITZGERALD T.J., *The band pass filter*, Working Paper No. 9906, Federal Reserve Bank of Cleveland, 1999.

[13] CLARK P.K., *The cyclical component of U.S. economic activity*, Quater. J. Econ., 1987, 102, 797–814.

[14] CROWLEY P., LEE J., *Decomposing the co-movement of the business cycle: A time-frequency analysis of growth cycles in the euro area*, Bank of Finland discussion papers, 2005, (12).

[15] FORNI M., HALLIN M., LIPPI M., REICHLIN L., *The generalized dynamic-factor model. Identification and estimation*, Rev. Econ. Stat., 2000, 82 (4), 540–554.
[16] FRANKEL J.A., ROSE A.K., *The endogeneity of the optimum currency area criteria*, Econ. J., 1998, 108 (449), 1009–1025.
[17] GRADZEWICZ M., GROWIEC J., HAGEMEJER J., POPOWSKI P., *Cycle of business in Poland. Conclusions from spectral analysis*, Bank Kredyt, 2010, 41 (5), 41–76 (in Polish).
[18] HAMILTON J.D., *Time Series Analysis*, Princeton University Press, 1994.
[19] HARDING D., PAGAN A., *Synchronization of cycles*, J. Econ., 2006, 132 (1), 59–79.
[20] HARVEY A.C., *Trends and cycles in macroeconomic time series*, J. Business Econ. Stat., 1985, 3, 216–227.
[21] HARVEY A.C., *Forecasting, Structural Time Series Models and the Kalman Filter*, Cambridge University Press, Cambridge 1989.
[22] HARVEY A.C., *Trends Analysis*, University of Cambridge, Faculty of Economics and Politics, 2000.
[23] HODRICK R., PRESCOTT E., *Post-war U.S. business cycles. An empirical investigation*, Working paper, Carnegie Mellon University, 1980.
[24] HODRICK R.J., PRESCOTT E.C., *Postwar U.S. business cycles. An empirical investigation*, J. Money Credit Bank., 1997, 29 (1), 1–16.
[25] KONOPCZAK K., MARCZEWSKI K., *Why so different from other CEECs – Poland's cyclical divergence from the euro area during the recent financial crisis*, Bank Kredyt, 2011, 42 (2), 7–30 (in Polish).
[26] KYDLAND F., PRESCOTT C., *Business cycles. Real facts and a monetary myth*, Federal Reserve Bank of Minneapolis, Quarterly Review (Spring), 1990, 3–18.
[27] LEE J., *Measuring business cycle co-movements in Europe: Evidence from a dynamic factor model with time-varying parameters*, Econ. Lett., 2012, 115 (3), 438–440.
[28] LUCAS R.E., *Understanding business cycles*, [in:] K. Brunner, A.H. Meltzer (Eds.), *Stabilization of the Domestic and International Economy*, Carnegie-Rochester Conference Series on Public Policy 5, North Holland, Amsterdam 1977, 7–29.
[29] NELSON C.R., PLOSSER C.I., *Trends and random walks in macroeconomic time series. Some evidence and implications*, J. Monetary Econ., 1982, 10, 139–162.
[30] PARZEN E., *Mathematical considerations in the estimation of spectra. Comments on the discussion of Messers, Tukey, and Goodman*, Technometrics, 1961, 3, 167–190, 232–234.
[31] PRIESTLEY M.B., *Spectral Analysis and Time Series*, Vols. 1 and 2, Academic Press, London 1981.
[32] SARGENT T.J., *Macroeconomic Theory*, 2nd Ed., Academic Press, London 1987.
[33] SKRZYPCZYŃSKA M., *Measurement of the business cycle in Poland. Comparative analysis*, Bank Kredyt, 2011, 42 (4), 31–54 (in Polish).
[34] SKRZYPCZYŃSKI P., *Fluctuations in economic activity in Poland and in the euro zone*, Mater. Studia 227, Narodowy Bank Polski, Warsaw 2008 (in Polish).
[35] SKRZYPCZYŃSKI P., *Spectral methods in the analysis of the business cycle*, Mater. Studia 252, Narodowy Bank Polski, Warsaw 2010 (in Polish).
[36] STOCK J.H., WATSON M.W., *Variable trends in economic time series*, J. Econ. Persp., 1988, 2, 147–174.
[37] STOCK J.H., WATSON M.W., *Business cycle properties of selected US economic time series 1959–1988*, NBER working paper 3376, 1990.
[38] WARNER R.M., *Spectral Analysis of Time Series Data*, The Guilford Press, New York 1998.
[39] WATSON M.W., *Univariate detrending methods with stochastics trends*, J. Mon. Econ., 1986, 18 (1), 49–75.
[40] ZARNOWITZ V., OZYILDIRIM A., *Time series decomposition and measurement of business cycles, trends and growth cycles*, Economics Program Working Paper Series, The Conference Board, New York 2001.

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