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USING AN ANALYTIC HIERARCHY PROCESS TO DEVELOP A SCORING SYSTEM FOR A SET OF CONTINUOUS FEASIBLE ALTERNATIVES IN NEGOTIATION

The use of an Analytic Hierarchy Approach (AHP) for scoring offers in continuous negotiation problems has been studied. AHP has already proven its usefulness in constructing a ranking of alternatives in discrete decision making problems. In negotiations, however, some issues may have a quantitative character and be defined by feasible ranges, which results in uncountably large sets of feasible offers. This is a problem to which AHP cannot be applied in its original form. Therefore we propose an approach to building a scoring system that operates within AHP and a predefined discrete subset of feasible alternatives, then a method for determining global scores for all the feasible alternatives is proposed. When this subset has been built, the notion of border alternatives is applied. Assuming that these border alternatives have been ranked, single-issue utility functions are constructed using linear interpolation over the set of selected border alternatives. Single-issue utility functions are then aggregated using issue weights in order to form the final utility function. The issue weights are also determined using AHP. Such an approach means that a relatively small number of comparisons are required for a negotiator in AHP process to build a comprehensive scoring system, which makes the process of eliciting the negotiator's preferences simple and rapid.

Keywords: *negotiation analysis, evaluation of negotiation template, negotiation offer scoring system, AHP, Rembrandt*

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1. Introduction

Negotiation is a way of solving conflict situations that may occur in everyday life. Frequently, the negotiation process is perceived as an iterative process of exchanging offers and messages until a solution satisfying the preferences of all the interested parties is found [25]. In the case of such a protocol, it is desirable to operate with an evaluation function (scoring system), which can be used both to evaluate potential offers and track the negotiation progress, and is a great aid to the negotiation process. For this reason, it is frequently implemented in various Negotiation Support Systems (NSS) and Electronic Negotiation Systems (ENS) [9, 23, 24] which require the definition of such a scoring system in the pre-negotiation phase. The process of evaluating an offer results in the computation of an overall score for an alternative. Most ENS and NSS, such as INSPIRE [9], Negoisst [22], and SmartSettle [23] employ Multi-Attribute Value Theory (MAVT) [8] to form a scoring system, namely the negotiator is required to specify weights for each issue subject to negotiation, and the score of an alternative is computed in the form of a weighted sum of all the issue scores. Other approaches try to overcome the limitations of additive score functions [3] by proposing non-additive scoring systems.

Several multi-criteria decision-making methods are available, and the MAVT-based method is only one of the possible ways of specifying and eliciting preferences. Since negotiators may have different types of intuition and experience with decision-making approaches, it is worth evaluating other decision-making approaches to the analysis of preferences in the negotiation problem. Methods such as ELECTRE [15, 16], PROMETHE [1, 2] or the Analytic Hierarchy Process (AHP) [17–22] are approaches that usually enable ranking a set of predefined alternatives. However, it is worth looking at these methods from another viewpoint and check what aspects of these methods can be employed for the analysis of preferences in negotiation problems. In this paper, we focus on AHP, since it is perceived as a user friendly, systematic approach to decision making, that is based on evaluation instead of the simple assignment of scores, operates with an intuitive linguistic scale and reduces the complexity of the problem by decomposing it into its atomic elements, which are easy to compare [5, 17].

A typical application of AHP involves a series of comparisons of the issues considered and predefined alternatives, in order to derive a ranking of these alternatives. The basic strength of AHP, motivating its use, is the specific activity of pairwise comparison, which is intuitive to many decision-makers and enables avoiding direct assignment of a score to an alternative. In the application context considered, the number of issues is finite, meaning that the AHP is used in a standard way when comparing issues. However, in the case of an infinite number of alternatives resulting from the set of feasible alternatives forming a continuous space, the application of

AHP is limited if we consider the standard use of such a method. Since the basic activity of AHP is the pairwise comparison of alternatives, the number of such comparisons is strongly limited by the abilities of decision-makers, as well as the time needed to analyze preferences. Although, AHP can be used to rank larger sets of alternatives by splitting the set into consecutive clusters and ranking alternatives within clusters [5], we aim to avoid a large number of comparisons. The procedure based on clustering (involving many comparisons) makes the process of analysis troublesome and time-consuming for a decision-maker. In order to avoid a large number of comparisons, we propose to use AHP in its classical form for selected marginal alternatives, whose number may be initially small, without excluding the possibility of modifying preferences by adding new alternatives in later negotiation stages. This approach is complemented by the idea of interpolation to form continuous marginal scoring functions (in the case of qualitative issues), which are then aggregated into an overall scoring function. On the other hand, the procedure of preference modification enables increasing the accuracy of the elicited scoring function at any later stage by changing the function only locally (in a close neighborhood of the new reference alternative).

Another approach worth considering in further work is the extension of the comparison matrix into a continuous function that would constitute the kernel of the homogenous Fredholm equation. The numeric solution of such an equation would also lead to a continuous marginal scoring function. However, such an approach is not transparent to a decision-maker and hard to understand. Moreover, the computation of the function is complicated and more time consuming.

When considering the negotiation problem, there are some applications of AHP in this domain. Chen and Huang [4] proposed supporting the negotiation process with the AHP method. However, AHP was applied to select the optimal service provider before the actual negotiation phase. Osuna and Coello [12] propose to use AHP for resolving conflict situations. However, their approach uses the AHP method in the ex-post analysis of some small number of proposals. Kim [7] proposed using AHP to select a negotiation package from some small set of packages, before proposing it to the counterpart. Although these applications of the AHP method support the negotiation process, they do not involve the formation of a full scoring system over the set of feasible alternatives. Roszkowska, Brzostowski, Wachowicz [14] used AHP together with fuzzy TOPSIS to build a scoring system in ill-structured negotiation problems.

