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OPTIMIZATION OF AN M/M/1/N FEEDBACK QUEUE WITH RETENTION OF RENEGED CUSTOMERS

Customer impatience has become a threat to the business world. Firms employ various customer retention strategies to retain their impatient (or reneged) customers. Customer retention mechanisms may help to retain some or all impatient customers. Further, due to unsatisfactory service, customers may rejoin a queue immediately after departure. Such cases are referred to as feedback customers. Kumar and Sharma take this situation into account and study an M/M/1/N feedback queuing system with retention of reneged customers. They obtain only a steady-state solution for this model. In this paper, we extend the work of Kumar and Sharma by performing an economic analysis of the model. We develop a model for the costs incurred and perform the appropriate optimization. The optimum system capacity and optimum service rate are obtained.

Keywords: *reneging, retention of reneged customers, revenue, queuing system, optimization*

1. Introduction

Queues with impatient customers can be observed everywhere, in business or elsewhere. For instance, customers arrive at the office of an insurance company to purchase policies, on arriving at the front of the queue they are served and then they depart. Some customers may not complete the process of initiating a policy due to various reasons (e.g. lack of money to pay for a premium, a better option existing elsewhere). Also, they may get impatient and resign from obtaining a policy. Compa-

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nies try to retain such customers. They apply various strategies (counseling, discounts on premiums, etc.) to retain impatient customers. Eventually, they may or may not be able to retain a reneged customer. There are situations when customers are not satisfied with the initial service (unsatisfactory settlement amount, false cheques, etc.). They rejoin the system with some probability and are referred to as feedback customers in the queuing literature. There are lots of other businesses where the same phenomenon can be observed, for example vehicle service stations, restaurants etc. In this process, a customer's impatience seems to be a very significant threat due to the highly competitive business environment. A popular saying in business is: a customer gone once is a customer gone forever. This paper contributes towards the economic analysis of such situations. It suggests an optimum strategy for a firm to maximize its profit, functioning under the constraints mentioned above.

Kumar et al. [7] analytically study an M/M/1/N feedback queue with retention of reneged customers and obtain the steady-state solution recursively. They simply derive the steady-state solution of the model. They do not perform any economic analysis of the model. In this paper, we extend the work by Kumar et al. [7] through developing a model where the total expected cost, total expected revenue and total expected profit functions are derived.

We present the optimization of various parameters in the model such as system capacity and the service rate. A sensitivity analysis for this model has been carried out with respect to the probability of retention, rate of reneging and arrival rate. The optimum service rate and optimum system capacity are obtained based on various parameters such as the probability of retention of reneged customers, rate of reneging and arrival rate. A comparative analysis has been presented to gain deep insight into expected and optimum costs, revenues and profits.

Pattern search and classical techniques of optimization are used for optimization based on the above mentioned model. Analysis of the model is performed in MS EXCEL and MATLAB. MATLAB programs and Spread Sheets are constructed and executed as and when needed.

2. Literature review

Customer impatience results in loss of business for any firm. Choudhary et al. [2] study customer impatience in multi-server queues. They consider both balking and reneging as functions of the state of the system by taking into consideration situations where the customer is aware of his/her position in the system. Kapodistria [5] studies a single server Markovian queue with impatient customers and considers situations where customers abandon the system simultaneously. She considers two abandonment scenarios. In the former one, all the present customers become impatient and perform synchronized abandonments, while in the latter scenario, the customer presently being served is excluded from the abandonment procedure. She also extends this analysis to an M/M/c queue under the second abandonment scenario. The phenomenon of customer impatience in single-server queues is discussed in the work of Wu et al. [14] as well. Pan [11] studies a model of an M/M/1/N queue with variable input rates. Jain et al. [4] consider a multi-server queuing system in which additional servers are used when the queue is long, in order to reduce the likelihood of customers balking and reneging. They obtain the equilibrium distribution of the queue size along with other performance measures. Altman et al. [1] study a system operating as an M/M/∞ queue. They discuss the case in which whenever the queue is empty, a server is assigned some other task, say U. While performing this additional task, a new customer arrives, finds the server busy and becomes impatient. They analyze both multiple and U-task scenarios and derive the probability generating function (PGF) of the number of customers present. Dudin et al. [3] analyze a multi-server queuing system with a finite buffer and impatient customers. They give an algorithm for finding the stationary distribution of the state of the system and derive basic performance characteristics. Wu et al. [15] focus on an M/M/s queue with multiple vacations, such that the server works with different service rates rather than no service during a vacation period. They generalize an M/M/1 queue with working vacations. A cost function is formulated to determine the optimal number of servers subject to given stability conditions.

