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Grażyna PETRICZEK[1]
Jan STUDZIŃSKI[1]
Lucyna BOGDAN[1]

MODELLING THE STEADY STATE OF SEWAGE NETWORKS AS A SUPPORT TOOL FOR THEIR PLANNING AND ANALYSIS

Fundamental questions connected with the modelling of communal sewage networks have been considered and formulas used to model the functioning of the basic network have been analyzed. The problem described concerns gravitational sewage networks divided by nodes into branches and sectors. Simulation of the steady state functioning of sewage networks is commonly carried out on the basis of nomograms in the form of charts, in which the relations between network parameters like channel diameters, flow rates, hydraulic slopes and flow velocities are described. In traditional design, the values of such parameters are simply read from such nomogram chart tables. Another way of simulating the functioning of a network is the use of professional software, like SWMM, that models sewage flows along the channels by means of differential equations de-scribing the movement of fluids. In both approaches, the user is a mechanical operator of a "black box" procedure. In this paper, another way of simulating the functioning of sewage net-works has been presented. Numerical solutions of nonlinear equations describing the physical phenomena of sewage flows are applied and explained. The presented algorithms were developed to model the steady state of a sewage network enabling a quick analysis of the network parameters and the possibility of fast, simple and comprehensible network modeling and design.

Keywords: *hydraulic formulas for describing sewage networks, mathematical modeling, optimization and design of sewage networks*

1. Introduction

Modelling and design of communal sewage networks is a difficult problem because of the complexity of the mathematical equations used to describe precisely wastewater flows in network channels and also because of the variety of networks. In the case of

[1]Systems Research Institute, Polish Academy of Sciences, 01-447 Warsaw, ul. Newelska 6, Poland, e-mail addresses: petricz@isbpan.waw.pl, studzins@ibspan.waw.pl, bogdan@ibspan.waw.pl

drinking water networks which are pressure systems, their main parameters are water flows and pressures, depending on both the diameters of the pipes and water pressures produced by pump stations within the network. Contrary to this, wastewater networks are gravitational and their main hydraulic parameters are sewage flows and the depth in the channels. The factors determining these values are the channel diameters, slopes and profiles. *A* classic approach of designing sewage systems consists of using nomograms, which are diagrams illustrating relations between the channel diameters, slopes and depths, as well as the sewage flow intensities and velocities. Appropriate values of these parameters are read from nomograms which are based on calculations made using formulas commonly applied to modelling sewage networks (equations of Chezy, Colebrooke–White and of Manning) [1, 2, 4, 6, 14].

A more advanced approach to modelling communal sewage systems involves the use of hydraulic models of wastewater networks such as the SWMM software developed by the Environmental Protection Agency, which however requires some knowledge of and skills in informatics [9]. *A* classical approach to modelling sewage systems is very mechanical and approaches like SWMM are very complicated.

In this paper, an indirect approach to calculate the hydraulic parameters of wastewater networks is proposed. An algorithm for finding a simple numerical solution of the nonlinear equations resulting from the main hydraulic formulas and rules describing a network have been presented. This approach enables fast analyses of main network parameters, i.e. of channel depths and sewage flow velocities. It also enables fast and simple simulation of the sewage system investigated. The algorithms presented have been used to simulate and design an example wastewater network of a sanitary type. Changing the values of the intensities of sewage inflows into the network through chosen network nodes, one can simply calculate new values of the depths and flow velocities of the sewage passing through the channels.

2. Basic assumptions

In general, the following kinds of sewage can be distinguished: housekeeping (sanitary) sewage, industrial sewage, rain wastewater, drainage sewage and groundwater. The following sewage networks can be distinguished depending on the kind of wastewater transported [4]:

- rainwater network,
- housekeeping network,
- combined network.

In a combined network, various kinds of wastewater flow through common channels. At present, most commonly separated sewage systems are used, in which the rainwater and housekeeping networks are not interconnected. In this paper, the following basic assumptions are made [3]:

- Only housekeeping or combined sewage networks are considered, divided by nodes into branches and segments.
- Nodes are points at which several segments/branches of the network intersect, the network parameters change, or sewage flows into the network (sink basins, rain inlets, connecting basins).
- Segments are characterized by constant hydraulic parameters like shape, channel size, channel slope and the roughness coefficient. Sewage flow into the network is realized pointwise at network nodes. The sewage flow in channels is stable, uniform and continuous.
- In the connecting nodes, the flow balance equations and the conditions of level consistency are satisfied.

The modelling and design of sewage networks means solving the following tasks:

- *Modelling the behaviour of the network for known cross-sections and channel slopes.* The calculation of channel depths and flow velocities depending on the sewage flow rates must be done. This calculation is carried out for each net segment in turn, using the flow values obtained earlier.
- *Designing new segments of the network.* This concerns the case when new segments of the network are to be added to the existing ones. In this situation, the diameters and slopes for the new channels must be chosen. It is assumed that the sewage inflows are known.

3. Fundamental hydraulic equations for modelling the steady state of a sewage network

It is assumed that all the segment parameters such as shape, channel dimension, slope or roughness are constant. Based on these assumptions, all the following relations describe the steady state. According to Manning's formula [4], the flow velocity of sewage depends on the hydraulic radius R and the radius R depends on the depth H. Manning's formula for the velocity v is of the form:

$$v = \frac{1}{n} R^{2/3} J^{1/2} \tag{1}$$

where: R – hydraulic radius [m], J – channel slope [%], n – roughness coefficient (s·m$^{1/3}$], v – flow velocity [m/s].