In this paper, we propose applying some of the AHP concepts to form a full scoring system in a continuous negotiation problem with an uncountably large set of alternatives. In section 2, we describe the fundamentals of AHP and then in section 3, we analyze the possibilities of using AHP in the context of negotiation. In section 4, our novel procedure for building a scoring system for negotiation offers is presented in detail. Finally, in section 5, we show a numerical example of using our approach in a simple negotiation problem.

2. The AHP methodology

AHP is one of the multi-criteria decision-making methods enabling decomposition of a complex decision problem and forming a ranking of the alternatives considered. The AHP method was proposed by Saaty in 1980 [17] (Fig. 1) and it is used in various fields such as: management, sociology and transportation [6, 11, 21, 22, 26]. The decomposition of the decision problem starts with the pairwise comparison of criteria, in order to derive their priority levels. In the next stage of analysis, the alternatives considered are compared with respect to each criterion in order to derive single-criterion scores. In the final stage, the criteria priorities and single-criteria scores are aggregated to form the final scores of all the alternatives. Based on the values of these scores, a ranking of alternatives is formed.

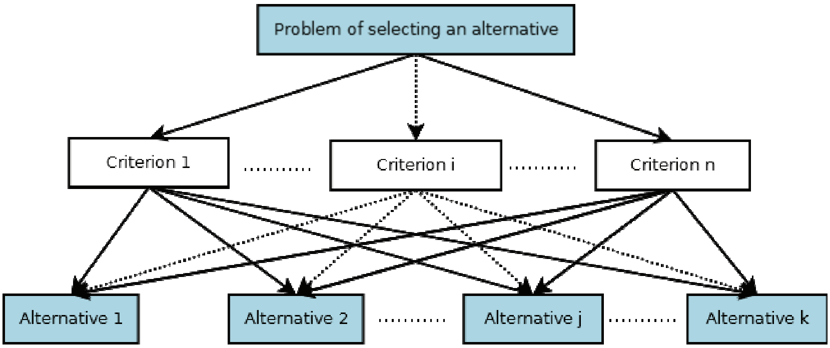

Fig. 1. Illustration of the AHP decision-making approach. Source: Saaty [17]

Our problem differs from classical decision problems to which AHP is usually applied, namely we aim to form a scoring system for a continuous set of feasible alternatives that can be later used in the negotiation process to evaluate alternatives Fig. 2). In such a situation, we do not have all the alternatives at our disposal for comparison, but instead we have sets of options for each issue to be compared with respect to this criterion/issue. In other words, for a selected criterion/issue the decision-maker specifies a range of options that is further discretized. A small number of options is selected from this range that is then used for pairwise comparison, in order to derive their single-issue scores. In the next step, continuous single-issue score functions are formed based on the values of the scores derived for the options selected. Therefore, such an application of the AHP approach differs from its classical applications, since we do not consider a predefined set of alternatives to be ranked, but aim to form a full scoring system for a continuous set of alternatives.

One cycle of AHP applied to a selected set of options or a set of criteria/issues is based on the pairwise comparisons of the elements considered. The decision-maker is required to perform the comparisons using a ratio scale, where the ratios indicate how many times one element is better or worse in terms of preferences. Alternatively, he may operate with the corresponding verbal scale proposed by Saaty [17] (see Table 1).

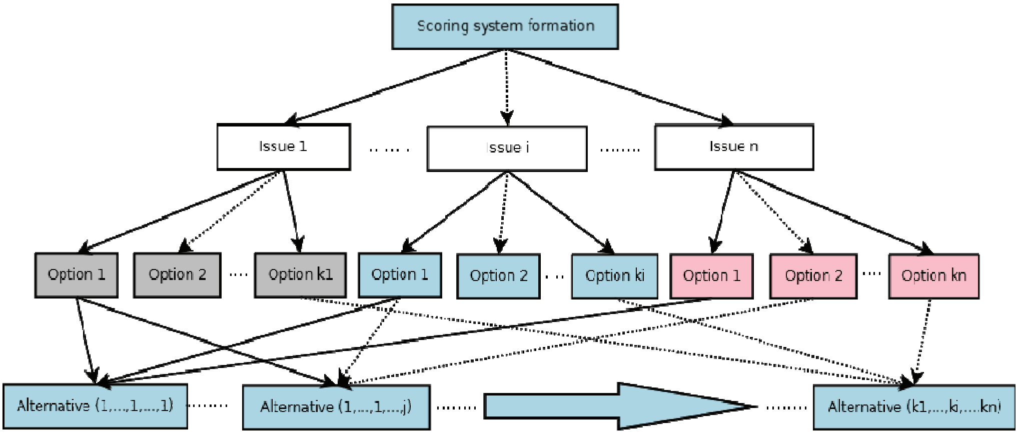

Fig. 2. Illustration of the AHP approach for defining a scoring system

Table 1. Judgement scale for relative importance in pairwise comparisons

Intensity of importance	Definition	Explanation
1	equal importance	two activities contribute equally to the objective
3	moderate importance	experience and judgment slightly favour one activity over another.
5	strong importance	experience and judgment strongly favour one activity over another
7	very strong or demonstrated importance	an activity is favoured very strongly over another, its dominance demonstrated in practice
9	extreme importance	the evidence favouring one activity over another is of the highest possible order of affirmation
2, 4, 6	for interpolation between the above values intermediary values	sometimes one needs to interpolate a compromise judgment numerically because there is no good word to describe it

Source: Saaty [17].