Tadj et al. [12] use a vacation queuing model and develop a set of quantitative performance measures for a two-parameter time allocation policy. Based on renewal cycle analysis, they derive an expression for the average cost and propose a search algorithm to find the optimal time allocation policy that minimizes the average cost. Ke et al. [6] analyze the cost in an M/M/R machine repair problem with balking, reneging and server breakdowns. A cost analysis for a finite M/M/R queuing system with balking, reneging and server breakdown is discussed in Wang et al. [13]. Mishra et al. [10] perform a cost analysis for a machine interference model with balking, reneging and spares. Furthermore, in [16] Yue et al. present an analysis for an M/M/R/N queuing system with balking, reneging and server breakdowns. Yue et al. [17] present an analysis for an M/M/c/N queuing system with balking, reneging and synchronous vacations of partial servers. They formulate a model for the costs to determine the optimal number of servers on vacation. Kumar et al. [8] optimize revenue in an insurance business facing customer impatience. They develop a model of the costs in an M/M/1/N queuing system with retention of reneged customers and balking and then minimize them by applying a pattern search algorithm and classical optimization techniques. They also optimize service rate and system capacity with varying rates of reneging and arrival and probability of retention. Kumar et al. [10] further optimize an M/M/1/N queuing system with retention of reneged customers by developing a model for the costs and using optimization techniques through soft computing.

The literature review discussed above provides a sufficient understanding of queuing models with customer impatience. The modeling of costs and optimization are crucial to the design of optimal queuing systems.

3. Description of the model

The model considered in this paper is based on following assumptions:

- Arrivals occur in a Poisson stream one by one with an average arrival rate of λ. The inter-arrival times are independently, identically and exponentially distributed with the parameter λ.
- There is only one server and service times are exponentially distributed with the parameter μ.
- The queue discipline is first-come, first-served (FCFS).
- The capacity of the system is finite (say N).
- Each customer upon arriving in the queue will wait a certain time (reneging time) for service to begin. If it has not begun by then, with probability p he becomes impatient and leaves the queue without being served and with probability $q = 1 - p$ remains in the queue until service is complete. The reneging times follow the exponential distribution with parameter ξ.
- With probability p_1 any service obtained is incomplete. After getting incomplete service, a customer rejoins the queue. Hence, after being served, with probability p_1, a customer rejoins the system as a feedback customer to receive another regular service. Otherwise, he leaves the system, i.e. with probability q_1, where $p_1+q_1 = 1$.

The differential-difference equations for this model are given by:

$$\frac{dP_0(t)}{dt} = -\lambda P_0(t) + \mu q_1 P_1(t) \tag{1}$$

$$\begin{aligned}\frac{dP_n(t)}{dt} &= -\left(\lambda + \mu q_1 + (n-1)\xi p\right)P_n(t) \\ &\quad + \left(\mu q_1 + n\xi p\right)P_{n+1}(t) + \lambda P_{n-1}(t), \quad 1 \le n \le N-1\end{aligned} \tag{2}$$

$$\frac{dP_N(t)}{dt} = \lambda P_{N-1}(t) - \left(\mu q_1 + (N-1)\xi p\right)P_N(t), \qquad n = N \tag{3}$$

4. Model for the costs

In this section, we develop a model for the costs incurred in the queuing system using the following symbols:

$1/\lambda$ – mean inter-arrival time
$1/\mu$ – mean service time
L_s – expected number of customers in the system
R_r – average rate of reneging
R_R – average rate of retention
C_s – cost of service per unit time
C_h – unit holding cost per unit time
C_L – cost associated with each lost unit
C_r – cost associated with each reneged unit
C_R – cost of retaining a reneged customer
C_{s1} – cost of serving a feedback customer
R – revenue earned by providing service to a customer
TEC – total expected cost per unit time of the system
TER – total expected revenue per unit time of the system
TEP – total expected profit per unit time of the system