The equations presented in the following concern channels with circular cross-sections. From Manning's formula, taking into account the geometry of the channels, we obtain the following equations [3]:

For $H \leq 0.5d$:

$$A = \frac{d^2}{8}(\varphi - \sin\varphi) \tag{2a}$$

$$\varphi = 2\arccos\left(1 - 2\frac{H}{d}\right) \tag{2b}$$

$$R = \frac{1}{4}d\left(1 - \frac{\sin\varphi}{\varphi}\right) \tag{2c}$$

For $H > 0.5d$:

$$A = \frac{\pi d^2}{4} - \frac{d^2}{8}(\varphi - \sin\varphi) \tag{3a}$$

$$\varphi = 2\arccos\left(2\frac{H}{d} - 1\right) \tag{3b}$$

$$R = \frac{d}{4} + \frac{d}{8}\frac{\sin\varphi}{\pi - 0.5\varphi} \tag{3c}$$

where: A – cross-sectional area [m^2], H – depth [m], r – radius of the circular channel [m], φ – central angle [rad], d – interior diameter of the channel [m].

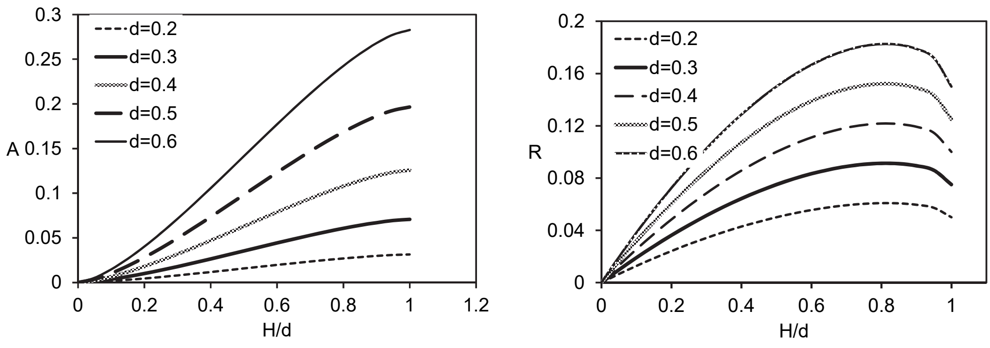

Fig. 1. Dependences of the cross section area A [m^2] (left) and of the hydraulic radius R [m] (right) on the relative depth H/d for various diameters d [m]

From the above expressions, one can see that, for circular channels, the cross-sectional area *A* and hydraulic radius *R* depend on the channel depth *H* and, as a result, the sewage flow velocity *v* depends on the channel depth *H* when the channel slope *J* and diameter *d* are given. We define the relative depth to be *H*/*d*. In Fig. 1, the dependences of *A* and *R* on *H*/*d* are shown for various values of *d*. The figure shows that the cross-sectional area *A* increases monotonically in the relative depth *H*/*d*. The increase in cross-sectional area is fastest for intermediate values of the relative depth. The greatest value of *A* occurs when the channel is full and equals $\pi d^2/4$. The hydraulic radius *R* increases from zero and achieves its maximum at a relative depth of 81.3% and then decreases to a value equal to half of the channel height. When the channel is completely or half full, then the value of the radius is *d*/4. For greater diameters *d*, the hydraulic radius also increases but the shape of the curve does not depend on *d*.

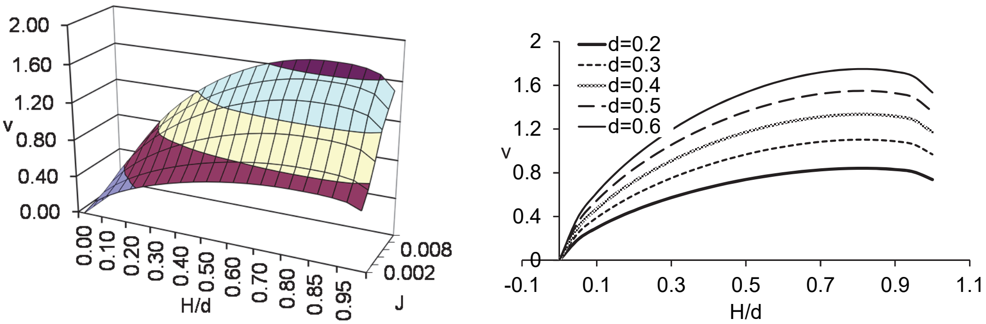

Fig. 2. Dependences of the flow velocity *v* [m/s] on the relative depth *H*/*d* and channel slope *J* [%] for the roughness coefficient *n* = 0.013 and the channel diameter *d* = 0.3 m (left), and of *v* on *H*/*d* for *n* = 0.013 and *J* = 0.5% for various values of *d* [m] (right)

The sewage velocity depends on channel parameters such as the diameter, channel slope and the roughness coefficient and also on the relative dept. Cross-sections of the surface from Fig. 2, left obtained by fixing *J* to be constant are presented in Fig. 2 (right). They show that the function describing the velocity *v* depending on the relative depth *H*/*d* has a similar shape to the function describing the hydraulic radius *R*. The sewage velocity increases from zero and achieves its maximum at the relative depth of 81.3% and then decreases to a value equal to the velocity when the relative depth is 50%. The velocity *v* is increasing in the diameter *d*, but the shape of the curves presented does not depend on *d*.

An exemplary cross-section of the surface from Fig. 2, left obtained by fixing the relative depth is shown in Fig. 3, left. This shows that the flow velocity increases monotonically in the channel slope for a given relative depth. Fig. 3, right shows the relation between the flow velocity *v* and the relative depth *H*/*d* for various slope values *J*. One can see from this figure that the channel slope influences the velocity, but does not influence the shape of the velocity curve as a function of the relative depth.