3. Possible ways of employing AHP to define a scoring system

As noted in the previous section, the AHP is suitable for defining a scoring system for a predefined set of alternatives. In order to apply the AHP approach, the set of alternatives has to be limited, since it requires comparisons of each pair of alternatives from the perspective of each issue considered and requires comparisons of each pair of issues. For a large number of alternatives (n), at least $(n^2 - n)/2$ comparisons are required.

Let us also observe that during the negotiation process we should be able to add new alternatives to the predefined set or remove some alternatives from this set. In such a situation, a decision maker can, of course, build a new scoring system based on the AHP method. However, if the scoring system is recomputed from scratch the process will not only be time consuming, but may also lead to rank reversal. Therefore, it is desirable to maintain the stability of the scoring system without excluding the possibility of adding new reference alternatives.

If we have to cope with issues modelled by a continuous set of options or we allow modification of the predefined finite set of alternatives, we have to select a discrete set of options for each issue in order to apply the AHP. After selecting the discrete options for each issue, the set of discrete alternatives "covering" the whole set of alternatives is determined as the Cartesian product of the discrete options corresponding to each issue. If the application of AHP to computing the score for all discrete alternatives is possible, in the next step a tool is needed to complete the scoring system by defining the values of the scores for all the remaining alternatives in the continuous space of alternatives. Therefore, we face two problems, namely the problem of applying the AHP approach in the case of a large set of discrete alternatives chosen from a continuous set of feasible alternatives, and the problem of completing the scoring system in terms of determining the values of the scores for all the other alternatives not belonging to the discrete reference set.

Let us consider possible ways of applying the AHP method assuming that the discrete set of alternatives has been selected out of the continuous space of all feasible alternatives.

3.1. Application of AHP once for all possible alternatives from the discrete set representing alternatives

Such a procedure requires performing a large number of comparisons. In such a case, if the number of options of the i-th issue is equal to k_i, then the number of all alternatives to be considered is equal to the product of the numbers of options for each issue $k_1 k_2 \cdots k_r$ (assuming r issues). In the case of r issues, the decision-maker has to perform $\frac{(k_1 k_2 \cdots k_r)^2 - k_1 k_2 \cdots k_r}{2}$ comparisons with respect to each issue. Therefore, such a procedure may be infeasible because of a very large number of comparisons.

Moreover, according to Saaty [17, 18] AHP is appropriate when the number of compared elements is not higher than nine. One of the methods of reducing the number of comparisons is to cluster the set of alternatives and perform the comparisons for each cluster. However, such a procedure requires splitting the set of alternatives into consecutive clusters, which assumes the ability of the decision-maker to form series of clusters.

3.2. Application of AHP for options of each issue separately

Application of AHP for options of each issue separately means that the AHP approach is applied the same number of times as the number of issues. For each issue, the AHP is applied to compare single-issue options to form single-issue scores for each option. Such a procedure is quite simple and for r issues requires application of the AHP r times to single issues. The problem with such an approach is that it assumes the additive structure of preferences. Such an assumption holds in cases where the criteria/issues are preferentially independent of each other [8]. This assumption has to be kept in mind while defining the scoring system. The assumption of preferential independence simplifies the process of analysis, since single-issue value functions can be formed separately for each criterion/issue.

Such a procedure of constructing a scoring system is depicted in Fig. 3 as part of the whole negotiation process. The process of structuring the negotiation problem (Step 1) involves determination of issues by both negotiation parties. In the next stage, both parties analyse their preferences starting with the pairwise comparison of issues (Step 2). Next, the borders of each issue range are elicited (reservation value and aspiration level). Each party sets its agenda for eliciting the value functions for single-issues.

The value functions for single issues are elicited by specifying the single-issue scores for selected resolution levels of an issue (Steps 3 and 4). Based on this partial knowledge of the value function for a single-issue, the scores for all issue resolution levels are computed in order to form a continuous value function for the issue considered (Step 5). After all the value functions for single-issues have been elicited, the full scoring system is determined by aggregating the value functions for single-issues using weights describing the importance of issues (Step 6). After defining the scoring system, the parties can proceed to the intention phase which involves the iterated exchange of offers and counter-offers (Steps 7–11), which ends if the parties either find an offer which is mutually satisfactory (Step 12) or they withdraw from negotiations (Step 13) In this phase, a partner may reject an incoming offer (Step 10) after its evaluation (Step 9), meaning that in the next step he/she will propose the counter-offer (Step 7).