Here we consider a single server, finite capacity Markovian queuing model as studied by Kumar et al. [7], where the steady state probabilities are given by:

$$P_n = \prod_{k=1}^{n} \frac{\lambda}{\mu q_1 + (k-1)\xi p} P_0, \qquad 1 \le n \le N-1 \tag{4}$$

also, for $n = N$, we get

$$P_N = \prod_{k=1}^{N} \frac{\lambda}{\mu q_1 + (k-1)\xi p} P_0 \tag{5}$$

Using the normalization condition, $\sum_{n=0}^{N} P_n = 1$, we get

$$P_0 = \frac{1}{1 + \sum_{n=1}^{N} \prod_{k=1}^{n} \frac{\lambda}{\mu q_1 + (k-1)\xi p}} \tag{6}$$

and the expected number of customers in the system is:

$$L_s = \sum_{n=1}^{N} n \left(\prod_{k=1}^{n} \frac{\lambda}{\mu q_1 + (k-1)\xi p} \right) P_0 \tag{7}$$

Here, we derive various functions like the total expected cost per unit time, total expected revenue per unit time and total expected profit per unit time. The total expected profit per unit time is then optimized by using pattern search and classical optimization techniques as mentioned above.

The total expected cost (TEC) per unit time is given by:

$$\text{TEC} = \mu(C_s + p_1 C_{s1}) + C_h L_s + C_L \lambda P_N + C_r R_r + C_R R_R$$

$$\begin{aligned}\text{TEC} = {} & \mu(C_s + p_1 C_{s1}) + C_h \sum_{n=1}^{N} n \left(\prod_{k=1}^{n} \frac{\lambda}{\mu q_1 + (k-1)\xi p} \right) P_0 \\ & + C_L \lambda \prod_{k=1}^{n} \frac{\lambda}{\mu q_1 + (k-1)\xi p} + C_r \left(\sum_{n=1}^{N} (n-1)\xi p \prod_{k=1}^{n} \frac{\lambda}{\mu q_1 + (k-1)\xi p} \right) P_0 \\ & + C_R \left(\sum_{n=1}^{N} (n-1)\xi q \prod_{k=1}^{n} \frac{\lambda}{\mu q_1 + (k-1)\xi p} \right) P_0 \end{aligned} \tag{8}$$

where the average reneging rate R_r and the average retention rate R_R are given by:

$$R_r = \sum_{n=1}^{N} (n-1)\xi p P_n$$

$$R_R = \sum_{n=1}^{N} (n-1)\xi q P_n$$

Let R be the revenue earned for providing service to a customer, then $R\mu(1 - P_0)$ is the rate of earning revenue for providing service to customers in the system. Hence, total expected revenue (TER) of the system is given by:

$$\text{TER} = R\mu(1 - P_0) \tag{9}$$

Now, total expected profit (TEP) of the system is defined as:

$$\begin{aligned}\text{TEP} = {} & R\mu\left(1-P_0\right)-\mu\left(C_s+p_1C_{s1}\right)\\ & -C_h\sum_{n=1}^{N}n\left(\prod_{k=1}^{n}\frac{\lambda}{\mu q_1+\left(k-1\right)\xi p}\right)P_0-C_L\lambda\prod_{k=1}^{n}\frac{\lambda}{\mu q_1+\left(k-1\right)\xi p}P_0\\ & -C_r\left(\sum_{n=1}^{N}\left(n-1\right)\xi p\prod_{k=1}^{n}\frac{\lambda}{\mu q_1+\left(k-1\right)\xi p}\right)P_0\\ & -C_R\left(\sum_{n=1}^{N}\left(n-1\right)\xi q\prod_{k=1}^{n}\frac{\lambda}{\mu q_1+\left(k-1\right)\xi p}\right)P_0\end{aligned} \tag{10}$$

Thus, we have the TEC, TER and TEP functions in terms of various parameters involved. The economic analysis of the model is performed numerically by using these functions and the results are discussed accordingly. The optimization of the model is also carried out in order to obtain the optimal service rate and optimum system capacity.