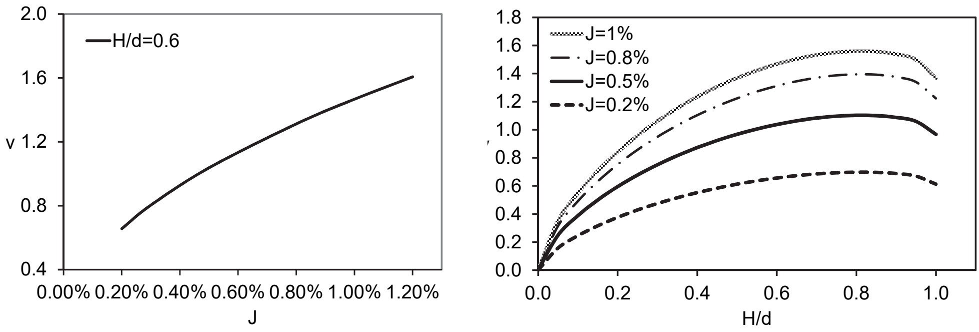

Fig. 3. Dependences of the flow velocity v [m/s] and channel slope J [%] for a given relative depth H/d (left), and of v [m/s] on H/d for the roughness coefficient $n = 0.013$ and the diameter $d = 0.3$ m for various J [%]

4. Algorithms for calculating the steady state of wastewater networks

4.1. An algorithm for modelling channel depths and flow velocities

The algorithm presented requires the following input data for its implementation:

• Type of network – housekeeping sewage network or combined sewage network.

• Structure of the network, i.e. the number of nodes NW, number of segments N, set of nodes $W = \{j = 1, ..., NW\}$, set of segments $U = \{i = 1, ..., N\}$.

• Maximal sewage inflow into the network and the corresponding number of input nodes.

• Set of diameters $\{d_i\}$, set of slopes $\{J_i\}$ and set of the roughness coefficients $\{n_i\}$ for segments $i = 1, ..., N$. Based on the rates of inflow $\{Q_i\}$ for each segment, the goal of the algorithm is to determine the following values:

– the depths in each segment of the wastewater network,

– the flow velocity in each segment of the network.

The algorithm presented below is designed for channels with circular cross-sections. The most important part of the algorithm is the calculation of the depths H_i (or, equivalently, the relative depths $x = H_i/d_i$) and of the flow velocities v_i for each segment of the wastewater network (for the given values of the rates of inflow Q_i at particular nodes). The problem is to solve the nonlinear algebraic equations which are derived from the fundamental relations and hydraulic formulas. At each node, the flow satisfies the balance equation:

$$Q_i = \sum_{j \neq i} Q_j + q_i$$

where: q_i – sewage inflow to the i-th channel [m^3/s], Q_j – sewage outflow from the channels connected to the i-th channel.

I. From Manning's formula, taking into account the geometryof channels, one obtains the following relations with $x = H/d$ [1, 3, 4, 22]:

For $H/d \leq 0.5$:

$$\beta F_1(x) - Q = 0 \tag{4a}$$

$$F_1(x) = \frac{\left(\varphi_1(x) - \sin\left(\varphi_1(x)\right)\right)^{5/3}}{\varphi_1(x)^{2/3}} \tag{4b}$$

$$\varphi_1(x) = 2\arccos\left(1 - 2x\right) \tag{4c}$$

For $H/d > 0.5$:

$$\beta F_2(x) - Q = 0 \tag{5a}$$

$$F_2(x) = 2\frac{\left(\pi - 0.5\varphi_2(x) + 0.5\sin\left(\varphi_2(x)\right)\right)^{5/3}}{\left(\pi - 0.5\varphi_2(x)\right)^{2/3}} \tag{5b}$$

$$\varphi_2(x) = 2\arccos\left(2x - 1\right) \tag{5c}$$

$$\beta = 0.5 \times \frac{1}{n} d^{8/3} \times \left(\frac{1}{4}\right)^{5/3} J^{1/2} \tag{6}$$

where: H – depth [m], φ – central angle [rad], d – interior diameter of the channel [m], J – channel slope [%], n – roughness coefficient [$s/m^{1/3}$], Q – rate of inflow [m^3/s], H/d – relative depth, β – a parameter [m^3/s].

The β parameter in Eq. (6) depends purely on the channel diameter d and channel slope J. Solving Eqs. (4a)–(4c) or (5a)–(5c), we obtain the relative depth H/d as a function of the flow rate Q. These equations are nonlinear and thus in order to solve them, some standard numerical method should be applied. In order to determine the appropriate solution to this set of equations, some conditions on the value of the parameter β and the sewage flow Q must be fulfilled as discussed below.

For fixed values of the network parameters, the solutions of the equation $\beta F(x) - Q = 0$ depend on the sewage flow Q. The function $F(x) = F_1(x) + F_2(x)$ is continuous for $x \in (0; 1]$. When $x = 1$, i.e. the channel is full, then $F = 2\pi$ and for $x = 0.5$ we obtain $F = \pi$. For $x \in (0; 0.8]$, the function $F(x)$ grows monotonically. For $x \in (0.8; 1]$, the function reaches its maximum $F_{\max} = 6.7588$ at $x = 0.9381$. It is decreasing for $x \in (0.9381; 1]$. The following analysis is carried out for $d = 0.6$ m, $J = 0.35\%$ and $n = 0.013$ s/m$^{1/3}$. For the three given values of the network parameters, the solutions of the equation $\beta F(x) - Q = 0$ depending on the sewage flow Q are shown in Fig. 4.

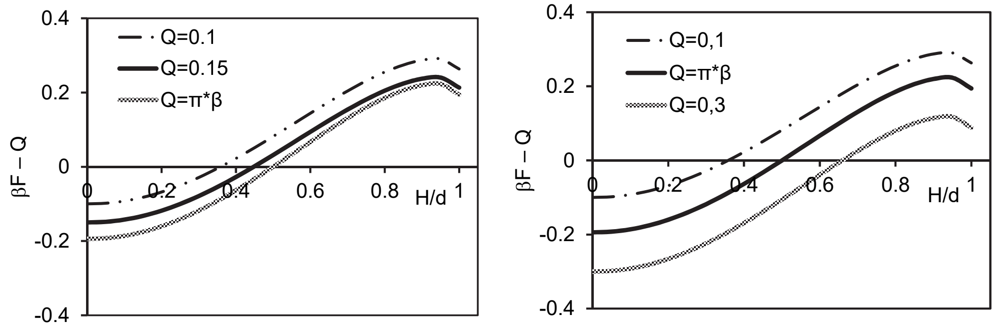

Fig. 4. Plots of the function ($\beta F(x) - Q$) [m^3/s] for various values of Q [m^3/s] in the range $(0; \pi\beta]$ (left) and in the range $(0; 2\pi\beta]$ (right)

The equation $\beta F(x) - Q = 0$ as a function of x has the following roots:

1. When $0 < Q \leq \pi\beta$, then there is one root where $x \in (0; 0.5]$. This inequality defines a range of possible values for the sewage flows Q for fixed channel diameters d and channel slopes J.