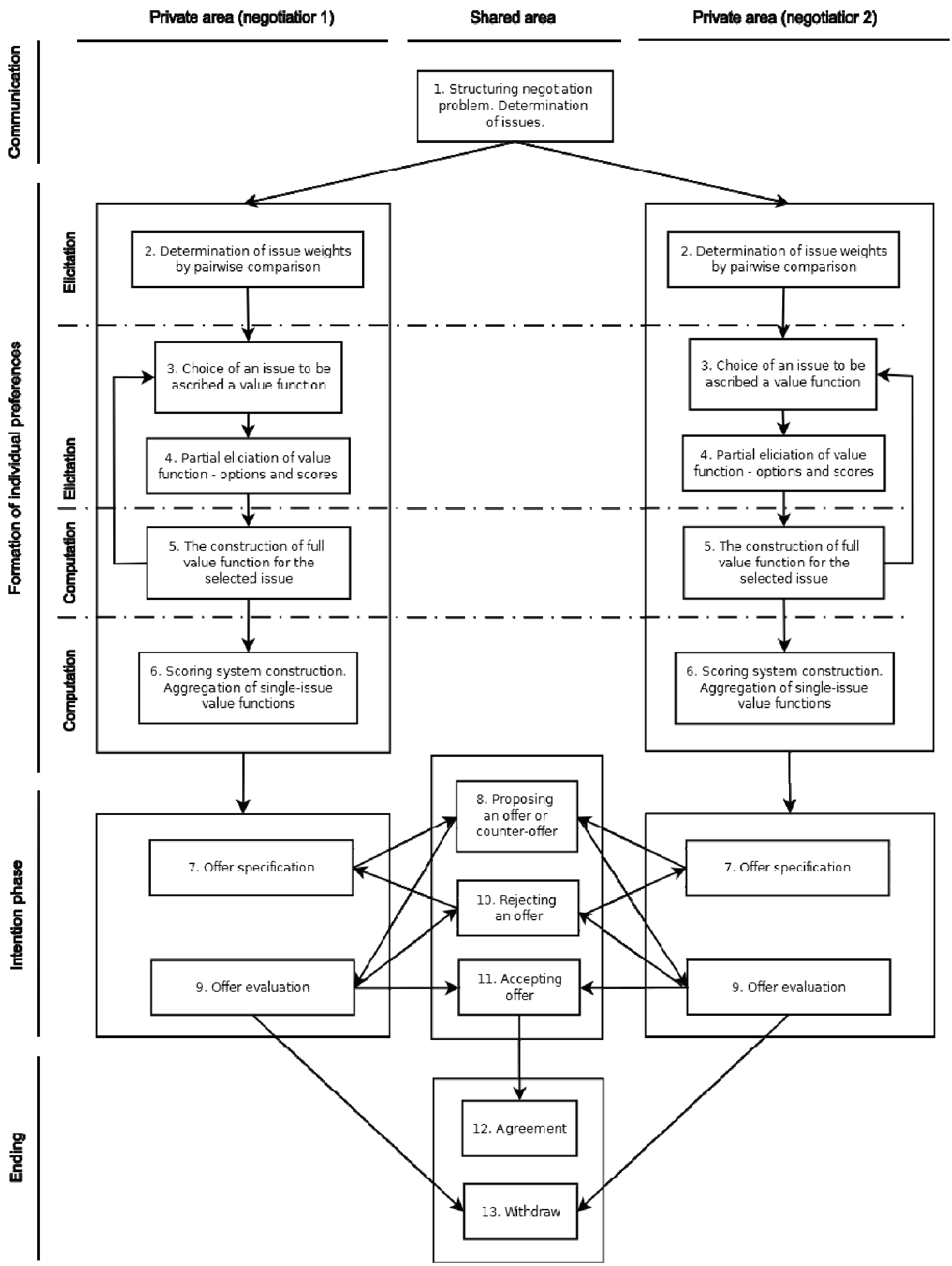

Fig. 3. A description of the negotiation process employing
the proposed method of preference elicitation

4. The definition of a scoring system

A typical application of AHP results in determining the score of a finite set of alternatives. Moreover, such a set of alternatives cannot be too large since each pair of alternatives has to be compared in order to derive the value on a ratio scale. The number of such alternatives should be limited to 9 as Saaty suggests [17] meaning that when there are many criteria it is impossible to span the value function over a continuous set of feasible alternatives. On the other hand, the formation of unidimensional value functions for each of the considered criteria is a relatively easy task that enables later aggregating such functions to form the overall value function. However, such a procedure assumes that the value function has an additive form, which is only applicable when all the criteria are mutually preferentially independent.

In the procedures proposed, the decision-maker will operate on a set of options O_i selected for the i-th issue/criterion from the range of feasible options $[a_i, b_i]$ (determined in the pre-negotiation phase, e.g. the number of issues is finite and usually limited to no more than several issues) of the following form:

$$O_i = \{o_1^i, o_2^i, \ldots, o_{k_i}^i\} \in [a_i, b_i] \tag{1}$$

In order to determine the single-criterion value function, the decision-maker (negotiator) may use one of the following procedures:

4.1. Bisection of the scoring range [8]

Using this procedure, the decision-maker is asked to specify the certainty equivalent of a simple gamble. In the case of five options, the decision-maker is first asked to specify a midpoint option o_3^i which is the certainty equivalent of a lottery in which both the worst option and best option occur with the probability of 0.5. The next two questions enable splitting the score ranges resulting from the first split, namely:

1. Which option is the certainty equivalent of a lottery in which both the worst option and the option o_3^i occur with the probability of 0.5?

2. Which option is the certainty equivalent of a lottery in which both the best option and the option o_3^i occur with the probability of 0.5?