5. Optimization of the model

In this section, the optimization of the model is performed. First pattern search algorithm is used to optimize the system capacity. Pattern search optimization technique is a hit and trial technique which states that the function under consideration shall be checked for various values. In this paper, we check the value of the profit function for various values of the system capacity starting from minimum ($N = 2$) by keeping all other variables fixed. The value of the profit function increases initially and then starts decreasing after a certain value of N. The value after which the value of TEP starts decreasing is considered as optimized value of N. We obtain the optimum value of the service rate at which the total expected profit of the system is maximum. We study the variation in total optimum profit in function of the probability of customer retention associated with a particular customer retention strategy. The total optimum cost and total optimum revenue are also computed. The numerical results are presented for cost-profit analysis of the model.

5.1. Determination of optimal service rate

Computational algorithm

Step 1. Define variables.
Step 2. Write the formula of function TEP in terms of μ.

Step 3. Obtain critical values for TEP.
Step 4. Find the value of μ at which TEP is maximum (let it be μ^*).
Step 5. Compute the values of TEC, TER and TEP at μ^*.

Results of the comparative analysis of the average system size (L_s) with respect to the rate of reneging (when no retention and when there is a certain probability of retention) are given in Table 1 and Fig. 1.

Table 1. Average system size when no retention strategy is followed and when some customer retention strategy for reneged customers in applied

ξ	L_s at $q = 0$	L_s at $q = 0.6$
0.05	1.3812	1.4053
0.06	1.3734	1.4020
0.07	1.3657	1.3987
0.08	1.3582	1.3955
0.09	1.3508	1.3923
0.10	1.3435	1.3891
0.11	1.3364	1.3859
0.12	1.3293	1.3827
0.13	1.3224	1.3796
0.14	1.3155	1.3765
0.15	1.3088	1.3734

$\lambda = 4$, $\mu = 3$, $N = 4$, $q_1 = 0.9$.

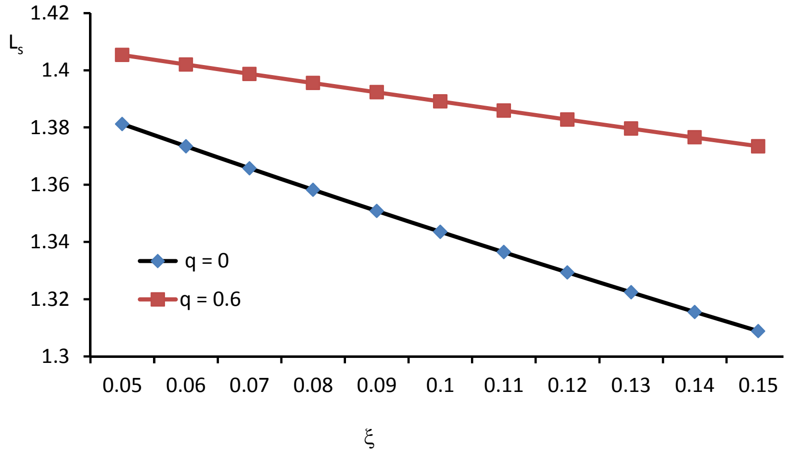

Fig. 1. L_s at $q = 0$ and $q = 0.6$ in function of ξ

It can be observed that the average system size remains high when the retention of reneged customers takes place with certain probability say, $q = 0.6$ in comparison to the system size when no retention of reneged customers take place. This affects the total profit and revenue of the firm as increase in system size results in more and more customers in the system and the revenue made goes high.

5.2. Optimization of the system capacity and service rate

In Table 2, first we optimize system capacity (N^*) by using search technique and then calculate optimal service rate (μ^*) for the obtained optimal system capacity by using classical optimization algorithm in MATLAB.

Table 2. Optimum system capacity

N	1	2	3	4	5	6^*	7	8	9	10
TEP	136.0567	194.7702	219.3722	230.6481	235.5184	236.9310	236.3410	234.5400	231.9905	228.9773

$\lambda = 4$, $\xi = 0.20$, $\mu = 3$, $C_s = 4$, $C_{s1} = 2$, $C_h = 3$, $q = 0.6$, $q_1 = 0.9$, $C_r = 8$, $C_R = 25$, $C_L = 12$.