2. The equation $\beta F(x) - Q = 0$ has:

- one root where $x \in (0.5; 1)$ when $\pi\beta < Q < 2\pi\beta$,
- two roots where $x \in (0.5; 1]$ when $2\pi\beta \leq Q < 6.7586936\,\beta$,
- when $Q = 2\pi\beta$, the roots are $x_1 = 1$ and $x_2 = 0.81963$.

The case where there are two roots of the equation $\beta F(x) - Q = 0$ is presented for $Q = 2\pi\beta$ and for $Q = 0.41$ m^3/s ($Q < 6.7588\beta$) (see Fig. 5, right). For fixed network parameters, the above relations allow us to decide what are the solutions for a given flow Q and whether the value of Q is not greater than the upper limit of $6{,}7586936\beta$, above which there are no solutions. In such a case, a change in one or both of the parameters d and J must be considered. From the above discussion, we may conclude that the flow Q depends on the parameter β. On the other hand, the parameter β depends on the channel diameter d and channel slope J. The equation describing the relationship between the relative depth and flow in the range $(0; 2\pi\beta)$ has only one solution and that is why this range is highly relevant. In Figure 6, the relationship between the solution to the equation $\beta F(x) - Q = 0$ and flow Q for $n = 0.013$ s/m$^{1/3}$ and $0 < Q < 2\pi\beta$ is illustrated for various d and $J = 1/d$.

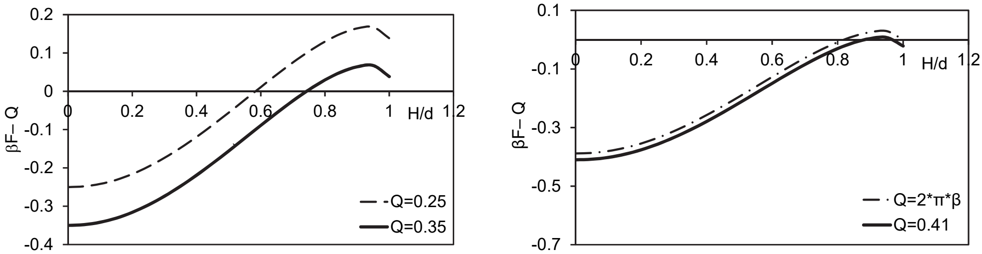

Fig. 5. Plots of the function $(\beta F(x) - Q)$ [m^3/s] for $\pi\beta < Q < 2\pi\beta$ (left) and for $2\pi\beta \le Q < \beta F_{max}$ (right)

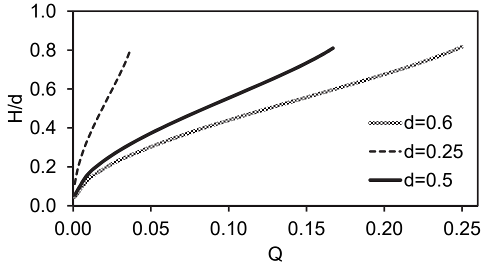

Fig. 6. Dependences of the solution of $(\beta F(x) - Q = 0)$ and flow Q [m^3/s] for various d [m]

II. For the relative depth H/d calculated above, the hydraulic radius R_i can be determined according to the formula:

For $H/d \le 0.5$:

$$R = \frac{1}{4} d \left(1 - \frac{\sin\varphi}{\varphi}\right) \tag{7a}$$

$$\varphi = 2\arccos\left(1 - 2\frac{H}{d}\right) \tag{7b}$$

For $H/d > 0.5$:

$$R = \frac{d}{4}\left(\frac{\pi - 0.5\varphi + 0.5\sin\varphi}{\pi - 0.5\varphi}\right) \tag{8a}$$