Such a procedure can be treated as the comparison of five options in terms of differences on the scoring range. The comparison matrix describing the differences between these options in terms of their scores is of the following form:

$$\begin{bmatrix} 0 & s_{21} & s_{31} & s_{41} & s_{51} \\ s_{12} & 0 & s_{32} & s_{42} & s_{52} \\ s_{13} & s_{23} & 0 & s_{43} & s_{53} \\ s_{14} & s_{24} & s_{34} & 0 & s_{54} \\ s_{15} & s_{25} & s_{35} & s_{45} & 0 \end{bmatrix} = \begin{bmatrix} 0 & f_2^i - f_1^i & f_3^i - f_1^i & f_4^i - f_1^i & f_5^i - f_1^i \\ f_1^i - f_2^i & 0 & f_3^i - f_2^i & f_4^i - f_2^i & f_5^i - f_1^i \\ f_1^i - f_3^i & f_2^i - f_3^i & 0 & f_4^i - f_3^i & f_5^i - f_3^i \\ f_1^i - f_4^i & f_2^i - f_4^i & f_3^i - f_4^i & 0 & f_5^i - f_4^i \\ f_1^i - f_5^i & f_2^i - f_5^i & f_3^i - f_5^i & f_4^i - f_5^i & 0 \end{bmatrix} \quad (2)$$

where: $f^i(o_j^i) = f_j^i$ for $j = 1, 2, 3, 4, 5$. In the specific case of Raiffa's procedure (bisection of the scoring range), the comparison matrix takes the following form:

$$\begin{bmatrix} 0 & 0.25 & 0.5 & 0.75 & 1 \\ -0.25 & 0 & 0.25 & 0.5 & 0.75 \\ -0.5 & -0.25 & 0 & 0.25 & 0.5 \\ -0.75 & -0.5 & -0.25 & 0 & 0.25 \\ -1 & -0.75 & -0.5 & -0.25 & 0 \end{bmatrix} \quad (3)$$

However, the bisection procedure can be generalized to consider various values of differences between the option's scores. For instance, the following comparison matrix is also valid:

$$\begin{bmatrix} 0 & 0.2 & 0.4 & 0.6 & 1 \\ -0.2 & 0 & 0.2 & 0.4 & 0.8 \\ -0.4 & -0.2 & 0 & 0.2 & 0.6 \\ -0.6 & -0.4 & -0.2 & 0 & 0.4 \\ -1 & -0.8 & -0.6 & -0.4 & 0 \end{bmatrix} \quad (4)$$

Such a procedure is related to the AHP, since the decision-maker is required to select options whose scores differ by a specific amount. The vector of scores assigned to consecutive options is of the following form:

$$[f_1^i, f_2^i, f_3^i, f_4^i, f_5^i] = [0, 0.4, 0.6, 0.8, 1.0] \quad (5)$$

4.2. The standard application of AHP

The decision-maker selects a finite set of options from the set of options ranging from the reservation value up to the aspiration level (including the reservation value $o_1^i \in O_i$ and aspiration level $o_{k_i}^i \in O_i$, where k_i is the number of selected options for the i-th criterion) that can be easily compared according to the criterion for which the value function is constructed. Pairwise comparisons of options (on a ratio scale) result in the comparison matrix, for which the eigenvector, $f_1^i, f_2^i, f_3^i, f_4^i, f_5^i \in [0, 1]$ constituting the criterion scores for each of the selected options, is calculated. If we assume five linearly ordered options: $o_1^i, o_2^i, o_3^i, o_4^i, o_5^i \in O_i$, the comparison matrix takes the following form:

$$\begin{bmatrix} 1 & a_{21} & a_{31} & a_{41} & 9 \\ \frac{1}{a_{21}} & 1 & a_{32} & a_{42} & a_{52} \\ \frac{1}{a_{31}} & \frac{1}{a_{32}} & 1 & a_{43} & a_{53} \\ \frac{1}{a_{41}} & \frac{1}{a_{42}} & \frac{1}{a_{43}} & 1 & a_{54} \\ \frac{1}{9} & \frac{1}{a_{52}} & \frac{1}{a_{53}} & \frac{1}{a_{54}} & 1 \end{bmatrix} \tag{6}$$

From the assumption of a linear ordering of the options, the comparison values should satisfy the condition: $\forall p \mid l < p < j \Rightarrow o_l \prec o_p \prec o_j \Rightarrow a_{jl} > a_{pl} \wedge a_{jl} > a_{pj}$. Moreover, the best option is assumed to be 9 times better than the worst option.

4.3. The procedure based on the REMBRANDT scale [10]

The decision maker follows the procedure of iterative division (bisection) of the range of feasible options. If we assume that five linearly ordered reference options are determined ($o_1^i, o_2^i, o_3^i, o_4^i, o_5^i \in O_i$), the decision-maker starts by answering the question: Which option is (definitely) more desirable (four times better) than the reservation value and (definitely) less desirable (four times worse) than the aspiration level. This gives us the midpoint $o_3^i \in O_i$ in the range of options of the i-th criterion resulting from the first division of the range. In the next step, the decision-maker answers questions enabling the sub-division of the two ranges obtained in the first step:

1. Which option is somewhat more desirable (two times better) than o_1^i (the reservation value) and somewhat less desirable (two times worse) than o_3^i (the midpoint)?

2. Which option is somewhat more desirable (two times better) than o_3^i (the midpoint) and somewhat less desirable (two times worse) than o_5^i (the aspiration level)?

In this case, only some options are pairwise compared using the imposed ratios, which enables partial filling of the comparison matrix [10]. The remaining comparison ratios are computed by assuming the perfect consistency of preferences. However, in this case, the eigenvector is not computed. Instead, the scores form a geometric sequence (starting with a small power of 2,

$$2^{-k_i} = \frac{1}{2^{k_i}} = \left(\frac{1}{32}\right) = 0.0625$$

which is close to 0, where k_i is the number of options for the i-th issue) with ratio 2, which is the comparison ratio for each pair of neighbouring options. The single-criterion scores are determined by reversing the AHP process, so the resulting comparison matrix has the following form:

$$\begin{bmatrix} 2^0 & 2^1 & 2^2 & 2^3 & 2^4 \\ 2^{-1} & 2^0 & 2^1 & 2^2 & 2^3 \\ 2^{-2} & 2^{-1} & 2^0 & 2^1 & 2^2 \\ 2^{-3} & 2^{-2} & 2^{-1} & 2^0 & 2^1 \\ 2^{-4} & 2^{-3} & 2^{-2} & 2^{-1} & 2^0 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 4 & 8 & 16 \\ 0.5 & 1 & 2 & 4 & 8 \\ 0.25 & 0.5 & 1 & 2 & 4 \\ 0.125 & 0.25 & 0.5 & 1 & 2 \\ 0.0625 & 0.125 & 0.25 & 0.5 & 1 \end{bmatrix} \quad (7)$$

which is consistent with the questions posed to the decision-maker, since the following conditions are satisfied for each pair of consecutive options:

$$a_{ii} = 1 \forall i \in \{1, ..., k_i\} \quad a_{(i+1)i} = 2 \; \forall i \in \{1, ..., k_i - 1\} \quad (8)$$

$$(m, p \in \{1, ..., k_i\} \wedge p < m) \Rightarrow \forall q \in \{p, ..., m\} \; a_{mp} = a_{mq} a_{qp} \quad (9)$$

where the values of i, p, m and q are the indices of the array elements.

The first condition means that all the ratios located on the superdiagonal (immediately above the leading diagonal) are equal to 2. The second condition assumes the perfect consistency of the ratio between neighbouring scores enabling the computation of all the ratios above the superdiagonal of the comparison matrix.

Moreover, the five linearly ordered options satisfying these comparison ratios are selected by the decision-maker. The vector of scores consistent with the comparison matrix can be determined using:

$$f_m^i = \frac{1}{2^{k_i}} = \frac{1}{32}, \qquad m = 1 \tag{10}$$

$$f_m^i = f_{m-1}^i \times 2, \qquad \forall m \in \{2, ..., k_i\} \tag{11}$$

which gives the following scores for each option:

$$\forall j \in \{1, ..., k_i\}\ f_j^i = f(o_j^i) = \frac{1}{2^{k_i}} \times 2^j = 2^{j-k_i} \tag{12}$$

In the case of the five elicited options, the precise values of the scores are computed as follows:

$$\left[f_1^i, f_2^i, f_3^i, f_4^i, f_5^i\right] = \left[\frac{1}{32}, \frac{1}{16}, \frac{1}{4}, \frac{1}{2}, 1\right] = [0.0625, 0.125, 0.25, 0.5, 1] \tag{13}$$

As in standard AHP, there is pairwise comparison of options associated with the comparison matrix describing the ratio between the scores of different options. The main difference is the verbal naming convention, meaning that the ratio levels are described using various verbal etiquettes.

The single-issue scores obtained for the selected options of each issue are the basis for constructing the single-issue value functions. Assuming that the vector of options and the corresponding vector of single-issue scores for the *i*-th issue are of the following form:

$$[o_1^i,\ o_2^i, ..., o_{k_i}^i]\,[f_1^i, f_2^i, ..., f_{k_i}^i] \tag{14}$$

we have the following knowledge regarding the value functions:

$$\forall i \in \{1, ..., n\} f_i : o_i^{m_i} \mapsto f_i^{m_i}, \qquad m_i \in \{1, ..., k_i\} \tag{15}$$

Additional knowledge on the value functions can be given in the form of the decision-makers' attitude to risk. Pratt [13] introduced the following measure of risk aversion:

$$A(o^i) = -\frac{f_i''(o^i)}{f_i'(o^i)} \tag{16}$$

Knowing some features of the decision-makers' attitude to risk, additional constraints can be imposed on the shape of the value function, in order to construct this function based on the elicited partial knowledge. However, at the current stage of our research, we do not impose any additional constraints on the shape of the function – simple linear interpolation is used to form a continuous single-issue value function over the range of feasible options.

An extension of the value function $f_i : o^i \mapsto [0, 1]$ of the following form is obtained:

$$o^i \in [o^i_{m_k}, o^i_{m_{k+1}}] \Rightarrow f_i(o^i) = f^i_{m_k} + \frac{f^i_{m_{k+1}} - f^i_{m_k}}{o^i_{m_{k+1}} - o^i_{m_k}}(o^i - o^i_{m_k}) \tag{17}$$

Figure 4 illustrates the interpolation procedure. The possible error of the resulting estimate of the continuous function can be computed by taking the distance between the actual (unknown) function v and the estimated function f_i:

$$d(v_i, f_i) = \int_{o_i^m}^{o_i^{m+1}} |v_i(o^i) - f_i(o^i)| do^i \le P_m \tag{18}$$

which is bounded by the area of the triangle P_m determined by the points:

$$(o^i_m, f^i_m),\ (o^i_{m+1}, f^i_{m+1}),\ (o^i_{m+1}, f^i_m) \tag{19}$$

The values $P_m, m \in \{1, ..., k_i\}$ can be easily computed and treated as indicators of possible improvement in the quality of estimation. The interval with the largest value of P_m can be further split (continuous elicitation) into two intervals: $[o^i_m, o^i_{m+1}] = [o^i_m, o^i_p] \cup [o^i_p, o^i_{m+1}]$ according to one of the three proposed procedures.

1. In the case of bisection of the scoring range, the following question is posed to the decision-maker: Which option is certainty equivalent to a lottery where both o^i_m and o^i_{m+1} occur with probability 0.5?

2. In the case of a procedure based on the Rembrandt scale, the following question is posed to the decision-maker: Which option is somewhat more desirable ($\sqrt{2}$ times better numerically) than o^1_m and somewhat less desirable than o^i_{m+1}?