It can be observed from the table that the TEP is maximum at $N = 6$ and then decreases successively, hence it can be identified that optimum system capacity in this case is at $N = 6$ and is represented as $N^* = 6$. To obtain optimum service rate (μ^*) for optimum system capacity (N^*) thus obtained is obtained by using a MATLAB program in which classical optimization technique has been followed. The algorithm gives us optimum service rate $\mu^* = 9.0674$ at which the TEP at $N^* = 6$ increases to 395.4931 from 236.9310 while all other parameters kept constant as they were.

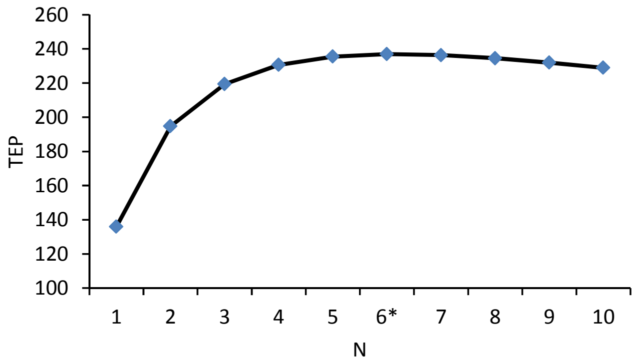

Fig. 2. Total expected profit in function of N

Figure 2 depicts that total expected profit is maximum at $N = 6$, therefore it can be observed that $N^* = 6$ for above mentioned data set and for $N^* = 6$ the optimum service rate is $\mu^* = 9.0674$.

Finding optimum triplet (N^*, q^*, μ^*)

Now we optimize the total expected revenue (TER), total expected cost (TEC) and total expected profit (TEP). MATLAB programming is used to obtain optimum service rate for each value of q.

Table 3. Values of TEC, TER and TEP for various probabilities of customer retention, q

q	C_R	$\mu = 3$			$\mu = \mu^*$				
		TER	TEC	TEP	$N = N^*$	μ^*	TER*	TEC*	TEP*
0.2	8	291.5763	44.8471	246.7292	9	8.0336	433.7637	38.0540	395.7098
0.3	12	290.8458	45.7609	245.0849	8	8.3571	435.5537	39.2616	396.2921
0.4	14	289.3779	45.9429	243.4350	7	8.2562	435.4111	39.1587	396.2524
0.5*	20	290.1051	49.8856	240.2196	7	8.6304	437.5155	40.7156	396.7999
0.6	25	287.4766	50.5456	236.9310	6	8.8156	437.3708	41.7725	395.5982
0.7	32	288.1573	55.8861	232.2712	6	8.8178	438.2098	42.4101	395.7998
0.8	36	283.6454	54.5200	229.1255	5	9.3245	437.2923	44.4659	392.8265
0.9	40	284.2787	58.6724	225.6062	5	9.3271	437.9756	44.9841	392.9916
1.0	45	276.4612	55.3030	221.1582	5	9.3472	438.7125	45.6896	393.0229

$\lambda = 4$, $\mu = 3$, $q_1 = 0.9$, $\xi = 0.2$, $C_s = 4$, $C_{s1} = 2$, $C_h = 3$, $C_L = 12$, $C_r = 8$, $R = 100$