$$\varphi = 2\arccos\left(2\frac{H}{d} - 1\right) \tag{8b}$$

III. The flow velocity is to be calculated from Eq. (1).

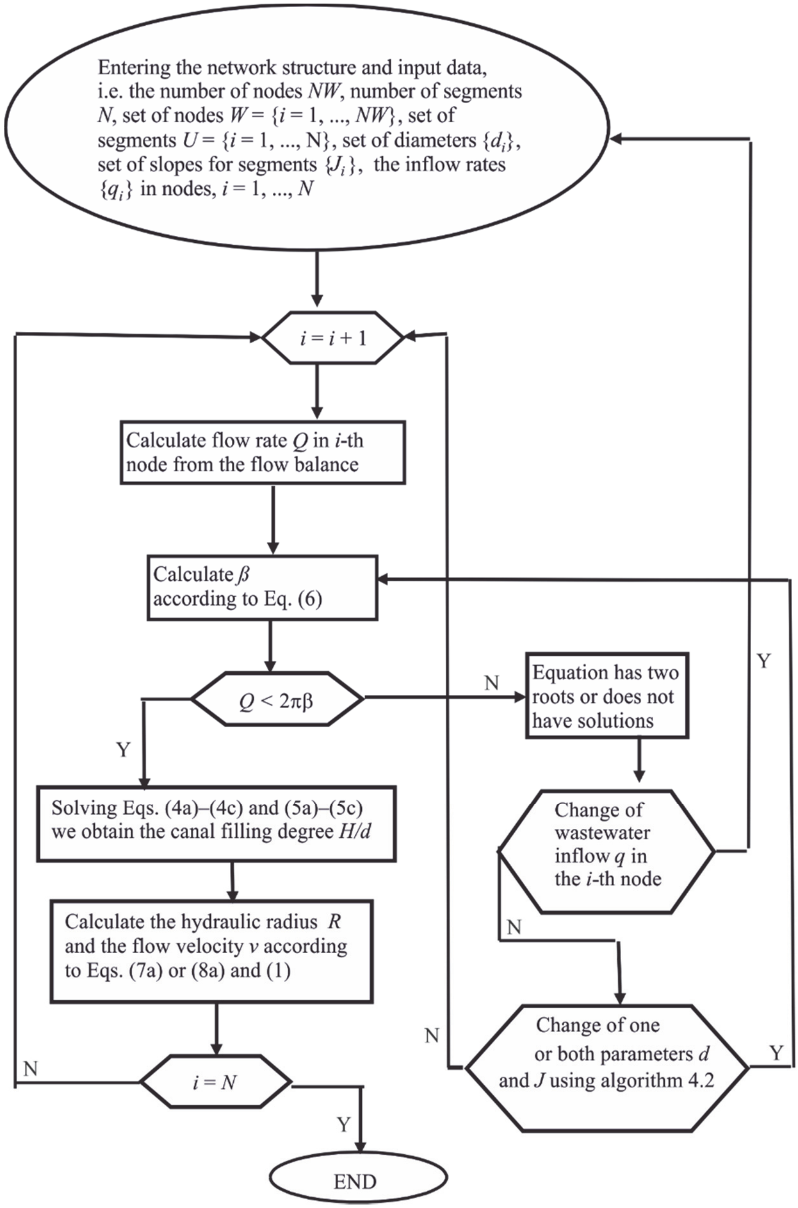

Fig. 7. The algorithm for calculating channel depths and flow velocities

Knowing the network geometry, i.e. the slopes, shapes and diameters of the channels, as well as the wastewater inflows Q_i, one can calculate the depths and flow velocities in each network channel. These calculations are carried out for each network segment in turn, beginning from the one farthest to the wastewater treatment plant and finishing with the nearest one. The algorithm is illustrated in Fig. 7.

The state of the whole network can be calculated once again with the wastewater inflows changed. Under the assumption of constant sewage flows in the network segments, the simulation of the evolution of the sewage system over a sequence of time steps, representing a couple of hours or days, can be executed. In such a case, the change in the wastewater inflows occurring within this time frame must be considered.

4.2. The algorithm for designing channel diameters for given flow values

It follows from the relationships presented above that the flow Q depends on the parameter β, which in turn depends on the channel diameter d and channel slope J. For flows in the range $(0; 2\pi\beta)$, the equation describing the relationship between the channel depth and the sewage flow has only one solution and hence this range is very important for further discussion. The dependence of the parameter β on the channel diameter d and channel slope J is shown in Fig. 8.

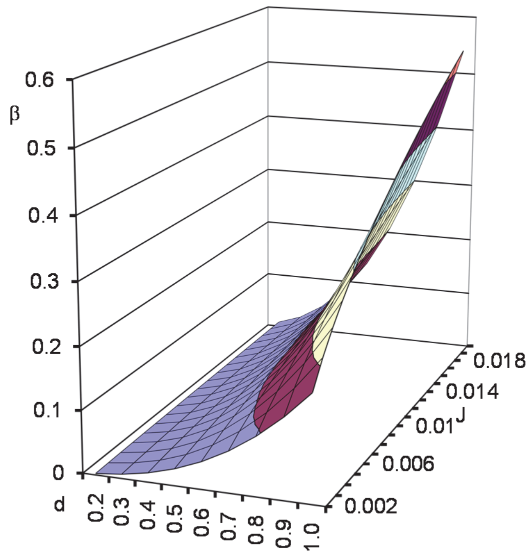

Fig. 8. Dependence of the parameter β [m^3/s], on d [m] and J [%]

In particular, the calculation procedure shown above covers the following cases (see Fig. 9):

• The flow Q exceeds the upper boundary of $6{,}7586936\beta$. In this case, a change in the values of the channel diameters d and slopes J has to be considered.

• Designing new network channels, which means that some new fragments have to be added to the already existing network. This problem consists of choosing the diameters of the new channels and in calculating the channel slopes. It is assumed that the forecasted sewage inflows Q are known in advance.