In the case of the procedure based on classical AHP, the decision-maker selects an option o^i_p and performs pairwise comparison of the three options: o^i_m, o^i_p, o^i_{m+1}.

After the new option is added to the set of options, in the case of the first two procedures the following condition will be satisfied:

$$f_i(o_m^i) \le f_i(o_p^i) \le f_i(o_{m+1}^i) \tag{20}$$

meaning that the single-issue score of the new option is consistent with the linear order of all options and the previous scores of neighbouring options do not change during the adjustment procedure.

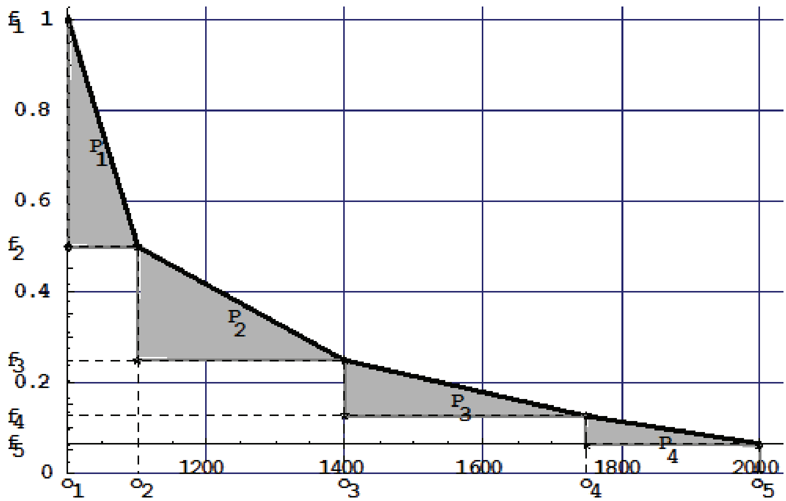

Fig. 4. Five options corresponding to one issue were assigned single-issue scores. The continuous utility function is formed by linear interpolation

However, when the third procedure is used (standard AHP), the scores $h(o_m^i), h(o_p^i), h(o_{m+1}^i)$ obtained for options: o_m^i, o_p^i, o_{m+1}^i will differ from the scores obtained in the previous phase of elicitation. Therefore, we propose to transform these scores linearly according to the proportion formula:

$$\frac{h(o_{m+1}^i) - h(o_p^i)}{h(o_{m+1}^i) - h(o_m^i)} = \frac{f_i(o_{m+1}^i) - f_i(o_p^i)}{f_i(o_{m+1}^i) - f_i(o_m^i)} \tag{21}$$

As a result of this transformation, we obtain the score for the new option:

$$f_i(o_p^i) = f_i(o_{m+1}^i) - \frac{\left(f_i(o_{m+1}^i) - f_i(o_m^i)\right)\left(h(o_{m+1}^i) - h(o_p^i)\right)}{\left(h(o_{m+1}^i) - h(o_m^i)\right)} \tag{22}$$

After the single-issue utility functions have been constructed, they can be aggregated to form the final scoring function using issue weights:

$$u(o_1, o_2, \ldots, o_n) = \sum_{i=1}^{n} w_i f_i(o_i) \tag{22}$$

In the next stage of analysis, the overall scores of some reference alternatives can be displayed to the decision-maker in order to allow tuning of parameters, such as issue weights. After preparing the scoring system, the parties can proceed to the intention phase in which an agreement is determined.

5. Numerical example

Let us assume that three issues are to be determined via buying/selling negotiations, namely the price, delivery and payment time with the following ranges (Step 1. see Fig. 3):

The buyer: price [€]: [1000, 2000] delivery time [days] [7, 28] payment time [1, 8].

The seller: price [€]: [1500, 2600] delivery time [days] [14, 42] payment time [1, 8].

The buyer compares the three issues and determines their weights according to standard AHP, (Step 2 in Fig. 3 performed according to Eq. (6)) as follows: $[w_1^b, w_2^b, w_3^b] = [0.7, 0.2, 0.1]$. The seller compares these issues and obtains the following weights: $[w_1^s, w_2^s, w_3^s] = [0.6, 0.2, 0.2]$. During the process of eliciting preferences, the decision makers use one of the procedures to determine the reference options and their single-issue scores according to the bisection procedure (next two Steps 3, 4 in Fig. 3 employing Eq. (2)). As a result, the buyer assigns the following scores to the selected price options:

$$\{(o_1^i, f_1^i)\ i \in \{1, \ldots, 5\}\} = \{(1000, 1), (1250, 0.8), (1500, 0.6), (1750, 0.4), (2000, 0)\}$$

to the delivery time options:

$$\{(o_2^i, f_2^i)\ i \in \{1, \ldots, 5\}\} = \{(7, 1), (10, 0.8), (16, 0.6), (22, 0.4), (28, 0)\}$$

and to the payment time:

$$\{(o_3^i, f_3^i)\ i \in \{1, \ldots, 4\}\} = \{(1, 0), (2, 0.25), (4, 0.45), (8, 1)\}$$

The seller's preferences are elicited and the following scores are assigned to the selected options with respect to the three criteria. In the case of price:

$$\{(o_1^i, f_1^i)\ i \in \{1, ..., 5\}\} = \{(2600, 1), (2000, 0.7), (1800, 0.6), (1600, 0.4), (1500, 0)\}$$

in the case of delivery time:

$$\{(o_2^i, f_2^i)\ i \in \{1, ..., 5\}\} = \{(42, 1), (36, 0.85), (24, 0.65), (20, 0.5), (14, 0)\}$$

in the case of payment time:

$$\{(o_3^i, f_3^i)\ i \in \{1, ..., 4\}\} = \{(1, 1), (2, 0.7), (6, 0.3), (8, 0)\}$$

Table 2 illustrates the scores of the packages proposed in consecutive negotiation rounds by the parties. These scores are computed by interpolating between the nearest options for each issue and aggregating the single-issue scores with the weights obtained in the first step of analyzing preferences.