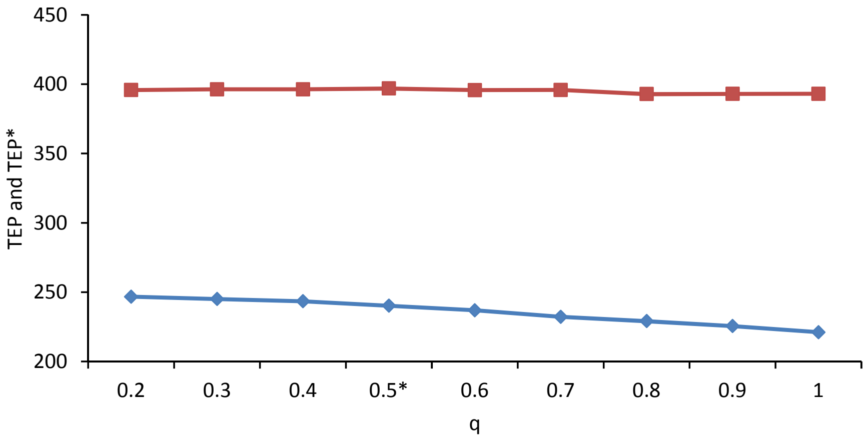

Fig. 3. TEP (lower curve) and TEP* (upper curve) in function of q

When no optimum strategy is followed, the TEP is lower and when the optimum strategy is followed the profit is higher. We obtain an optimum triplet $q^* = 0.5$, $N^* = 7$ and $\mu^* = 8.6304$ (Table 3). For the system capacity 7, retention strategy is applied in such a way that 50% of reneged customers are retained and the customers are provided a service at the rate of 8.6304, the profit obtained is maximum. From Figure 3, it can be observed easily that the profit obtained is much higher if the optimum policy is followed and is maximum at optimum triplet i.e., at N^*, q^*, μ^* while keeping all other parameters constant.

Finding optimum triplet (N^*, ξ^*, μ^*)

Table 4. Variation in total optimum profit in function of ξ

ξ	$\mu = 3$			$\mu = \mu^*$				
	TER	TEC	TEP	$N = N^*$	μ^*	TER*	TEC*	TEP*
0.1*	292.1964	49.8478	242.3485	7	8.2299	439.4631	39.1478	400.3153
0.2	287.4766	50.5456	236.9310	6	8.7302	437.1208	41.5149	395.6058
0.3	286.0972	54.1401	231.9571	6	8.9857	436.2713	42.9632	393.3081
0.4	279.8442	52.5270	227.3173	5	9.6575	428.4839	46.3513	382.1326
0.5	278.5869	55.1180	223.4688	5	9.8459	433.9795	46.7389	387.2406
0.6	277.3391	57.5592	219.7798	5	10.0271	433.4630	47.7594	385.7035
0.7	276.1031	59.8593	216.2438	5	10.2014	432.9774	48.7274	384.2500
0.8	274.8810	62.0269	212.8540	5	10.3691	432.5193	49.6478	382.8715
0.9	266.9599	57.0897	209.8703	4	10.9868	430.2480	51.9871	378.2609
1	265.9804	58.7161	207.2642	4	11.1120	429.9423	52.7032	377.2391

$\lambda = 4$, $q = 0.6$, $q_1 = 0.9$, $C_{s1} = 2$, $C_s = 4$, $C_h = 3$, $C_R = 25$, $C_L = 12$, $C_r = 8$, $R = 100$

In the case of variation of ξ and no optimum strategy followed, the total expected profit is lower as compared to the case when the optimum strategy is followed (Table 4). We obtain an optimum triplet $\xi^* = 0.1$, $N^* = 7$ and $\mu^* = 8.2299$. It is observed that if the system capacity is 7, 10% of the customers are reneging due to whatever reason and the customers are provided with a service at the rate 8.2299, the total expected profit obtained is maximum for the values taken in this scenario.

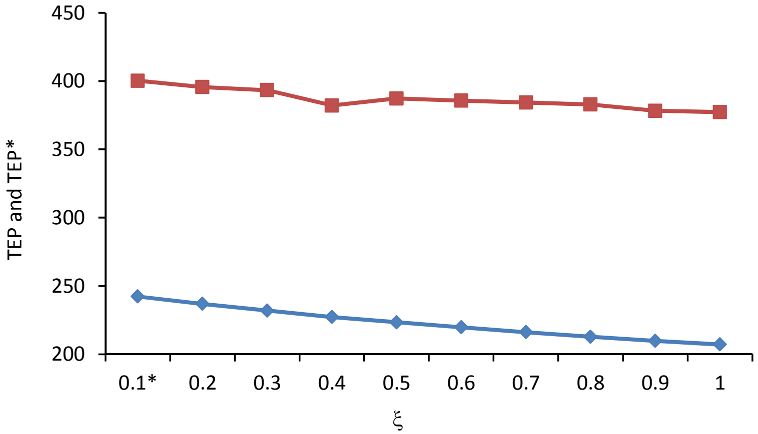

Fig. 4. TEP (lower curve) and TEP* (upper curve) in function of ξ

From Figure 4 it can be observed easily that the profit obtained is much higher if the optimum policy is followed and is maximum at optimum triplet i.e., at N^*, ξ^*, μ^* while keeping all other parameters constant.