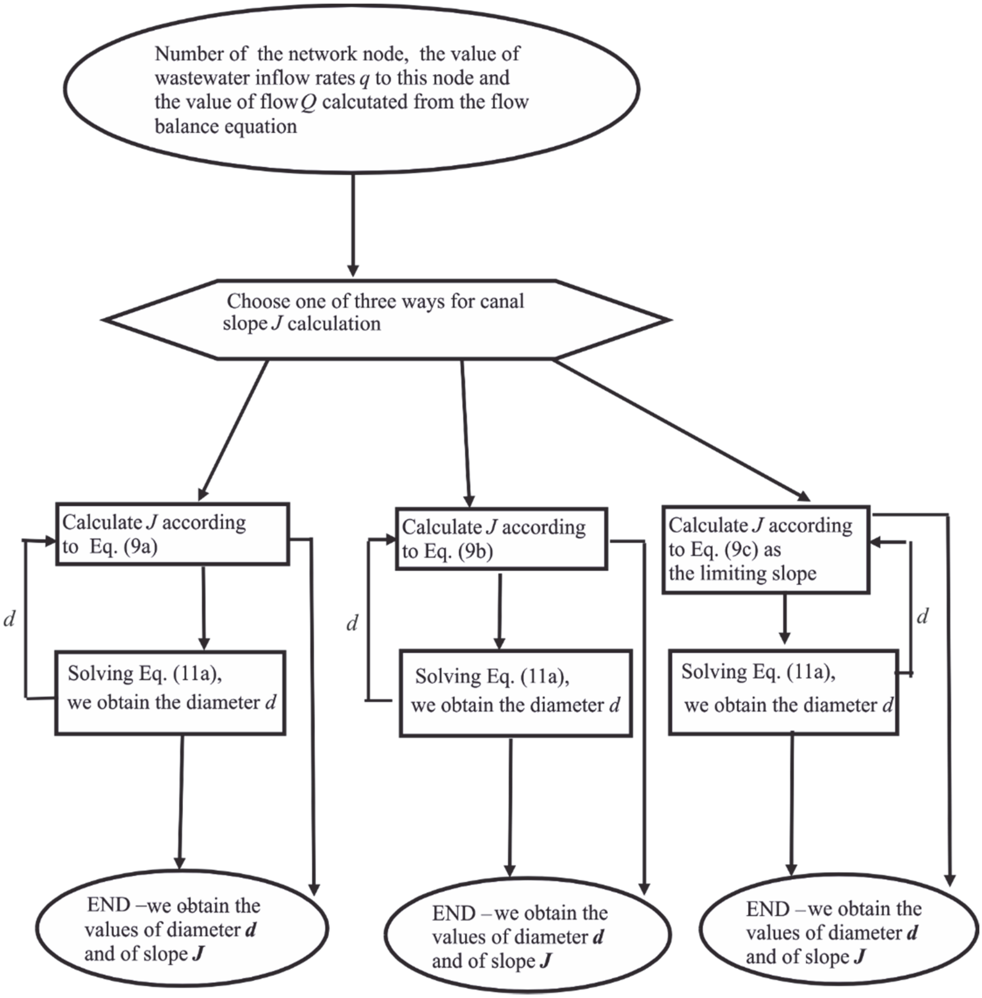

Fig. 9. The algorithm for calculating channel diameters for given flow values

In both cases, while calculating the diameters and slopes of the new channels for given flows Q, the inequality $2\pi\beta - Q > 0$ has to be considered. When this inequality is satisfied, then the equation describing the relation between channel slope J and channel flow Q only has one solution. The calculation procedure consists of the following steps, which are carried out for the planned flow values Q:

Step 1. Determination of the channel slope *J*. This value can be determined according to existing technical standards, or calculated using formulas for minimal slopes which are known from the literature [7, 11, 18, 19]:

$$J = \frac{a}{d} \tag{9a}$$

where *a* is a parameter depending on the type of the sewage system.

The minimal channel slope ensuring channel self-purification is given by:

$$J_s = \frac{\tau_{\min}}{\rho R} = \frac{4\tau_{\min}(\pi - 0.5\varphi)}{\rho\left(\pi - 0.5\varphi + 0.5\sin\varphi\right)} \times \frac{1}{d} \tag{9b}$$

where $\varphi = 2\arccos\left(2\frac{H}{d} - 1\right)$, J_S is the minimal channel slope ensuring channel self-purification, $\tau_{\min}$ – tangential tension [kg/m^2], with $\tau_{\min} > 0.225$ kg/m^2 for communal and industrial wastewater, ρ – sewage density [kg/m^3], *R* – hydraulic radius [m].

After assuming that the relative depth *x* for housekeeping networks is 60% [19], after some transformations the minimal channel slope *J* ensuring self-purification takes the form:

$$J_s = \frac{3.602\tau_{\min}}{\rho d}$$

On the other hand, there is also another important value of the slope (borderline slope) that ensures the laminar flow of sewage, namely:

$$J_g = \frac{2.279gn^2}{d^{1/3}} \tag{9c}$$

where: *g* – gravitational constant [m/s^2], *n* – roughness coefficient [s/m$^{1/3}$].

This means that the channel slope must be between the minimal and borderline slopes, which ensures self-purification and laminar rather than turbulent flow.

Step 2. Solving the following equations:

$$\varsigma d^{8/3} - Q = 0, \qquad \varsigma = \frac{\pi}{n}\left(\frac{1}{4}\right)^{5/3} J^{1/2} \tag{10}$$

where ζ is the friction coefficient [m$^{1/3}$/s].

A. For $J = \dfrac{a}{d}$:

$$\alpha_1 d^{13/6} - Q = 0, \qquad \alpha_1 = \frac{\pi}{n} \times \left(\frac{1}{4}\right)^{5/3} a^{1/2} \tag{11a}$$

where α_1 is a coefficient [$m^{5/6}/s$].

B. For the minimal slope J ensuring channel self-purification:

$$\alpha_2 d^{13/6} - Q = 0, \qquad \alpha_2 = \frac{\pi}{n} \times \left(\frac{1}{4}\right)^{5/3} \left(\frac{3.602\tau_{\min}}{\rho}\right)^{1/2} \tag{11b}$$

where α_2 is a coefficient [$m^{5/6}/s$].

C. For the borderline slope J ensuring laminar wastewater flow:

$$\alpha_3 d^{3/2} - Q = 0, \qquad \alpha_3 = \pi \times \left(\frac{1}{4}\right)^{5/3} \times (2.279g)^{1/2} \tag{11c}$$

where α_3 is a coefficient [$m^{1/2}/s$].

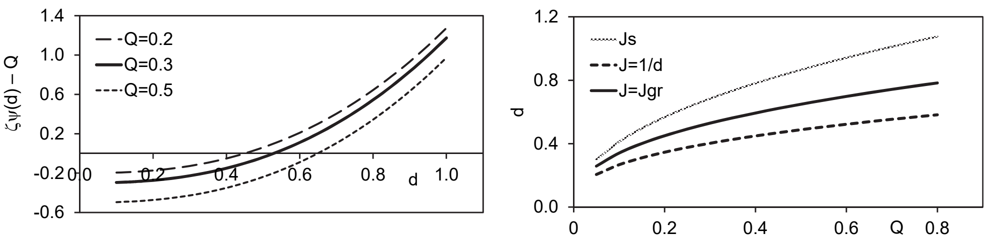

Fig. 10. Plots of the function ($\xi d^{5/2} - Q$) for various values of Q (left) and the dependence of the channel diameter d on the channel flow Q for various channel slopes J

From the above equations, one can calculate the channel diameter d for a planned flow value Q. Solving Eq. (10) for known Q, we obtain the minimal diameter d_* such that the inequality $\zeta d^{8/3} - Q > 0$ is fulfilled for all $d > d_*$. Diameters lower than or equal to this minimal value are forbidden. For the borderline slope value (case 3), the resulting relationships are presented in Fig. 10, left. If the channel slope J lies between its minimal and borderline values (see Eqs. (9a)–(9c)) but the value of d is lower than d_*, then one has to increase the channel diameter, return to Step 1 and the channel slope has to be calculated again. If no solution of Eq. (10) exists, then one must return to Step 1, change the value of J and solve Eq. (10) once again. Figure 10, right illustrates the relationship between the solution of Eq. (10) (as an equation for the channel diameter d) and the channel flow Q for various channel slopes J.