Table 2. Negotiation history with the scores of consecutive proposals from both parties

Source of offer	Package	Price	Delivery time	Payment time	Buyer's score	Seller's score
B	PC_1^B	1000	7	8	1.0000	0.0000
S	PC_1^S	2600	42	1	0.0000	1.0000
B	PC_2^B	1500	16	6	0.6125	0.0630
S	PC_2^S	1800	22	4	0.3490	0.5850
B	PC_3^B	1600	20	4	0.5023	0.4300
S	PC_3^S	1650	20	4	0.4743	0.4650
B	PC_3^S	1650	20	4	Agreement	

Let us compute the buyer's score for package $PC_2^B = (1500, 16, 6)$. The price score is as follows: $w_1^b f_1(1500) = 0.7 \times 0.6 = 0.42$. For the delivery time, we have the following score: $w_2^b f_2(16) = 0.2 \times 0.6 = 0.12$. In the case of payment time, the score is located in the interval: $[w_3^b f_3(4), w_3^b f_3(8)] = [0.1 \times 0.45, 0.1 \times 1] = [0.045, 0.1]$. Assuming the score has a linear form in this range, the issue score can be computed by applying the interpolation: $(0.1 - 0.045)/(w_3^b f_3(6) - 0.045) = (8 - 4)/(6 - 4) = 2$ (Step 5 in Fig. 3 performed according to Eq. (21)). As a result, the payment time score for option 6 is obtained:

$w_3^b f_3(6) = 0.055 \cdot 0.5 + 0.045 = 0.0725$ (according to Eq. (22)). The overall package score from the buyer's perspective is then computed (Step 6 in Fig. 3 performed according to Eq. (23)): $w_1^b f_1(1500) + w_2^b f_2(16) + w_3^b f_3(6) = 0.42 + 0.12 + 0.0725 = 0.6125$.

The next phase is the intention phase (Steps 7–12, see Fig. 3) involving the exchange of proposals illustrated in Fig. 5.

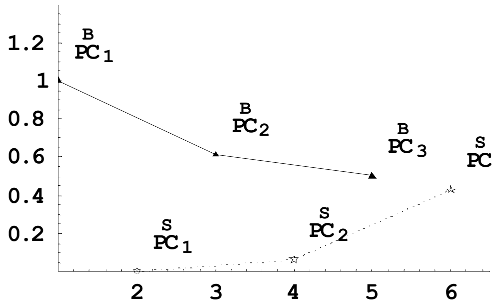

Fig. 5. The negotiation history (from the buyer's perspective)

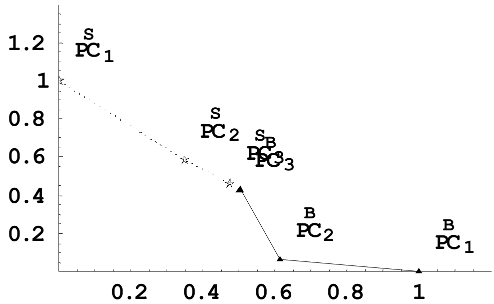

Fig. 6. The negotiation dance

Figure 5 illustrates the negotiation history from the buyer's perspective (the x-axis corresponds to the round number and the y-axis corresponds to the package's score computed according to the buyer's scoring system). The negotiation dance (Fig. 6) illustrates the negotiation process by taking into account the scores of both parties.

6. Conclusions

AHP is usually applied to constructing a ranking of predefined alternatives. In this paper we have proposed a completely different application of the AHP method, which enables the definition of a scoring system over a continuous set of feasible alternatives. Therefore, there was a need for a new scoring algorithm to be developed that uses some notions of the classical AHP approach, but also applies other formal mechanisms that enable a score to be ascribed to the offers not analyzed directly during the AHP analysis. We proposed to focus on border alternatives or options and to perform the AHP method for each criteria/issue separately. Using such an approach, the number of comparisons is reduced only to the most necessary comparisons. Then linear interpolation is applied to define the single-issue scoring functions, which are further aggregated to form the overall scoring function.

Among the strengths of the proposed approach are its simplicity, low computational effort required to construct the scoring system and its transparency. It preserves the intuitiveness of classical AHP and therefore should be easy to use for negotiators who are not skilled in decision theory. On the other hand, it is also very flexible, as it allows modification of the scoring function (by introducing new reference alternatives) during the intention phase, which affects the scoring function only locally (in the neighborhood of the alternative introduced), It may also be easily implemented in the form of a spreadsheet to create a software support tool without using any sophisticated technologies.

Regarding its limitations, the approach assumes the preferential independence of issues, which restricts the scoring function to having an additive form. Also, the quality of the scoring function depends on the number of reference alternatives and in the case of a low number of such alternatives, the approximation of the scoring function may be inaccurate. Moreover, the introduction of new reference alternatives during the intention phase affects the scoring function (although only locally in the space of feasible alternatives) which may result in a change in the overall score of previously evaluated offers.

In future research, an Excel add-in will be implemented, in order to enable the practical use of the approach. The tool will be tested by different types of users, in order to evaluate its strengths and weaknesses. Moreover, an alternative approach based on the continuous extension of the comparison matrix leading to a homogenous Fredholm equation will be considered.

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