Finding optimum triplet (N^*, λ^*, μ^*):

Table 5. Variation in total optimum profit w. r. t. λ

λ	$\mu = 3$			$\mu = \mu^*$				
	TER	TEC	TEP	$N = N^*$	μ^*	TER*	TEC*	TEP*
3.5	282.9488	46.2032	236.7456	7	7.4557	382.9180	36.3204	346.5976
3.6	284.9410	47.8883	237.0526	7	7.6428	393.8968	37.1552	356.7416
3.7	286.6998	49.5599	237.1400	7	7.8299	404.8762	37.9885	366.8877
3.8	288.2506	51.2160	237.0346	7	8.0169	415.8560	38.8202	377.0357
3.9	286.1037	49.0582	237.0455	6	8.5313	426.1667	40.6421	385.5246
4.0	287.4766	50.5456	236.9310	6	8.7302	437.1208	41.5149	395.6058
4.1	288.7063	52.0247	236.6816	6	8.9290	448.0748	42.3866	405.6882
4.2	289.8076	53.4949	236.3127	6	9.1277	460.7082	43.1978	417.5104
4.3	290.7941	54.9560	235.8381	6	9.3264	469.9829	44.1269	425.8561
4.4	291.6779	56.4078	235.2701	6	9.5251	480.9373	44.9959	435.9414
4.5*	288.3666	53.6270	234.7397	5	10.3369	490.3083	47.9925	442.3159

$\xi = 0.2$, $q = 0.6$, $q_1 = 0.9$, $C_{s1} = 2$, $C_s = 4$, $C_h = 3$, $C_R = 25$, $C_L = 12$, $C_r = 8$, $R = 100$

In the case of variation of λ and no optimum strategy is followed, the total expected profit is lower as compared to the case when the optimum strategy is followed (Table 5). We obtain an optimum triplet $\lambda^* = 4.5$, $N^* = 5$ and $\mu^* = 10.3369$. If the system capacity is 5, arrival rate – 4.5 customers per unit time and the customers are provided with a service at rate 10.3369, the profit obtained is maximum for the values taken in this scenario.

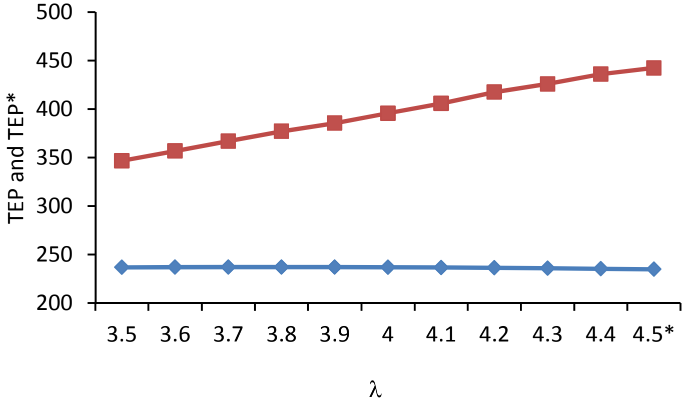

Fig. 5. TEP lower curve) and TEP* (upper curve) in function of λ

From Figure 5 it can be observed easily that the profit obtained is much higher if the optimum policy is followed and is maximum at optimum triplet i.e., at N^*, λ^*, μ^* while keeping all other parameters constant.

6. Conclusions

Economic analysis of an M/M/1/N feedback queuing system has been performed with retention of reneged customers. Average system sizes with and without retention of reneged customers have been analyzed, the cost model developed and various parameters of the system optimized such as capacity of the system and average service rate.

Three optimum triplets (N^*, q^*, μ^*), (N^*, ξ^*, μ^*) and (N^*, λ^*, μ^*) have been obtained and the total optimum profit presented against total expected cost in all three cases.

The results obtained in this paper are useful for any firm operating in the field of finance, supply chain, manufacturing, etc.

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