4.3. Modelling and designing an example wastewater network

The algorithms presented for modelling and designing sewage networks have been tested on an examplary housekeeping network consisting of 27 nodes connected by 26 segments (Fig. 11). The network consists of 15 input nodes (W_6, W_7, W_8, W_{10}, W_{11}, W_{14}, W_{15}, W_{16}, W_{19}, W_{20}, W_{21}, W_{23}, W_{25}, W_{26}, W_{27}) and of 1 output node W_1. Other nodes correspond to junctions between various segments of the network. The arrows in Fig. 11 show the direction of sewage flow.

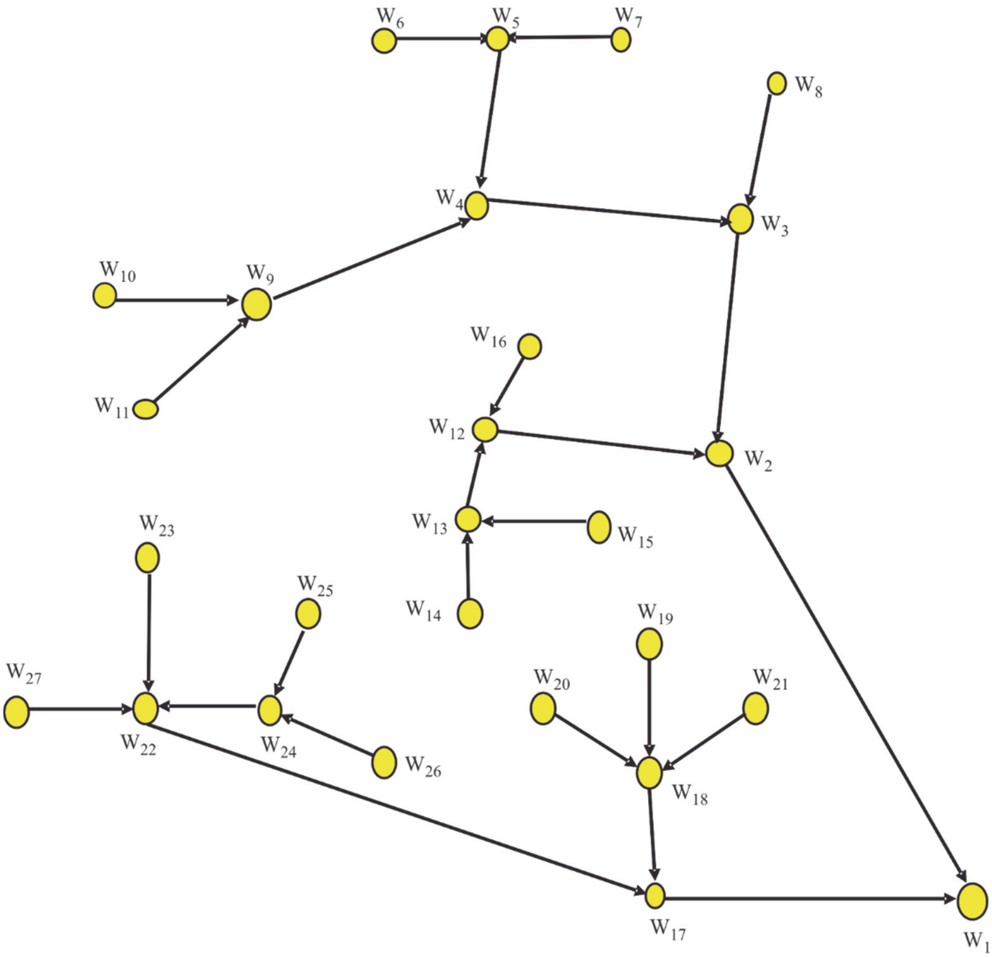

Fig. 11. Structure of the sewage network investigated

The flow rates of sewage at the input nodes were given in advance. The flow rates in the connection nodes were calculated from the balance equations. For each segment, the diameter $d = 0.2$ m and the channel slope $J = 0.5\%$. The relative depth H/d and flow velocities v in each segment were calculated based on this network structure. The steady state of the network was also calculated using the MOSKAN system implemented in IBS PAN [17]. This system is based on the SWMM5 hydraulic model developed by EPA [20]. The results obtained from both approaches are presented in Table 1.

Table 1. Results of modelling the examplary network shown in Fig. 11

Upper node	Lower node	Segment	Input flow at a node	Flow in segments Q [dm^3/s]	H/d [%]	v [m/s]	MOSKAN	
							H/d [%]	v [m/s]
W_6	W_5	1	0.56	0.56	10.72	0.309	11	0.29
W_7	W_5	2	0.31	0.31	8.09	0.259	8	0.26
W_5	W_4	3	0.27	1.14	15.08	0.383	15	0.38
W_{10}	W_9	4	0.36	0.36	8.69	0.271	9	0.27
W_{11}	W_9	5	1.13	1.13	15.02	0.382	14.6	0.39
W_9	W_4	6	0.64	2.13	20.48	0.460	20	0.46
W_4	W_3	7	0.64	3.91	27.78	0.549	28	0.55
W_8	W_3	8	0.11	0.11	4.98	0.189	5	0.19
W_3	W_2	9	0.1	4.12	28.53	0.557	29	0.56
W_{14}	W_{13}	10	0.11	0.11	4.98	0.189	5	0.19
W_{15}	W_{13}	11	0.32	0.32	8.22	0.261	8	0.26
W_{13}	W_{12}	12	0.23	0.66	11.59	0.325	12	0.33
W_{16}	W_{12}	13	0.24	0.24	7.17	0.240	7	0.24
W_{12}	W_2	14	1.86	2.76	23.29	0.497	23	0.49
W_2	W_1	15	0.73	7.61	39.42	0.661	39	0.66
W_{23}	W_{22}	16	4.56	4.56	30.06	0.574	30	0.58
W_{27}	W_{22}	17	4.4	4.4	29.51	0.568	30	0.57
W_{25}	W_{24}	18	4.81	4.81	30.90	0.582	31	0.58
W_{26}	W_{24}	19	3.53	3.53	26.37	0.533	26	0.53
W_{24}	W_{22}	20	3.69	12.03	51.10	0.745	51	0.75
W_{22}	W_{17}	21	1.53	22.52	79.47	0.841	79	0.84
W_{19}	W_{18}	22	0.83	0.83	12.94	0.348	13	0.35
W_{20}	W_{18}	23	0.3	0.3	7.97	0.256	8	0.26
W_{21}	W_{18}	24	0.19	0.19	6.43	0.223	6	0.22
W_{18}	W_{17}	25	0.22	1.54	17.46	0.419	17	0.42
W_{17}	W_1	26	0.57	24.63	89.27	0.832	89	0.83
W_1	Sewage plant							

Analysis of these results shows that in two network segments, 21 (W_{22}–W_{17}) and 26 (W_{17}–W_1), the relative depths exceed the maximal acceptable value of 70% [10, 14].

The sewage inflows are fixed and thus some new channel diameters and channel slopes have to be calculated which has been done using the design algorithm proposed in 4.2.

Three possible ways of calculating the slope *J* have been applied as follows:

A. *J* is proportional to the inverse of the diameter *d* according to Eq. (9a).

B. *J* is the minimal slope J_s ensuring self-purification in the sewage channel according to Eq. (9b).

C. *J* is the borderline slope J_g according to Eq. (9c).

The values of *d* were obtained by calculating *J* from Eqs. (11a), (11b) or (11c), as appropriate. For these new values of *d* and *J*, the new relative depths *H*/*d* and flow velocities *v* can be computed. The results obtained from these approaches are presented in Table 2.

Table 2. Results from the design algorithm for the example network shown in Fig. 11

Upper node	Lower node	Q [dm^3/s]	Approach A				Approach B				Approach C			
			d [m]	J [%]	H/d [%]	v [m/s]	d [m]	J_s [%]	H/d [%]	v [m/s]	d [m]	J_g [%]	H/d [%]	v [m/s]
W_{22}	W_{17}	22.52	0.3	0.33	44.32	0.744	0.4	0.225	32.5	0.637	0.3	0.57	38.23	0.864
W_{17}	W_1	24.63	0.3	0.33	46.63	0.762	0.4	0.225	34	0.657	0.3	0.57	40.13	0.93

To ensure safe functioning of the network, some restrictions must be put on the slopes of the channels in order to secure laminar sewage flows, as well as self-purification. Any appropriate slope for a given channel diameter must be greater than the minimal slope J_s and less than the borderline slope J_g. Analysis of the results presented in Table 2 shows that for given flow values (for example, Q = 22.52 dm^3/s for segment 21), the least relative depth H/d = 32.5% is obtained for the minimal slope J_s = 0.225%, the least slope ensuring self-purification (case B). For slope J = 0.33%, based on the inverse of the diameter *d*, the relative depth H/d = 44.32% is greatest (case A).

The results obtained using the algorithm proposed are very similar to those obtained using the MOSKAN software. The only differences result from the rounding of numbers used in MOSKAN. This means that the algorithms proposed here are more reliable and dependable than classical ones that use nomograms and not less reliable than the complicated approach that uses the SWMM algorithm to model sewage networks.

5. Conclusions

A new practical approach to modelling sewage networks has been proposed. It differs from the approaches commonly used in present day practice. The classical and most commonly applied method of modelling sewage networks consists of using nomogram diagrams which enable us to calculate appropriate parameters such as channel diameters and channel slopes, when designing networks based on estimated sewage inflow rates in a purely mechanical way. The results obtained depend strongly on the quality of the diagrams used. The modern approach in this field consists of applying advanced computer programs like SWMM [12, 13] developed by EPA [20], or MIKE URBAN developed by DHI [22]. They use hydraulic models of sewage networks in their computations. This approach requires advanced knowledge from program users. The most important obstacle to using this software in operational practice is the necessity of having a calibrated hydraulic model of the network investigated. To calibrate such a model, a GIS system to generate a coordinate map of the network and an appropriately dense monitoring system to collect measurement data have to be installed in the sewage network, which generates large costs [16]. Most Polish waterworks are municipal enterprises and they commonly do not have enough money to buy such costly systems. It seems that the approach to modelling sewage networks presented in this paper could be currently an ideal tool for modelling such networks, as it is a healthy compromise between the classical and modern approaches. It has all the advantages of both these approaches, but none of their drawbacks. It uses analytical relationships concerning the hydraulics and geometry of sewage networks and transforms them into nonlinear equations, from which the required relative depths and sewage speeds or channel diameters and slopes can be directly calculated. Analysis of these equations enables us to determine the maximal sewage inflows at the network nodes. These calculations can be carried out quickly and precisely avoiding the need to use a complicated hydraulic model. The example presented here illustrating the modelling of the steady state of sewage networks and its application in network design is rather simple, but the algorithms proposed are practical for modelling and designing more complex municipal sewage systems. For the algorithms presented, one significant problem is the need to determine the inflow rates of sewage into individual network segments. In the case of communal and industrial sewage, these inflows are relatively simple to define given data regarding water consumption by end users of the water network connected to the sewage network. Another problem arises from the need to model the flow of rainfall into the sewage channels. Rainwater inflows can be defined directly by means of a function based on specific field investigations, or indirectly by means of functions describing rainfall and the drainage basin considered. In the second case, several parameters describing the surface and shape of the terrain, water retention, density of buildings in the drainage basin, soil type etc., must be defined, which greatly complicates the problem of modelling inflow into the system [7, 21].

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